quasiparticle bands are parallel.

Numerical solutions of Eq. (6) give a binding energy of 0.9 meV at $N = 10^{12}$ cm⁻² increasing with N to 1.7 meV at 3×10^{12} cm⁻². Since the static approximation is quite bad for the calculation of self-energies one probably would not expect the calculated binding energy to be very exact. It is, however, of an order of magnitude which is not entirely negligible. The energy difference between the two sets of maxima measured by Wheeler and Goldberg is considerably smaller than one would expect for the difference between the first- and second-excited sub-bands, so one might speculate whether the assignment of transitions should be changed so that the lower points correspond to exciton formation and the higher to transition from the lowest to the first-excited sub-band. But to substantiate this conjecture one would have to calculate the relative efficiencies of those processes.

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Isotopic Composition of Cosmic-Ray Nitrogen at 1.5 GeV/amu*

Robert Dwyer and Peter Meyer

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637 (Received 16 June 1975)

We have determined the cosmic-ray ${}^{15}N/({}^{14}N + {}^{15}N)$ ratio with a detector sensitive to velocities of particles in the penumbra of Earth's magnetic field. Our instrument was flown twice on high-altitude balloons from Palestine, Texas, obtaining an exposure factor of 20 m² sr h. We measure the fraction of ${}^{15}N$ to be 0.45 ± 0.07 at 1.5 GeV/amu consistent with an origin for nitrogen primarily due to spallation in the interstellar medium.

Knowledge of the isotopic composition of cosmic-ray nuclei is of fundamental importance in understanding their origin and history. To date much has been learned about elemental abundances but information on specific isotopic abundances has been limited to a few elements at low energies [up to a few hundred MeV per amu). It is isotopic, rather than elemental, abundance measurements that place the strongest constraints on the models of galactic confinement and propagation and on the characteristics of the sources. Also, isotope results at higher energies ($\gtrsim 1 \text{ GeV}/$ amu) are little influenced by extrapolation out of the solar-modulation region and benefit from the energy independence of the spallation cross sections and negligible interstellar-ionization loss of the particles. A method applicable for higher energies and using the geomagnetic field has been proposed and extensively developed by Peters¹ and his co-workers. At a given location, the geomagnetic field transmits all particles above a certain rigidity R_{max} and none below another rigidity R_{min} . Between these two values lies the rigidity range called the penumbra. This method involves comparing velocity spectra in the penumbra of two elements, one considered a

reference element whose isotopic composition is known or assumed. For a given nuclear charge Z, the signal C in a Cherenkov radiator of refractive index *n* depends on velocity: $C/Z^2 = 1 - (p_0^2/2)$ p^{2}), where $p_{0}^{2} = (n^{2} - 1)^{-1}$, $p^{2} = \beta^{2} \gamma^{2}$, and $\gamma^{2} = (1 - 1)^{-1}$ $-\beta^2)^{-1}$, $\beta = v/c$. Recall the relation between rigidity R and momentum per amu p: R = (A/Z)p, where A is the nucleon number. Thus if one element has a larger average A/Z than another, its Cherenkov-signal spectrum will show the presence of lower-velocity particles allowed by the geomagnetic field. We have built and flown an instrument designed to use the differential geomagnetic method¹ for the determination of the mean mass of various elements and we describe here our first results which yield the $^{15}N/N$ ratio at energies around 1.5 GeV/amu.

A cross section of the apparatus used in this experiment is shown in Fig. 1. It consists of two scintillators, T1 and T3, and a liquid Cherenkov counter T2, each enclosed in a white high-reflectance light-integration box. T1 and T3 are identical scintillators made of 1 g/cm^2 Pilot Y and are the basic charge-measuring elements of the instrument. The liquid Cherenkov counter T2, filled with Freon fluorocarbon liquid (E-2) and a wave shifter, has an index of refraction of 1.26 and is therefore very sensitive for measuring velocities around 1 GeV/amu. Its resolution is limited by photoelectron statistics due to the low yield of Cherenkov light.

Three multiwire proportional chambers A, B, and C are used for determining the particle trajectory and for rejecting background. The guard counter G is a scintillator in the form of an annulus serving to eliminate side showers and other background events. The geometrical factor of



FIG. 1. Schematic cross section of the instrument. MWPC is the multiwire proportional chamber.

the instrument is $0.25 \text{ m}^2 \text{ sr.}$

Our instrument was flown from Palestine, Texas (vertical cutoff rigidity ~4.5 GV), in September 1973 and May 1974. In both flights combined, it spent 77 h under $3-5 \text{ g/cm}^2$ of residual atmosphere yielding an exposure factor of 20 m² sr h.

The analysis begins by applying three selection criteria. We require (1) a unique straight line in the proportional chambers, (2) agreement between the top and bottom scintillator pulse heights within 15%, and (3) a low guard-counter signal. The velocity dependence of the scintillator response for velocities less than that of a minimum ionizing particle is corrected for, using the Cherenkov-counter information. A charge histogram of a portion of the float data to which these criteria have been applied is shown in Fig. 2. The charge resolution in the C, N, O region is 0.10 charge units and thus these elements are very clearly separated, a prerequisite for measurements of isotopic composition.

By summing nitrogen's neighboring elements carbon plus oxygen as a composite reference element, Z-dependent effects are minimal since C +O has the same average charge as N and yet, for these abundant elements, good statistical accuracy is obtained.

If we assume for the moment that carbon and oxygen have A/Z = 2.0 exactly, i.e., only ¹²C and ¹⁶O, then we note that if nitrogen consisted exclusively of ¹⁴N, the N and C +O Cherenkov-signal spectra would be identical since ¹⁴N has A/Z= 2.0 also. However, an increasing contribution by ¹⁵N in the cosmic rays would show an increasing difference between the N and C +O spectra in the signal range affected by the penumbra.

In order to quantitatively obtain the $^{15}N/N$ ratio



FIG. 2. Charge histogram of a portion of flight data after selection criteria were applied, but without at-mospheric correction.

we construct another reference spectrum corresponding to nuclei with $A/Z = \frac{15}{7}$ (i.e., pure ¹⁵N) from the known C+O distribution. Given the momentum per amu p_1 , for each particle in the C +O spectrum, we know its rigidity ($R = 2.0p_1$). Hence we know the momentum per amu p_2 it would have if it's A/Z were $\frac{15}{7}$: $p_2 = \frac{7}{15}R = \frac{14}{15}p_1$. This procedure is valid as long as the momentum per amu p_1 can be obtained from the Cherenkov signal C/Z^2 with negligible error due to finite instrumental resolution. Calibrations on the ground verified that this error is indeed negligible for C, N, O nuclei in the range of momentum per amu corresponding to signals below 0.9 (90% of the signal due to a fully relativistic particle).

The arguments given above are valid only if the interplanetary energy spectra of C+O and N have the same shape in the range used for isotope analysis. To establish the relative shape of the spectra, we measured the energy dependence of the N/(C+O) ratio in the same experiment when the balloon drifted north into a region of lower geomagnetic cutoff, and used results of Juliusson.² Figure 3 shows that, within errors, the N/(C+O)ratio is indeed constant from 1 to 4 GeV/amufalling to lower values at higher energies.³ We then normalize the nitrogen Cherenkov-signal spectrum using the measured N/(C+O) ratio at 1-4 GeV/amu (0.148±0.003) to present our results from the first balloon flight in Fig. 4. The figure shows how, for any energy in the penumbra region, i.e., for $C/Z^2 \leq 0.7$, the ratio E/Dyields the desired ¹⁵N fraction, and it exhibits pictorially the principle of the differential method devised by Peters. It is already qualitatively evident from these raw data that nitrogen is a



FIG. 3. The N/(C+O) ratio versus energy at balloon altitude. For comparison points are shown from Juliusson at 1 GeV/amu (Ref. 2) and above 20 GeV/amu (Ref. 3).



FIG. 4. Cherenkov-signal spectra observed in the first flight. These spectra are influenced by the geomagnetic field for $C/Z^2 \lesssim 0.7$.

mixture of isotopes. The fact the N lies below the C+O distribution in the region where $C/Z^2 \approx 1$ is due to the energy dependence of the N/(C +O) ratio at higher energies.

Figure 5 gives the final results for the ratio $k_{15} = {}^{15}\text{N/N}$ based on the two balloon flights. In arriving at these values, a 5–6% admixture of ${}^{13}\text{C}$ to carbon and of ${}^{17}\text{O} + {}^{18}\text{O}$ to oxygen was assumed based on calculations of Tsao, Shapiro, and Silberberg.⁴ We obtain from our first balloon flight, $k_{15} = 0.43 \pm 0.07$ (at 1.2 GeV/amu) and from our second flight, $k_{15} = 0.47 \pm 0.08$ (at 1.7 GeV/amu) after correction to the top of the atmosphere.

An extrapolation of these results to the cosmicray sources using a steady-state model of galac-



FIG. 5. The observed fraction $k_{15} = {}^{15}\text{N}/({}^{14}\text{N} + {}^{15}\text{N})$. The low-energy result is from Ref. 5.

tic confinement and propagation with an exponential distribution of particle pathlengths, mean pathlength = 5 g/cm² of hydrogen, and including the recent spallation cross sections of Lindstrom *et al.*,⁶ yields a ratio of nitrogen to carbon in the sources of 0.07 ± 0.02^7 which is lower than previous estimates.^{8,9} The need for reducing the amount of source nitrogen based on new crosssection measurements has been pointed out by Shapiro, Silberberg, and Tsao.¹⁰

This result represents the first conclusive highenergy ($\geq 1 \text{ GeV/amu}$) measurement of isotopic composition. Together with further results for other elements at these high energies it is expected to contribute significantly to the problems of confinement, propagation, and the nature of the cosmic-ray sources.

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COMMENTS

Comment on High-Energy Behavior of Non-Abelian Gauge Theories

Barry M. McCoy*

Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794

and

Tai Tsun Wu†

Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts 02138 (Received 18 June 1975)

The high-energy behavior of the sixth-order fermion-fermion scattering amplitude in the Yang-Mills theory is recalculated, and found to be qualitatively different from that given previously.

A year ago, Nieh and Yao¹ studied the high-energy behavior of the fermion-fermion scattering amplitude in the Yang-Mills theory² and stated that, for large center-of-mass energy \sqrt{s} and fixed $t = -\vec{\Delta}^2$, this amplitude behaves as $s \ln^3 s$ in sixth order and as $s \ln^5 s$ in eighth order. We have carried out in detail the required computation in this case for the sixth order and found their result qualitatively incorrect; instead, for $s \rightarrow \infty$ with fixed $t \leq 0$, the spin-nonflip amplitude is determined to be given approximately by

$$\mathfrak{M}^{(6)} \sim -2^{-4}g^6 m^{-2}s \big[(\ln^2 s - \pi i \ln s) \vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)} f_1(t) + 3\pi i \ln s f_2(t) \big],$$

(1)