with $1/\delta = (3\pi/4)^{2/3} - 1/\gamma$ and ρ given by (5) and (9). A lower bound is obtained by minimizing $\mathcal{E}_\delta$ over all ρ such that $\int\rho = N$. For simplicity we shall only use the absolute minimum of $\mathcal{E}_\delta$. By (12)

$$\langle\psi|H_N|\psi\rangle \geq -3.68(N\gamma + \delta\sum_{j=1}^{M} Z_j^{7/3})^2. \qquad (15)$$

Optimizing (15) with respect to γ yields

$$\langle\psi|H_N|\psi\rangle \geq -2.08\,N\left[1+\left(\sum_{j=1}^{M}\frac{Z_j^{7/3}}{N}\right)^{1/2}\right]^2. \qquad (16)$$

Remarks.—(1) If the fermions are of q species (instead of 2 as in the electron case), then the right-hand side of (10) would acquire a factor $(2/q)^{2/3}$ and the right-hand side of (16) a factor $(q/2)^{2/3}$.

(2) If all $Z_j = Z$, our result (16) gives a $Z^{7/3}$ dependence instead of the known Z^2 bound.[3] If $MZ \leq N$ then $MZ^{7/3}/N \leq Z^{4/3}$, which is an improvement over Ref. 3. If $MZ > N$ then we have to use the $\int\rho = N$ condition in (14). The TF no-binding theorem also holds in the subneutral case. By convexity of the TF energy in $\int\rho$, the minimum occurs for M atoms with equal electron charge N/M. If $MZ \gg N$ the energy per atom is proportional to $(N/M)^{1/3}Z^2$. Then $\langle\psi|H_N|\psi\rangle$ is bounded below by $-aN - bZ^2M^{2/3}N^{1/3}$. While this has the correct Z dependence, it has the wrong M dependence[3]; $M^{2/3}$ should be replaced by $N^{2/3}$. This difficulty is inherent in TF theory. What one needs is a simple proof that if $MZ \gg N$, then one can remove most of the surplus nuclei without affecting the energy. Even for $N=1$ this is not a simple problem. Nevertheless, our present bound is proportional to the *total* particle number, and this is sufficient for proving the existence of the thermodynamic limit.[10]

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[1]The notation is $\hbar = e = 2m = 1$; x_i and p_i are electron variables; R_k and $Z_k > 0$ are nuclear coordinates and charges ($\frac{1}{4} = 1$ Ry).

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[9]By numerically solving the three-dimensional variational equation for K, Eq. (7), when $N=1$, J. F. Barnes has shown that (8) holds with the TF constant $\frac{3}{5}(6\pi^2)^{2/3}$ when $N=1$ (private communication).

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Simple Model for $1/f$ Noise*

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The noise produced by thermodynamic fluctuations (e.g., in carrier number) is shown to diverge for ideal contact (spreading) resistors. For diffusion-controlled fluctuations the frequency spectrum is shown to be proportional to ω^{-1} (i.e., $1/f$) for high frequencies. This behavior is shown to be approached by resistors whose surfaces have sharp corners. The relation to observed noise is briefly discussed.

Noise power proportional to the square of the applied voltage (V) with a frequency spectrum $S(\omega) \propto \omega^{-1}$ over a wide range of ω has been observed in a variety of electrical devices.[1,2] Several models for this noise, invoking either complicated processes (e.g., hydromagnetic turbulence[3]) or purely mathematical constructs (e.g., correlated pulses[4]), have not given successful quantitative predictions. Attempts to derive this noise from simple thermodynamic fluctuations in such variables as local carrier concentration or local temperature (with the time course determined by diffusion) have also not been very successful.[5,6] The ω^{-1} law and the corresponding infinite mean square noise have been predicted only for a resistive layer with a special distribution of thicknesses.[7] In this communication I show that diffusion noise from the singular region of the field in a contact resistor should give $S(\omega) \propto \omega^{-1}$ as $\omega \to \infty$.

The treatment is similar to that of Richardson[7] except that I explicitly evaluate the spatial weighting function for fluctuations in terms of the electric field $\vec{E}$. I then derive the results directly from the known geometry of spreading resistances, not from assumptions about a possible surface layer. For fluctuations in conductivity σ caused by spontaneous fluctuations in carrier or impurity concentration, local temperature, etc., $\langle(\Delta\sigma/\sigma)^2\rangle = v/d^3r$ in a small volume d^3r, where v is a scaling volume which depends on the source of the fluctuations[5] (i.e., the magnitude of the fluctuations is inversely proportional to the size of the region observed). To first order in the fluctuations (for a linear theory[7]) we may assume constant V and $\vec{E}$ and consider the effect of fluctuations in σ on the resistance R. Using the expression $E^2\sigma$ for the power dissipation density, we obtain (for uniform σ and v)

$$\left\langle\left(\frac{\Delta R}{R}\right)^2\right\rangle = \left\langle\left(\frac{\Delta(V^2/R)}{V^2/R}\right)^2\right\rangle = v\frac{R^2\sigma^2}{V^4}\int E^4(\vec{r})\,d^3r, \quad (1)$$

where the integral is evaluated over the conducting region. This result may be extended[7] to obtain the autocorrelation function:

$$G(\tau) = \left\langle\frac{\Delta R(t+\tau)\Delta R(t)}{R^2}\right\rangle = \frac{vR^2\sigma^2}{V^4}\iint E^2(\vec{r})E^2(\vec{r}\,')c(\vec{r},\vec{r}\,',\tau)\,d^3r'\,d^3r,$$

where $c(\vec{r},\vec{r}\,',\tau)$ is the space-time correlation function for the fluctuating variable. For uniform diffusion in three dimensions with diffusion coefficient D

$$c(\vec{r},\vec{r}\,',\tau) = (4\pi D\tau)^{-3/2}e^{-(\vec{r}-\vec{r}\,')^2/4D\tau} \quad (2)$$

(see Ref. 7). One may rewrite $\int E^2(\vec{r}\,')c(\vec{r},\vec{r}\,',\tau)\times d^3r'$ as $\overline{E}^2(\vec{r},\tau)$, which is roughly the average of E^2 in a sphere of radius $(D\tau)^{1/2}$ around $\vec{r}$.

A simple contact resistor consists of an infinitely thin insulating sheet with a hole of radius a separating two conducting regions, with $R = 1/2a\delta$. The potential and field for this case have been solved analytically.[8] Near the edge of the hole the field diverges:

$$E^2(x) \to (V^2/2a\pi^2)x^{-1}, \quad (3)$$

where x is the distance to the hole's edge for any point within a solid torus around the edge. Then the torus of radius $x_0 \ll a$ contributes to $\int E^4 d^3r$ a term

$$(V^4/4a^2\pi^4)2\pi a\int_0^{x_0} x^{-2}2\pi x\,dx \to \infty, \quad (4)$$

where $2\pi a\,2\pi x\,dx$ is the volume of a toroidal shell of minor radius x. Intuitively, one may say that the divergence results from the inverse proportionality of the fractional noise to the size of the system, which emphasizes singularities.

For short correlation times (high frequencies) the noise is dominated by contributions from the singular region. Qualitatively, diffusion has the effect of truncating $\overline{E}^2(x,\tau)$ at a maximum about equal to $E^2((D\tau)^{1/2})$. This produces a leading term for small τ given by

$$G(\tau) \to (v/8\pi^2 a)\ln(x_0^2/D\tau). \quad (5)$$

Taking the Fourier transform [first terminating this term in $G(\tau)$ at $\tau = x_0^2/D$] one gets

$$S(\omega) \to v(8\pi^2a^3)^{-1}\omega^{-1}. \quad (6)$$

Although I have assumed a circular hole, any hole with finite curvature has the same order field divergence at the edge, so that the results are quite general except for the coefficient $(8\pi^2\times a^3)^{-1}$. The ω^{-1} law holds only for $\omega \gg D/a^2$. For $\omega \leq D/a^2$ the singularity is unimportant, so that in this range an ordinary diffusion spectrum[5] with finite $S(0)$ is predicted.

Physical contact resistors usually have narrow wedges rather than planes of insulator (see Fig. 1). Near a wedge with appropriate boundary conditions, conformal mapping gives

$$E^2(x) \propto x^{2(\theta-\pi)/(2\pi-\theta)},$$

with θ as illustrated. Then $\int E^2(\vec{r})\,d^3r$ converges, so that for small τ we may write

$$G(\tau) = G(0)(1-\psi\tau^{\gamma}). \quad (7)$$

Again, the effect of diffusion within the resistor is to truncate $\overline{E}^2(x,\tau)$ at about $E^2((D\tau)^{1/2})$, producing a term with $\gamma = \theta/(2\pi-\theta)$.[9] Terminating this term conveniently at $\tau = \psi^{-1/\gamma}$ we obtain the Fourier transform which has a limiting behavior

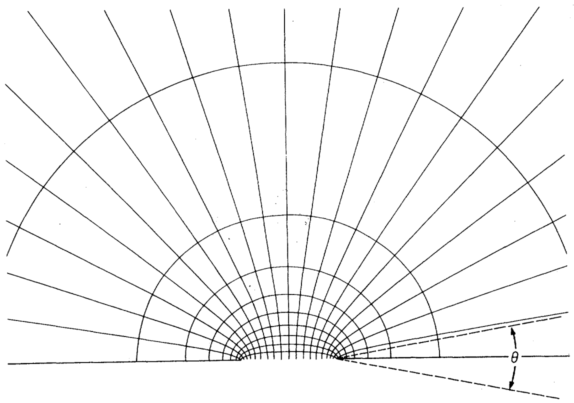

FIG. 1. The field and equipotential lines in half a cross section of a spreading resistor. The resistance between consecutive equipotentials is $dR' = R/20$. The current density is inversely proportional to the spacing between field lines. A wedge angle is shown without its field lines.

as $\omega \to \infty$ of $S(\omega) \propto \omega^{-\alpha}$, where $\alpha = 1 + \gamma = 1 + \theta/(2\pi - \theta)$. For $\theta < \pi/10$, $\alpha \lesssim 1.05$. For another case of interest, that of a spreading resistance at the end of a channel, $\theta = \pi/2$, giving $\alpha = \frac{4}{3}$.

Several physical effects other than finite wedge angles can cause a deviation from $S(\omega) \propto \omega^{-1}$ at high frequencies. Replacing the ideal insulating plane by a sheet of finite thickness l with a hole with rounded edges should lead to a rolloff steeper than ω^{-1} for $\omega > D/l^2$. Even for an ideal geometry, such effects as the inherent time constant ($RC = \xi/\sigma$, where ξ is the dielectric constant) of the conductor, the finite carrier relaxation time, and higher-order terms in the thermodynamic-fluctuation-probability expansion prevent the ω^{-1} law from holding for infinitely large ω. Although such effects are likely to occur above the observable frequency range, they must be invoked to justify the self-consistent use of a linear theory, which requires $\langle(\Delta R/R)^2\rangle \ll 1$.

I shall briefly consider next the relevance of this theory to some systems other than contacts per se, which are known sources of ω^{-1} noise.[6,7] Carbon resistors are known to exhibit ω^{-1} noise from about $\omega = 1.5\times10^{-3}$ sec^{-1} [10] to $\omega \approx 10^5$ sec^{-1}.[11] Since these resistors contain many small contacts, we expect them to give ω^{-1} noise. However, the persistence of this noise to extremely low frequencies can be explained by the present theory only by assuming a very slowly diffusing variable. The concentration of impurities has been proposed as such a variable.[12] An experiment on germanium[13] shows noise with $\alpha = 1.3$ to 1.35 extending from $\omega = 1.5\times10^{-5}$ sec^{-1} to audio frequencies. We may speculate that such noise could result from field singularities near the corners of lattice defects with $\theta = \pi/2$ and $\alpha = \frac{4}{3}$. In strained germanium samples with many lattice defects there is a dramatic increase in noise[14] consistent with this explanation. Different power laws may be produced by different types of defects.

Noise with a roughly ω^{-1} spectrum has been observed in ionic spreading resistors.[15] The noise scaled as $1/a^3$, as the present work predicts for this type of noise. The scaling factor v was $>1/n$, poorly reproducible and concentration independent, which may indicate that the noise resulted from fluctuations in concentrations of impurities, such as bubbles or dust, rather than in carrier concentrations. Other ionic systems have shown noises which deviate sufficiently from an ω^{-1} law to be accounted for by ordinary diffusion spectra.[15]

Temperature fluctuations have been established as the source of current noise in metal films[17] and one ionic system.[18] However, in the frequency ranges over which this noise has been studied, the contribution from the high-frequency tails of sharp corners should be small, so that the presence of this type of noise can not be determined.

This theory may be tested in ionic resistors for which walls can be made with θ from nearly 0 to nearly π. Suitable fluctuating variables could be either ion concentration or concentration of polystyrene spheres. Polystyrene spheres of radius r allow for changes in v and D with negligible changes in current, voltage, and resistance, since $D \propto 1/r$ and $v \propto r^3\times$(concentration in volume %). Such experiments should demonstrate that the "universal $\frac{3}{2}$-power law"[3] for diffusion spectra occurs only for the usually considered case of step-function singularities in E^2 within the region accessible to diffusion. Other singularities can produce any value of α between 1 and 2.

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New Charm-Current Effects in Neutrino Scattering*

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We investigate the quantitative implications for deep inelastic neutrino scattering of a proposed additional $V+A$ charm-changing current $\bar{c}\gamma_\mu(1-\gamma_5)d$. Quark-parton-model spectrum-averaged predictions are compared with available νN and $\bar{\nu}N$ y distributions as obtained at Fermilab. Agreement is achieved with an effective charm threshold $W_{\rm th}=4$ GeV. A characteristic feature is an enhanced sea contribution that can account for the anomaly in $d\sigma/dy$ for $\bar{\nu}N$. Predictions for sum rules are significantly changed at high energy.

A new charm-changing $V+A$ current $\bar{c}\gamma_\mu(1-\gamma_5)d$ has recently been proposed by De Rújula, Georgi, and Glashow[1] to explain the observed $\Delta I=\frac{1}{2}$ rule for nonleptonic decays and also to permit equal strange and nonstrange hadronic decays of charmed particles. The existence of this new current clearly has important implications for deep inelastic neutrino scattering. We investigate here the consequences and predictions in the quark-parton model (QPM). Agreement with available Fermi National Accelerator Laboratory (FNAL) $d\sigma/dy$ data[2] is achieved with a charm threshold $W_{\rm th}=4$ GeV. Charm production through this mechanism, that involves valence as well as sea quarks, also gives fast-muon distributions consistent with those from the dimuon events.[3,4] The presence of the new current also has significant implications for the x distributions and for sum rules.

Including the new term, the charged weak hadronic current in terms of quarks is[1]

$$J_\mu=\bar{u}\gamma_\mu(1+\gamma_5)(dC+sS)+\bar{c}\gamma_\mu(1+\gamma_5)(sC-dS)+\bar{c}\gamma_\mu(1-\gamma_5)d, \qquad (1)$$

where $C=\cos\theta_{\rm C}$ and $S=\sin\theta_{\rm C}$, with $\theta_{\rm C}$ the Cabibbo angle. Below threshold for producing charmed particles the differential cross sections $\sigma(x,y)\equiv d^2\sigma/dx\,dy$ per average nucleon $N=\frac{1}{2}(p+n)$ are

$$\sigma^{\nu N}(x,y)/x=(u+d)C^2+2sS^2+(\bar{u}+\bar{d})(1-y)^2,$$
$$\sigma^{\bar{\nu}N}(x,y)/x=(\bar{u}+\bar{d})C^2+2\bar{s}S^2+(u+d)(1-y)^2, \qquad (2)$$

in units of G^2ME/π. The quark densities are denoted by $u(x)$, $d(x)$, $s(x)$, and $c(x)$ and $x=Q^2/2M\nu$ and