that the decays will be isotropic.

[9]G. Snow, Nucl. Phys. B55, 445 (1973), and Ref. 3. It should also be pointed out that more recent theoretical estimates based on the charm model suggest that the branching ratio of charmed mesons into the $K\pi$ channel may be rather small. See M. B. Einhorn and C. Quigg, FNAL Report No. Fermilab-Pub-75/21-THY, February 1975 (unpublished).

Theorem for the Photon Asymmetry in Radiative Muon Capture*

Harold W. Fearing†
Nuclear Research Centre, Department of Physics, University of Alberta, Edmonton, Alberta, Canada
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I show, for the standard theory of radiative muon capture including all couplings except g_S and all usual diagrams, that the photon asymmetry α and circular polarization β satisfy $\alpha=+1+O(1/m^2)$ and $\beta=+1+O(1/m^2)$, where m is the nucleon mass. This may bear on current disagreement between α^{theor} and α^{expt} since $O(1/m^2)$ terms have never been calculated consistently and since other uncalculated corrections, some of which are discussed, may contribute in the same order.

In a recent Letter, DiLella, Hammerman, and Rosenstein[1] reported the results of a measurement of the photon asymmetry parameter α in the radiative capture of polarized muons, $\mu+{}^{40}\text{Ca}\to{}^{40}\text{K}+\nu+\gamma$. This important, previously unmeasured quantity depends strongly on the induced pseudoscalar coupling g_P and thus provides a sensitive way of measuring g_P and of elucidating the structure of weak interactions in nuclei. The quoted result, which however depends on the poorly known[2] neutron asymmetry in the related reaction $\mu+{}^{40}\text{Ca}\to{}^{39}\text{K}+\nu+n$, is $\alpha^{\text{expt}}\leqslant-0.32\pm0.48$ which is in serious disagreement with the prediction of the standard theory[3] $\alpha^{\text{theor}}=0.75$.

If the experimental result is correct, then there must be something seriously wrong with the theoretical prediction. The theory, however, seems to be a straightforward extension of standard muon-capture theory. Efforts toward improving it have been directed mainly towards improving the nuclear matrix elements,[3-5] which should not affect the asymmetry much since it involves a ratio of these matrix elements.

In the course of re-examining the theoretical situation, I have found a new general result which may indicate some ways in which the standard prediction for α may be wrong or incomplete. Specifically I show that $\alpha=+1+O(1/m^2)$, where m is the nucleon mass. Thus for $\alpha\equiv+1+\Delta\alpha$, the entire contribution to $\Delta\alpha$ comes from $O(1/m^2)$ terms which *a priori* one would expect to be small. This result is implicit in previous numerical calculations[3] which give $\Delta\alpha\simeq-0.25$. It apparently has never been noticed explicitly, although the fact that $\alpha=1$ for the muon radiating diagram with V and A couplings only is well known.[6]

The result is important because in the standard theory $O(1/m^2)$ terms have been presumed small and never calculated consistently. Thus in principle a large number of $O(1/m^2)$ terms contributing to $\Delta\alpha$ still need to be calculated. Furthermore, in view of the general result, $O(1/m^3)$ terms are the first-order corrections to $\Delta\alpha$ and so at least the largest should be examined.

Also very important is the fact that once one realizes that $\Delta\alpha$ is given in the standard theory by $O(1/m^2)$ terms it becomes much more worthwhile (and easier) to look for corrections and additional contributions to $\Delta\alpha$ which are of the same order. Until such additional terms have been examined and shown to be small, one must retain reservations about the correctness of the theoretical prediction which currently gives disagreement with experiment.

I now summarize the proof of the assertion above and discuss in somewhat more detail additional $O(1/m^2)$ terms which may contribute. My preliminary conclusion is that the most obvious of these extra terms will give contributions having $\alpha=\beta=+1$ or will be small. However such educated guesses are not a substitute for calculation. Hopefully the existence of my result and an enhanced appreciation of the importance of $O(1/m^2)$ terms will stimulate further thought about other possible corrections so that results of new experiments to be carried out at the Tri-University Meson Facility[7] can be compared with a theory in which we have complete confidence.

Let me first review the standard theory[3] so as to provide a basis for the proof of my general result. The Feynman amplitudes for the usual gauge-invariant set of diagrams (Fig. 1 of Ref. 4) are first reduced to two-component form. Terms of $O(1/m)$ in the result are kept, as well as some $O(1/m^2)$ terms, in particular those involving g_P which may be numerically large since g_P is large. This gives a two-component operator which is treated as a nonrelativistic, local, effective Hamiltonian H^{eff} to be taken between nuclear states.

The square of the resulting matrix element summed on appropriate spins and integrated on the neutrino direction is of the form $1+P_\mu\alpha\cos\theta$, where P_μ is the muon polarization, θ is the angle between $\vec{P}_\mu$ and the photon momentum $\vec{k}$, and α is the asymmetry parameter. Alternatively one can integrate on θ instead of summing on photon spin and define a photon circular-polarization parameter β. Doing both integration and summation gives the photon spectrum.

I now state the central result of this paper as the following theorem: In the context of standard theory[3] and in the absence of second-class induced scalar coupling g_S (but including the other five usual couplings g_V, g_A, g_M, g_P, and g_T) one has (i) $\alpha=+1+O(1/m^2)$, (ii) $\beta=+1+O(1/m^2)$, and, as a trivial consequence of (i) and (ii), (iii) $\alpha=\beta+O(1/m^2)$.

Only fragments of this theorem seem to have been previously noted. For example Manacher[8] determines, by enumerating a large number of terms, a formula for $\alpha-\beta$ which is $O(1/m^2)$, although he does not seem to observe this explicitly. Early workers[6] showed that for the muon radiating diagram only, with just g_V and g_A couplings, $\alpha=\beta=+1$, but the statement has been made that other couplings and diagrams destroy the result. Furthermore the importance of the $O(1/m^2)$ terms seems not to be known, or at least not fully appreciated as there are contemporary calculations[9] of the asymmetry which neglect even some $O(1/m)$ terms.

To prove the theorem I separate the amplitudes contributing to H^{eff} into two parts, $M=M_1+M_2$, where M_1 comes from the diagram [Fig. 1(a) of Ref. 4] where the muon radiates and M_2 comes from the remaining diagrams. We can write

$$M_1=-\tfrac{1}{2}T^\alpha\bar{u}_\nu\gamma_\alpha(1-\gamma_5)(p_\mu\cdot\gamma-k\cdot\gamma+m_\mu)\epsilon\cdot\gamma u_\mu/k\cdot p_\mu,$$

$$M_2=\epsilon_\beta R^{\alpha\beta}\bar{u}_\nu\gamma_\alpha(1-\gamma_5)u_\mu,$$

where p_μ, k, and ν are the respective four-momenta of the muon, photon, and neutrino; m_μ is the muon mass; and the Dirac notation is standard.[10] T^α and $R^{\alpha\beta}$ contain the nuclear matrix elements. One usually takes for the photon polarization vector ϵ the representation $\epsilon_0=0$, $\vec{\epsilon}=(\hat{i}-i\lambda\hat{j})/\sqrt{2}$ with $\lambda=\pm1$ so that $\lambda_+=\frac{1}{2}(1+\lambda)$ projects out circularly polarized photons corresponding to $\beta=+1$.

The square of the matrix element M summed on spins gives, after some simplification and partial two-component reduction, the result,

$$\sum_{\text{spins}}|M|^2\sim\operatorname{Tr}[(\nu\circ\gamma)\gamma_\alpha(1-\gamma_5)(p_\mu\cdot\gamma+m_\mu)\{\lambda_+\vec{\sigma}\cdot\vec{\epsilon}T^\alpha+m_\mu R^{\alpha\beta}\epsilon_\beta\}(1+\vec{\sigma}\cdot\vec{P}_\mu)\{\lambda_+\vec{\sigma}\cdot\vec{\epsilon}T^\delta+m_\mu R^{\delta\eta}\epsilon_\eta\}^+\gamma_\delta],$$

where $\vec{\sigma}$ is the usual Pauli matrix. Observe that both the $|M_1|^2$ and the $\operatorname{Re}M_1M_2^*$ pieces contain the expression $\lambda_+\vec{\sigma}\cdot\vec{\epsilon}(1+\vec{\sigma}\cdot\vec{P}_\mu)$ which can be reduced, using the explicit form for $\vec{\epsilon}$, to $\lambda_+(1+\hat{k}\cdot\vec{P}_\mu)\vec{\sigma}\cdot\vec{\epsilon}+\lambda_+\vec{\epsilon}\cdot\vec{P}_\mu(1-\vec{\sigma}\cdot\hat{k})$. The first of these terms has a factor $\lambda_+(1+\hat{k}\cdot\vec{P}_\mu)$ and thus has $\alpha=\beta=1$ to all orders in $1/m$ and for all six weak couplings. The second term vanishes for $|M_1|^2$ since $\lambda_+(1-\sigma\cdot\hat{k})\sigma\cdot\epsilon^*=0$. For $\operatorname{Re}M_1M_2^*$ it leads to a result, when combined with the other factors, of the general form $\vec{\epsilon}\cdot\vec{P}_\mu\vec{A}\cdot\epsilon^*$ where we must have $\vec{A}=a\hat{\nu}+b\hat{k}+c\hat{\nu}\times\hat{k}$ with a, b, and c functions of $|\vec{\nu}|$, $|\vec{k}|$, and $\hat{\nu}\cdot\hat{k}$. When integrated on neutrino angles this term vanishes also.

Consider finally the $|M_2|^2$ term. I show that it is $O(1/m^2)$. The simplest way to do this is explicitly to make the two-component reduction of M_2, keeping just the $O(1)$ terms. We find $M_2\sim(1-\vec{\sigma}\cdot\hat{\nu})\vec{\sigma}\cdot\vec{\epsilon}g_S+O(1/m)$. Thus if $g_S=0$, $\sum|M_2|^2$ is $O(1/m^2)$ and our theorem is proved. If $g_S\neq0$ we get a contribution to $\sum|M_2|^2$ proportional to $g_S^2(1+\lambda\hat{k}\cdot\vec{P}_\mu)$ which will contribute at $O(1)$ to both $\alpha=\pm1$ and $\beta=\pm1$.

The substance of the theorem I have proved is thus the following. The diagram in which the muon radiates and its interference with *all* of the other diagrams contributes only terms with $\alpha=\beta=1$ to *all* orders in $1/m$ and for all six couplings. The $|M_2|^2$ term however contributes terms having both positive and negative α and β. In the standard theory these terms are all $O(1/m^2)$ with the exception of terms involving g_S.

Consider now the important implications of this result. We ask which new terms if any could significantly change the value of α. In previous calculations, terms of $O(1/m^2)$ both in H^{eff} and in $|M|^2$ have usually been assumed to be small and treated rather cavalierly. We now know from the theorem that it is these $O(1/m^2)$ terms in $|M|^2$ which give the *entire* interesting contribution to α (and β). This contribution, $\Delta\alpha$, contains all of the dependence on coupling constants. Furthermore those $O(1/m^2)$ terms kept [the most important of which is the square of a g_P/m term in H^{eff} coming from Fig. 1(b) of Ref. 4] give $\Delta\alpha \sim -0.25$. This is not particularly small and one does not know *a priori* whether other terms of $O(1/m^2)$ which have been left out might be of comparable size and effect the theoretical prediction for α.

Thus we must examine the possible $O(1/m^2)$ terms in $|M|^2$. These arise either from the square of the $O(1/m)$ terms in H^{eff} or from the interference of the $O(1)$ and $O(1/m^2)$ terms. The former terms are known, but the latter are not since a consistent expansion of H^{eff} to $O(1/m^2)$ has never been carried out. Terms such as g_P/m^2 have usually been kept in H^{eff} but other $O(1/m^2)$ terms have been dropped. For example, in principle a term $\mu g_M/m^2$, where $\mu = 1 + \kappa_P - \kappa_n = 4.7$ involves the nucleon magnetic moments, may appear in H^{eff}. Since $\mu g_M \sim 17 g_V$ while g_P may reach $\sim 20 g_V$ this term may be comparable to the g_P/m^2 term kept.

The theorem, however, makes general statements about the importance of these terms. Recall that all contributions to $|M|^2$ coming from the $|M_1|^2 + 2\,\mathrm{Re} M_1 M_2{}^*$ piece contribute only terms having $\alpha = \beta = 1$ to *all* orders in $1/m^2$. Hence the only new $O(1/m^2)$ terms of interest in H^{eff} are those originating from M_2. If we then assume (a) standard theory and (b) $g_S = 0$, the leading term of M_2 is $O(1/m)$. Thus the new $O(1/m^2)$ pieces of H^{eff} contribute to $|M_2|^2$ and $\Delta\alpha$ only in order $O(1/m^3)$. In general one expects that such terms individually will be small, being of order $m_\mu/m \sim 10\%$ of the leading $(g_P/m)^2$ term. However in principle certain combinations might occur, e.g., $\mu m_\mu g_P{}^2/m^3$ which, while formally $O(1/m^3)$, are numerically significant. Also some terms might possibly end up with significant numerical coefficients. Thus these terms, which are now the first-order corrections to $\Delta\alpha$, analogous to the velocity terms of ordinary muon capture, really must be calculated even though our first guess is that they will probably be small.

It has been usual to take $g_S = 0$ which is a consequence of the conserved-vector-current theory if one neglects the neutron-proton mass difference. If $g_S \neq 0$, however, a very interesting situation arises since there then will be $g_S{}^2$ terms of $O(1)$ and possibly $g_S g_P$ terms of $O(1/m)$ which contribute to $\Delta\alpha$. Thus α and β may be quite sensitive to a nonzero value of g_S.

At the next level of sophistication one can attempt to include effects of Coulomb and nuclear potentials. A systematic way of doing this is to carry out a Foldy-Wouthuysen transformation[11] on the relativistic Hamiltonian. Only recently has such an approach been applied to nonelectromagnetic interactions; see, e.g., Friar.[12] For radiative muon capture it will give a large number of new terms proportional to the nuclear or Coulomb potentials. If all these terms are $O(1/m^2)$, however, one should be able to lump them with the $O(1/m^2)$ terms originating from M^2 so that they too will contribute only to the first-order correction to $\Delta\alpha$, which we recall is of $O(1/m^3)$.

To consistently include effects of exterior potentials one must also include them in the propagators of the intermediate particles. Rood, Yano, and Yano[13] have looked at some of these terms and find a change in α, due primarily to the propagation of the muon in the Coulomb potential of the nucleus, of only a few percent corresponding to a change in $\Delta\alpha$ of about 12%, roughly what one would expect if these are $O(1/m^3)$ corrections.

All of the terms mentioned above can be obtained from a complete Foldy-Wouthuysen reduction of the Hamiltonian with the consistent inclusion of exterior potentials. Such a calculation is well underway and the results will be reported elsewhere. It is clear, however, from the discussion, that probably the new terms one obtains will give contributions having $\alpha = \beta = 1$ or be of $O(1/m^3)$ and thus small.

Once $\Delta\alpha$ has been calculated consistently in the standard theory one must ask whether more exotic corrections might also contribute. As one of the purposes of this note is to stimulate thought on such corrections, we mention a few possibilities. One obvious approach is to include new diagrams which may add new $O(1/m)$ terms to H^{eff}. As the diagrams get more complicated, however, it becomes more difficult to include them in a relatively model-independent way and to extract the single quantity g_P in a convincing fashion. In a recent, very interesting calculation of this type, Ohta[14] includes diagrams containing a $\Delta(1236)$ resonance and obtains a rather large renormalization of g_P, which does improve the photon spectrum and presumedly changes α (cf., however, Beder[15]). Clearly other mesonic exchange contributions could be included, to the extent that they are not already included via an effective nuclear potential.

Another very interesting possibility results from the additional fundamental terms in the weak-interaction vertex which appear when the nucleons are off the mass shell. Such contributions should be small since they are proportional to the amount by which a nucleon is off shell. However for β decay some such terms,[16] when chosen in a natural way, reduce to nonrelativistic operators having one fewer powers of $1/m$ than some of the main terms. Thus some terms of this kind may be more important than one would *a priori* expect. A detailed study of such terms would clearly be a useful contribution.

The result of our theorem, namely that in the standard theory the entire interesting contribution to α is given by terms $O(1/m^2)$ which one *a priori* might have expected to be small, means that one should look rather carefully at a consistent calculation in the standard theory, which is being done, and at possible more exotic corrections, a few of which I have mentioned. I hope that the existence of this general result will stimulate further work on this problem.

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†Present address: TRIUMF, University of British Columbia, Vancouver, B.C., V6T 1W5, Canada.

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Saturated Two-Photon Resonance Ionization of He($2\,^1S$)*

G. S. Hurst, M. G. Payne, M. H. Nayfeh,† J. P. Judish, and E. B. Wagner
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 5 May 1975)

We have developed a photoionization method for complete conversion of a quantum-selected population to ionization, making possible sensitive and absolute measurement of the selected populations in a gas. Each photoionization involves the absorption of two photons (from a pulsed dye laser), one of which is resonant with an intermediate state. In this demonstration we measured the absolute number of He($2\,^1S$) states per ion pair following interaction of pulses of 2-MeV protons with He.

In noble-gas energy pathways research,[1] attempts are made to deduce the number of various excited species as a function of time after proton excitation. Photon-emission processes, when viewed over a range of gas pressure, are often so complex that unique kinetic models cannot be constructed even from time-resolved emission experiments.[1] In a search for more direct information, we conceived of a method in which each atom in a selected quantum state would be converted to an ion pair by the absorption of two photons, one of which is resonant with an intermediate state. Two-photon ionization processes are well known and have been used more recently