where

$$f_1(t) = (\lambda^2 + \vec{\Delta}^2)\left\{\int \frac{d^2k_\perp}{(2\pi)^3}\left[\left(\vec{k}_\perp - \frac{\vec{\Delta}}{2}\right)^2 + \lambda^2\right]^{-1}\left[\left(\vec{k}_\perp + \frac{\vec{\Delta}}{2}\right)^2 + \lambda^2\right]^{-1}\right\}^2, \tag{2}$$

$$f_2(t) = (\tfrac{5}{4}\lambda^2 + \vec{\Delta}^2)\left\{\int \frac{d^2k_\perp}{(2\pi)^3}\left[\left(\vec{k}_\perp - \frac{\vec{\Delta}}{2}\right)^2 + \lambda^2\right]^{-1}\left[\left(\vec{k}_\perp + \frac{\vec{\Delta}}{2}\right)^2 + \lambda^2\right]^{-1}\right\}^2$$
$$- \int \frac{d^2k_{1\perp}}{(2\pi)^3}\frac{d^2k_{2\perp}}{(2\pi)^3}(\vec{k}_{1\perp}^{\,2} + \lambda^2)^{-1}(\vec{k}_{2\perp}^{\,2} + \lambda^2)^{-1}[(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\Delta})^2 + \lambda^2]^{-1}, \tag{3}$$

λ is the mass of the Yang-Mills boson, m is the mass of the fermion, g is the coupling constant, and $\vec{\tau}^{(i)}$ are the Pauli matrices for isotopic spin of fermion i, $i = 1, 2$. Details of this computation will soon be submitted for publication.

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Color Gluons and the Decay of the $\psi(3700)$ into $\psi(3100)$*

H. Goldberg

Department of Physics, Northeastern University, Boston, Massachusetts 02115

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The decay of $\psi(3700)$ into $\psi(3100)$ plus hadrons constitutes a violation of Zweig's rule. On the assumption that the de-excitation proceeds through the emission of Yang-Mills gluons in a color-singlet state, I predict a width of 190 ± 40 keV for this process. All parameters entering the calculation are taken from previous fits to the decay widths of the $\psi(3100)$.

The discovery of the narrow mesonic resonances at 3.095[1] and 3.685[2] GeV [which we call the $\psi(3100)$ and $\psi(3700)$, or ψ and ψ', respectively] has stimulated some intriguing applications of the twin notions of asymptotic freedom[3] and quark confinement. On the assumption[4,5] that the $\psi(3100)$ is a lightly bound $^3S_1$ state of a heavy charmed quark[6] (c) and its antiquark ($\bar{c}$), its narrow width into hadrons (~ 70 keV) has been ascribed[4] to the necessity for de-excitation via annihilation into three vector gluons, each one a member of an SU(3) (color) octet, whose coupling to the charmed quarks in the $\psi(3100)$ is weak. In this model, the branching ratio $\Gamma(\psi \to e^+e^-)/\Gamma(\psi \to \text{hadrons})$ provides an estimate of ~ 0.20 for $\bar{g}^2/4\pi \equiv \alpha_s$, the effective coupling strength of the color gluons to charmed quarks in charmonium.

It is clearly of interest to provide independent corroboration for the role played by the color gauge theory in explaining the narrow width of the ψ. In practical terms, can one find other processes in this energy range whose rate depends on α_s in a calculable manner? In this note, I propose that the de-excitation $\psi(3700) \to \psi(3100) + \text{hadrons}$ is such a process, and the rate predicted on the basis of $\alpha_s \simeq 0.20$ lies within the present experimental limits.

At the time of the present writing, the parameters of the $\psi(3700)$ relevant to this discussion are given as $200\ \text{keV} < \Gamma_{\text{tot}} < 800\ \text{keV}$[7] and $0.48 \pm 0.06 \le \Gamma(\psi(3700) \to \psi(3100) + \text{hadrons})/\Gamma(\psi(3700) \to \text{all}) \le 0.57 \pm 0.08$.[8] The lower limit corresponds to saturation by the 2π channel, and is obtained by assuming that all π-π pairs are produced in an $I = 0$ state.

It is perhaps instructive to show first why conventional hadron phenomenology confronts some difficulty in accommodating this width. In order to remove the effects of phase space, the rate for $\psi(3700)\to\psi(3100)+\pi+\pi$ (the dominant mode) may be calculated in a model which has $\psi(3700)$ decay into $\psi(3100)$ plus a virtual scalar (σ) of mass ~ 1 GeV, with the latter decaying into two π's. If the same calculation is then performed for $\rho'(1600)\to\rho(770)+\pi+\pi$, one can obtain the ratio

$$\frac{\Gamma(\psi'\to\psi\pi\pi)}{\Gamma(\rho'\to\rho\pi\pi)}\simeq 0.066\left(\frac{g_{\psi'\psi\sigma}}{g_{\rho'\rho\sigma}}\right)^2. \qquad (1)$$

On the basis of $\Gamma(\rho'\to\rho\pi\pi)\sim 300$ MeV,[9] an accounting for the width $\Gamma(\psi'\to\psi\pi\pi)\sim 250$ keV requires the large dynamical suppression $g_{\psi'\psi\sigma}/g_{\rho'\rho\sigma}\simeq\frac{1}{9}$; this can be taken as a phenomenological expression of Zweig's rule, but gets us no closer to understanding the ψ' width.

In terms of quark line diagrams, the de-excitations $\psi'\to\psi+\pi+\pi$ and $\psi\to$ noncharmed hadrons involve similar violations of Zweig's rule. It may then be reasonable to tailor the Appelquist-Politzer model[4] for the annihilation to this situation along the following basic guidelines: (1) The rate for $\psi'\to\psi+$hadrons may be obtained from a calculation of the rate for $\psi'\to\psi+$[even number of color gluons in an SU(3′) color singlet state]. (2) The effective coupling strength, $\bar{g}^2/4\pi$, of a color gluon to a charmed quark in charmonium is ~ 0.20.[10]

To proceed most simply, consider the de-excitation $\psi'\to\psi+2$ gluons. Because (presumably) $m_{\psi'}\geq 2m_c$, intermediate (color-octet) states of unbound c-$\bar{c}$ pairs[11] degenerate (or nearly degenerate) with the ψ' may be reached via the emission of a soft color gluon. Hence the amplitude will contain nominally infrared-divergent terms, as shown in Fig. 1. We then *assume* that nature tames the infrared divergences, and that, as in quantum electrodynamics, some experimental lower limit $\omega_{\min}$ may be placed on the gluon energies ω_1 and ω_2 in all phase-space integrations. In the two-gluon case, $\omega_{\min}$ is fixed at ~ 35 MeV (in the rest frame of the ψ') by requiring the invariant mass of the two gluons to be $\geq 2m_\pi$, the hadron threshold.

Other terms will be required for gauge invariance. However, in the present case these can be safely neglected, since they are of order $\bar{\omega}/m_c$ ~ $\frac{1}{5}$ times the bremsstrahlung terms in Fig. 1. (Here $\bar{\omega}$ is the average gluon energy, ~ 300 MeV.)

The transition matrix element,

$$\int dx\,\exp(ik_2x)\langle\psi(3100)|T(j_\mu{}^a(x)j_\nu{}^b(0))|\psi(3700)\rangle,$$

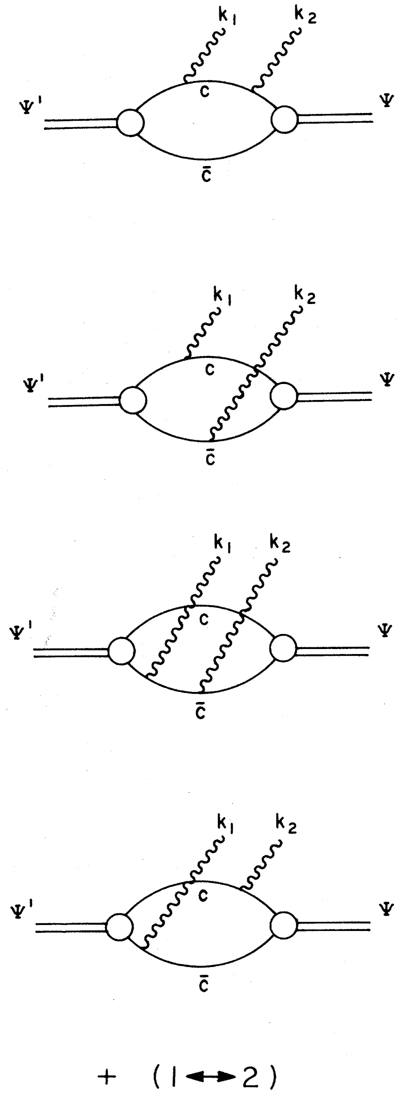

FIG. 1. Bremsstrahlung terms contributing to decay of $\psi(3700)$ into $\psi(3100)$ via gluon emission.

may then be evaluated in the charmonium model. Going to the limit $\omega_1,\omega_2\ll m_c$, and using the nonrelativistic approximation

$$\langle p'|\vec{\epsilon}\cdot\vec{j}\,|p\rangle=\vec{\epsilon}\cdot\vec{p}/m_c$$

for the quark current, we obtain the matrix element corresponding to Fig. 1[12]:

$$\mathfrak{M}=(2\omega_1)^{-1/2}(2\omega_2)^{-1/2}4\,\tfrac{1}{6}\bar{g}^2(\omega_1{}^{-1}+\omega_2{}^{-1})\bar{\mathfrak{M}}, \qquad (2)$$

$$\bar{\mathfrak{M}}\equiv\int d^3p\,\Psi_{3100}{}^*(\vec{p})\,\frac{\vec{p}\cdot\vec{\epsilon}_1}{m_c}\,\frac{\vec{p}\cdot\vec{\epsilon}_2}{m_c}\,\Psi_{3700}(\vec{p}). \qquad (3)$$

The factor $\frac{1}{6}$ in Eq. (2) comes from projecting the two gluons into a color-singlet state. Summing over gluons yields

$$\Gamma(\psi' \to \psi + \text{hadrons}) = (2\pi)^{-5}(8/2!)(4\pi/3)^2\alpha_s^2\Delta^2 \iint (d^3k_1/\omega_1^3)(d^3k_2/\omega_2^3)\delta(\omega_1+\omega_2-\Delta)\sum_{\text{pol}}|\mathfrak{M}|^2, \tag{4}$$

where $\Delta \equiv m_{\psi'} - m_\psi = 0.590$ GeV.

The s-wave nature of the states allows the replacement $p_i p_j \to \delta_{ij}p^2/3 = (m_c/3)\delta_{ij}H_0$, and hence

$$\mathfrak{M} = (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2/3m_c)\langle\psi(3100)|H_0|\psi(3700)\rangle = (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2/3m_c)\langle\psi(3100)|H - V|\psi(3700)\rangle$$
$$= -(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2/3m_c)\int d^3r\, \Psi_{3100}(r)V(r)\Psi_{3700}(r). \tag{5}$$

With a lower bound of $4m_\pi^2$ placed on $(k_1+k_2)^2$, Eqs. (4) and (5) give the result

$$\Gamma \simeq 252\alpha_s^2|\langle\psi(3100)|V|\psi(3700)\rangle/m_c|^2 \text{ MeV}. \tag{6}$$

For a linear potential[13] $V=Kr$, the matrix element can be numerically evaluated[14]:

$$\langle\psi(3100)|V|\psi(3700)\rangle = 0.653(K^2/m_c)^{1/3} = 0.653\Delta/(a_2 - a_1), \tag{7}$$

where $-a_1$ and $-a_2$ are the first and second zeros of the Airy function. With $\Delta = 590$ MeV, $m_c = 1.6 \pm 0.2$ GeV/c^2, and $\alpha_s = 0.20 \pm 0.02$, we find

$$\Gamma(\psi' \to \psi + \text{hadrons}) = 190 \pm 40 \text{ keV}. \tag{8}$$

Assuming a branching ratio of $(50 \pm 10)\%$ for the cascade process $\psi' \to \psi +$ hadrons, we are led to a total width of 380 ± 100 keV for the $\psi(3700)$.

As noted in the abstract, there are no parameters introduced in this calculation beyond those appearing in Ref. 4. Hence I regard the result (8) as supportive of the dynamical picture introduced in that work.

In closing, I shall comment on several points of interest.

(1) What about final states with more than two gluons? These can be ignored for several reasons: (a) The coupling for each additional pair effectively introduces a factor of $(\alpha_s/\pi)^2 \simeq 0.006$ in the cross section; (b) the coupling for a pair of transverse gluons is proportional to $\langle \vec{V}^2\rangle$ of the quarks, which is small in the case of charmonium; (c) there is a group-theoretic suppression in projecting out the color-singlet state from a final state of four or more gluons. All of these are more than sufficient to overcome the (logarithmic) enhancement of multigluon states as a result of a decrease of the lower limit of the gluon spectrum with rising multiplicity.

(2) What is the mass spectrum predicted for the emitted hadrons (mostly π-π pairs, in this case)? This is a very complicated matter, involving the final-state interaction of the gluons, $\gamma_c + \gamma_c \to \pi + \pi$.[15] Some partial information may be gleaned with the help of current algebra and partial conservation of axial-vector current. The pertinent analysis has been carried out[16] in the analogous case of $\eta' \to \eta + \pi + \pi$; the result for the decay amplitude $\psi' \to \psi + \pi + \pi$ is an expansion

$$(2\omega_1)^{1/2}(2\omega_2)^{1/2}\mathfrak{M}(\psi' \to \psi\pi\pi) \approx F_\pi^{-2}\langle\psi'|\Sigma|\psi\rangle + a(M_{\pi\pi}^2 - 2m_\pi^2)/4m_{\psi'}^2 + b(\omega_1 - \omega_2)^2/4m_{\psi'}^2. \tag{9}$$

The first term is the matrix element of the Σ commutator,

$$\Sigma \equiv \tfrac{1}{3}\sum_{a=1}^{3}[Q_5^a,[Q_5^a,\mathcal{H}(0)]].$$

In the charmonium model, where the operators are sandwiched between loosely bound $c\bar{c}$ bound states, the Σ term is equal to

$$F_\pi^{-2}\langle c'|\Sigma|c\rangle \int 4\pi r^2 dr\, \Psi_{3700}(r)j_0(\tfrac{1}{2}Qr)\Psi_{3100}(r),$$

where $\vec{Q} = \vec{k}_1 + \vec{k}_2$. The orthogonality of the radial wave functions introduces a large suppression (as a function of Q, the integral has a maximum value of ~ 0.06), so that even if the matrix element $\langle c'|\Sigma|c\rangle$ is as large as m_c, the contribution of the Σ term throughout the whole range of $M_{\pi\pi}$ is negligible. Thus the connection with the gluon result must be made through the second and third terms in Eq. (9). These contain the effects of the Adler zeros,[17] and will lead to a large suppression of the mass spectrum near $M_{\pi\pi} = 2m_\pi$. Hence one would expect the $M_{\pi\pi}$ distribution to peak near the middle to upper end of the spectrum, with a total normalization approximately given by Eq. (8).

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[10]In the present case the gluons carry low energy in the ψ' rest frame. Hence it may be questioned whether the decay rate is governed by α_s(3 GeV), as in the annihilation case. The following argument may be made in support of this assumption: In general, the renormalization group permits one to renormalize at an arbitrary mass, which one chooses judiciously in order to make the perturbation series converge. If we renormalize at 3 GeV, then the calculation of $\psi' \to \psi + X$ may contain correction terms of order $[\alpha_s(3)/\pi]\ln(M_\psi^2/M_X^2)$. For $M_X \simeq 500$ MeV, these corrections are of order $(0.2/\pi)\ln 36 = 0.23$. Hence the systematic error introduced by the use of asymptotic freedom in this region of M_X may be comparable to that incurred through the neglect of nonpole terms required for gauge invariance (see text). Finally, it should be noted that the process $\psi' \to \psi +$ anything, being inclusive, is free from mass singularities [T. Kinoshita, J. Math. Phys. (N.Y.) 3, 650 (1962)].

[11]As in all these discussions, it is assumed that such states never propagate into observability.

[12]Note that the four diagrams in Fig. 1 interfere constructively: In going from c to $\bar{c}$, both the coupling and the current change sign. Hence a factor of 4 in Eq. (2).

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[15]The mass spectrum of the gluons themselves is rather flat, falling by less than a factor of 2 between the threshold region (330 MeV) and the "high"-energy end (500 MeV).

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