layer was assumed to form pairs, and an additional paired half layer to bond to the remaining free bonds of the complete paired layer. Only single bonds were considered. The same local bonding arrangement could also produce a 2×2 structure, but not a 2×1.

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Measurement of Equilibrium Critical Velocities for Vortex Formation in Superfluid Helium*

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We have made measurements of the critical velocity for vortex formation in right cylinders of circular, elliptical, and rectangular cross section. The procedure used seems to produce the superfluid in its equilibrium state and the results are in good quantitative agreement with theoretical calculations based on equilibrium thermodynamics.

Since the predictions of Onsager and Feynman it has been widely demonstrated that quantized vorticity can exist in superfluids. Quantized vortex lines provide striking evidence for the fundamentally quantum nature of a superfluid, and the creation of quantized vortices apparently is the process which limits "super" or dissipationless flow of superfluids to small velocities. For these reasons the study of critical velocities for vortex creation is of significant theoretical and experimental interest.[1] This paper presents reproducible measurements of the critical velocity which are in quantitative agreement with theoretical calculations made by Fetter.[2]

The critical velocity Ω is generally difficult to calculate because of complicated geometries and it is difficult to measure because of the irreproducible metastability which is characteristic of flowing He II.[3] The most rigorous calculations performed up to now are those which treat vortex lines in rotating cylinders.[4] These calculations, which are based on equilibrium thermodynamics, predict the minimum angular velocity, Ω_t, at which the free energy will be minimized by the presence of one vortex line. Recently calculations have included cylinders without rotational symmetry,[2] although no quantitative experimental verification has been available. The main disparity between the free-energy calculations and actual measurements of critical velocities is that in most experimental situations the flowing He II exists in a metastable state rather than the equilibrium state. This metastability exists because of large energy barriers between flow states which are themselves local minima in the free energy.

For example, consider a circularly cylindrical bucket containing one quantized vortex in the center, and rotating with an angular velocity at which the equilibrium state would have no vortices. If the system passes to the equilibrium state via simple translation of the line to the wall, it requires an energy of $\sim 10^9$ K! Thus, as the vessel slows down the line may persist to speeds well below Ω_t, the critical velocity predicted from the equilibrium theory.

We have found a process which seems to put a sample of rotating He II into a true equilibrium state thus enabling us to verify quantitatively some of the equilibrium predictions of critical velocities. The basic process has been used previously by others[5] and simply consists of initially rotating the liquid above the λ point until internal mechanical equilibrium exists and then subsequently cooling the steadily rotating sample to below T_λ.

There are several reasons for expecting this process to yield the equilibrium state. The most likely explanation is that the energy barriers become vanishingly small near T_λ because they are proportional to the superfluid density ρ_s. If transitions between different flow states are thermally activated or are caused by random apparatus vibrations, equilibrium should be reached rapidly.

The apparatus used in our experiments must provide smooth, accurate rotation and it must be capable of detecting single vortex lines. To do this we have our experimental cell suspended at the bottom of a tube which rotates in two pairs of precision bearings—one at room temperature and one in the helium bath. The shaft passes through a rotating vacuum seal (*O* ring), and it is driven by a variable-speed motor through a positive-drive belt and pulley system. Although the motor speed can be regulated to better than 0.2%, the oscillations due to elasticity in the drive system and friction in the vacuum seal limit short-term regulation to about 2%.

The presence of a single vortex line is detected using the method of ion trapping.[3,6] The experimental cell, which is very similar to that described in Ref. 3, is sealed in a can so that the liquid level can be set independent of the bath level. The cylinder being studied is mounted in a holder which keeps it accurately coaxial with the axis of rotation. All the cylinders studied were constructed from methyl methacrylate, and the walls are coated with carbon resistive paint. This paint improves the uniformity of the electric fields used to manipulate ions and it prevents the buildup of surface charge on the cylinder walls.

The taking of data is a slow process. With the temperature above the λ point, the rotation speed of the cylinder is set, and the system is left undisturbed for 1–2 h—long enough to ensure that the normal, viscous He I is in mechanical equilibrium with its container. The bath temperature is

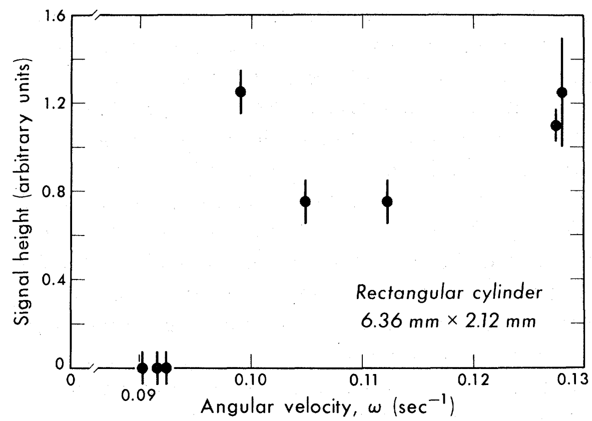

FIG. 1. Plot of signal height as a function of angular velocity for a rectangular cylinder. See text for explanation.

then slowly reduced, taking roughly 0.5 h to drop from ~2.8 to 1.2 K. This cooling process must be gradual because small heat flows or mechanical disturbances can sweep an otherwise stable vortex line to the wall of the cylinder, where it would be destroyed. Such events often have been observed. Once the operating temperature of 1.2 K is reached, the trapped-ion signal is measured for about 10^3 sec to detect the presence of any vortex in the cylinder. The system is then warmed above the λ point, the rotation speed is changed to a new value, and the whole process is repeated. After a series of such measurements the data are plotted as shown in Fig. 1; this is a plot of the data taken in a rectangular cylinder. In this plot of signal height as a function of ω,

TABLE I. Comparison of critical velocities predicted by equilibrium theory, Ω_t, and those found experimentally, Ω_e, in various cylinders. The length of all cylinders used is 3.05 cm.

Cylinder cross section	Ω_t (sec^{-1})	Ω_e (sec^{-1})
Circle, radius = 1.59 mm	0.103 ± 0.008	0.102 ± 0.0046
Ellipse,[a] $a = 1.19$ mm, $b = 0.40$ mm	0.855 ± 0.068	0.917 ± 0.036
Ellipse,[a] $a = 2.38$ mm, $b = 0.79$ mm	0.223 ± 0.018	0.204 ± 0.004
Ellipse,[a] $a = 3.18$ mm, $b = 1.06$ mm	0.128 ± 0.010	0.135 ± 0.0018
Rectangle, 6.36 mm × 2.12 mm	0.113 ± 0.009	0.0956 ± 0.0033

[a]The semimajor and semiminor axes are denoted a and b, respectively.

the signal height is roughly proportional to the length of vortex line present in the container. Therefore, we conclude that for $\omega \leqslant 9.23\times10^{-2}$ $\sec^{-1}$ there is no vortex. For $\omega \geqslant 9.90\times10^{-2}$ $\sec^{-1}$ there is a vortex in the cylinder, and we can say that the experimental critical velocity Ω_e is between the above velocities, or $\Omega = (9.56 \pm 0.33)\times10^{-2}$ $\sec^{-1}$. The error bars in the figure represent the amount of peak-to-peak noise observed in the signal over a period of ~30 min. The larger variations in signal heights between different points are probably due to variations in the gain of the low-temperature proportional counter used to detect the trapped charge.

Using the above procedures to determine Ω_e for a number of cylinders, we can make comparisons with the theoretically predicted critical velocities Ω_t. Such comparisons are made in Table I. Although the theoretical formulas are exact with no adjustable parameters, the formulas do depend on the lateral dimensions of the cylinders. Because of imperfections in the machining and coating of the cylinder walls, there are uncertainties in the lateral dimensions, and these produce the indicated uncertainties in Ω_t.

As shown in the table, Ω_e and Ω_t agree within the uncertainties, roughly 10%. We conclude from our observations that the method of sample preparation described above probably produces the equilibrium state, and that the calculations based on free-energy arguments predicts the correct equilibrium state.

Further refinements and more data are necessary before a more precise comparison between theory and experiment can be made.

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Localization and Migration of Energy among Sm^{3+} Ions in $CaWO_4$ Crystals

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We investigate energy transfer between Sm^{3+} ions in different crystal-field sites in $CaWO_4$ crystals by use of laser time-resolved spectroscopy techniques. The dependence of the results on time and temperature show the interaction to be electric quadrupole-quadrupole and the interaction strength to be much less than the inhomogeneous linewidth of the transition. This leads to the localization of the energy at low temperatures, confirming the predictions of Anderson.

It has been theoretically predicted by Anderson[1] that for a system exhibiting inhomogeneously broadened transitions, spatial localization of excitations can occur under certain conditions. Such localization occurs below a critical concentration of active sites when the interaction causing transfer falls off faster than the inverse third power of the distance between these sites. Lyo[2] found theoretically that the chromium-ion excitation energy in ruby crystals should be localized for concentrations less than about 0.3 at.% but it has been difficult to prove this experimentally.[3] Recently Orbach[4] has suggested time-resolved spectroscopy measurements of the energy migration among rare-earth ions as a method for observing the Anderson localization phenomenon.