of the compensating cloud to 4s-electron-shell dimensions would still be undetectable in Bragg neutron diffraction. Moreover it is to be recognized that the "viewing" time scale associated with neutron passage is less than the intrinsic fluctuation time of the magnetic center in contrast to the much longer time associated with the other techniques.

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# Effect of the Electron-Electron Interaction on the Excitation Energies of an *n*-Inversion Layer on Si

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The influence of the electron-electron interaction on the energy of electrons in the lowest and first-excited sub-bands of an inversion layer has been evaluated. When these corrections are included very good agreement with experimental results is obtained. A bound inter–sub-band exciton is predicted.

The exchange energy of electrons in the lowest sub-band of an inversion layer formed near the $SiO_2$-Si[001] interface in a metal-insulator-semiconductor structure was calculated by Chaplik[1] and by Stern[2,3] and is of the order of $-10$ meV. In a previous paper[4] I calculated the correlation energy and found that it further lowers the energy of electrons in the lowest sub-band by about 5 meV. Since the separation in energy between the two lowest sub-bands is of the order of 10 meV,[5] many-body effects strongly affect the energy of transition from the lowest to the first-excited sub-band.

For *p*-type inversion layers Ohkawa, Ando, and Uemura[6] have calculated the self-energy of a hole in the lowest sub-band. They found that the energy separation between the lowest and first-excited sub-bands is in much better agreement with measurements[7] when exchange and correlation in the lowest sub-band is taken into account.

In *n*-inversion layers, however, I have found that the calculated inter–sub-band splitting is much larger than measured if only the self-energy of the lowest sub-band is considered.

In this Letter I present results of a calculation of the correlation energy of an electron in the first-excited sub-band. It is also of the order of $-10$ meV and the energy separation as a function of inversion-layer density is found to agree very well with optical measurements.[8,9]

The bare Coulomb interaction between two electrons Fourier analyzed in the two dimensions

parallel to the interface is given by

$$v_{ijlm}(\vec{q}) = \frac{e^2}{2\epsilon_s q} f_{ijlm}(q),$$
$$f_{ijlm}(q) = \int_0^\infty \int_0^\infty dz\, dz' \zeta_i^*(z)\zeta_l^*(z')\left[e^{-q|z-z'|} + \frac{\epsilon_s - \epsilon_i}{\epsilon_s + \epsilon_i} e^{-q(z+z')}\right]\zeta_j(z)\zeta_m(z'), \tag{1}$$

where $\epsilon_s$ and $\epsilon_i$ are the permittivities of the semiconductor and insulator, respectively, the indices $i$ and $l$ denote the sub-bands of the electrons before scattering, $j$ and $m$ denote the sub-bands after scattering, and $\zeta_i(z)$ are the self-consistent envelope wave functions perpendicular to the surface.[5]

Since wave functions corresponding to different sub-bands are orthogonal, $f_{ijlm}(q) \to 0$ for $q \to 0$ when one of the electrons changes sub-band during the scattering. Furthermore, for an electron in the excited band Stern[3] calculated the exchange energy, which involves $v_{1001}$, to be small, about $-0.5$ meV. For these reasons and because it facilitates calculations greatly $f_{ijlm} = 0$ is used unless $i = j$ and $l = m$.

At absolute temperature $T = 0$ and at densities for which only the lowest sub-band is occupied the random-phase approximation gives a screened Coulomb interaction $U$ determined from

$$U_{iiii}(\vec{q},\omega) = v_{iiii}(\vec{q}) + v_{ii00}(\vec{q})\tilde{\chi}_{00}(\vec{q},\omega)U_{00ii}(\vec{q},\omega), \tag{2}$$

where

$$\tilde{\chi}_{00}(\vec{q},\omega) = -2in_v \int \frac{d^2k\, dE}{(2\pi)^3} G_0(\vec{k}+\vec{q}, E+\omega)G_0(\vec{k},E). \tag{3}$$

$G_0(\vec{k},E)$ is the Green's function for noninteracting particles in the lowest sub-band and $n_v$ is the valley degeneracy. For $\tilde{\chi}_{00}$ the Lundqvist[10]-Overhauser[11] approximation described earlier[4] has been used.

For the self-energies of the sub-bands one then has

$$M_{00}(\vec{k},E) = i\int \frac{d^2q\, d\omega}{(2\pi)^3} U_{0000}(\vec{q},\omega) \frac{1}{E-\omega-\xi(\vec{k}-\vec{q}) + i\,\mathrm{sgn}(|\vec{k}-\vec{q}|-k_F)\delta},$$
$$M_{11}(\vec{k},E) = i\int \frac{d^2q\, d\omega}{(2\pi)^3} U_{1111}(\vec{q},\omega) \frac{1}{E-\omega-E_{10}-\xi(\vec{k}-\vec{q}) + i\delta}, \tag{4}$$

where $\xi(\vec{k}) = (k^2 - k_F^2)/2m$, $k_F$ is the Fermi wave vector, $m$ is the bulk mass for motion parallel to the surface, $E_{10}$ is the noninteracting separation between the lowest and the first-excited sub-bands, and $\delta = 0^+$. The quasiparticle energies are found from the two Dyson equations:

$$E + \mu = \xi(k) + M_{00}(k,E),$$
$$E' + \mu = E_{10} + \xi(k) + M_{11}(k,E'), \tag{5}$$

where the quasiparticle energies are measured from the chemical potential $\mu = M_{00}(k_F, 0)$.

In Fig. 1 are shown solutions of those equations for $k = k_F$. For the wave functions $\zeta_0(z)$ and $\zeta_1(z)$ and the energy separation $E_{10}$ use was made of those determined numerically by Stern.[5,12] For the cases shown the parameters used can be found in Table I, and the Hartree potential included the contribution from the interaction of an electron with its image in the insulator.

It can be seen that the correlation energy of an electron in the excited sub-band is quite large, even larger than the correlation part of the self-energy in the lowest sub-band which is about $-5$ meV, and almost does not vary with $N$. It should be noted that any static-screening approximation would give $M_{11} = 0$ as long as scattering processes in which an electron changes sub-band are neglected.

The energy separation between the two quasi-

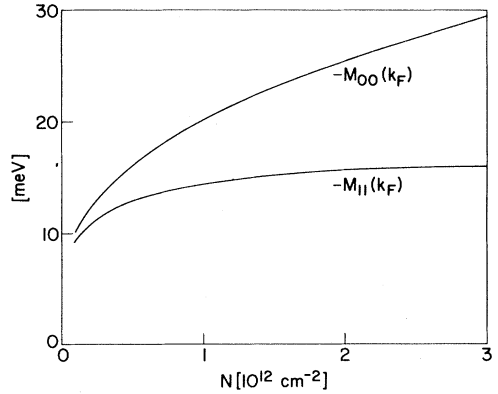


FIG. 1. The self-energy at the Fermi wave vector as a function of inversion-layer density, $M_{00}$, for electrons in the lowest sub-band, $M_{11}$, for electrons in the first-excited sub-band.

TABLE I. Parameters used in the calculations.

| | | |
|---|---|---|
| Valley degeneracy | $n_v$ | 2 |
| Mass perpendicular to surface[a] | $m_3$ | $0.916m_e$ |
| Mass parallel to surface | $m$ | $0.190m_e$ |
| Permittivity of Si | $\epsilon_s$ | $11.7\epsilon_0$ |
| Permittivity of $SiO_2$ | $\epsilon_i$ | $3.9\epsilon_0$ |
| Doping concentration[a] | $N_A-N_D$ | $7\times10^{14}$ cm$^{-3}$ |
| Depletion-layer concentration[a] | $N_{depl}$ | $1.01\times10^{11}$ cm$^{-2}$ |

[a]Only used in the Hartree calculations, Ref. 12.

particle bands is shown in Fig. 2. Curve *a* shows $E_{10}$ calculated by Stern.[12] He calculated wave functions and one-electron energies numerically (1) with the image potential included in the Hamiltonian and (2) without the image potential in the Hamiltonian and the image energy calculated as a perturbation. The effect of the exchange interaction, which tends to compress the wave functions as opposed to the image potential, was then roughly taken into account by comparing with variational results and interpolating linearly with $z_{av}$ between the results of Eqs. (1) and (2). These corrections are about 1 or 2 meV. If the wave functions from (2) are used the self-energies change less than 1 meV and the difference between the self-energies changes only a fraction of 1 meV.

Curve *b* of Fig. 2 shows the energy separation when the self-energies have been added. The experimental points are the results of photoresistance measurements by Wheeler and Goldberg.[8] For each photon energy they saw two peaks in the resistance as a function of inversion-layer density and associated them with transitions from the lowest to the first- and second-excited subbands. It is seen that the agreement between theory and experiment is very good, and that the exchange and correlation explain the difference between experiment and the simpler Hartree approximation.

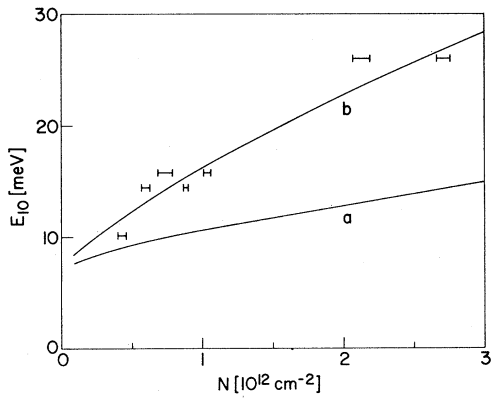


FIG. 2. Separation between lowest and first-excited sub-bands calculated (curve *a*) in the Hartree approximation (Ref. 12) and (curve *b*) including exchange and correlation. Experimental points: photoresistivity measurements (Ref. 8).

An infrared-absorption measurement on *n*-inversion layers has been performed by Kneschaurek, Kamgar, and Koch[9] on a sample with higher doping concentration but at lower densities. Results for this case give reasonable agreement with their measurements above $N=5\times10^{11}$ cm$^{-2}$.

It is important to note that sub-bands which correspond to the four valleys with a lighter mass perpendicular to the interface have minima in the Brillouin zone far from the minimum of the filled sub-band. The Coulomb interaction between an electron in one of those sub-bands with an electron in the filled sub-band is therefore very small and those sub-bands are not shifted appreciably by the electron-electron interaction. One would thus expect that they will only be populated at much higher densities or temperatures than found in the self-consistent Hartree calculations.

It has also been found that a bound exciton formed by states in the excited sub-band and states below the Fermi level in the lowest subband should be expected theoretically. The state is analogous to the exciton formed between Landau levels in Bi,[13] and the integral equation for the binding energy $E_B$ derived from the Bethe-Salpeter equation[14,15] in the static-screening approximation is

$$E_B u(\vec{p}) = \int \frac{d^2q}{(2\pi)^2} U_{1100}(\vec{p}-\vec{q},0)\theta(k_F-q)u(\vec{q}), \quad (6)$$

where it has been assumed that the self-energies are independent of momentum, i.e., that the

quasiparticle bands are parallel.

Numerical solutions of Eq. (6) give a binding energy of 0.9 meV at $N = 10^{12}$ cm$^{-2}$ increasing with $N$ to 1.7 meV at $3\times10^{12}$ cm$^{-2}$. Since the static approximation is quite bad for the calculation of self-energies one probably would not expect the calculated binding energy to be very exact. It is, however, of an order of magnitude which is not entirely negligible. The energy difference between the two sets of maxima measured by Wheeler and Goldberg is considerably smaller than one would expect for the difference between the first- and second-excited sub-bands, so one might speculate whether the assignment of transitions should be changed so that the lower points correspond to exciton formation and the higher to transition from the lowest to the first-excited sub-band. But to substantiate this conjecture one would have to calculate the relative efficiencies of those processes.

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# Isotopic Composition of Cosmic-Ray Nitrogen at 1.5 GeV/amu*

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We have determined the cosmic-ray $^{15}N/(^{14}N+^{15}N)$ ratio with a detector sensitive to velocities of particles in the penumbra of Earth's magnetic field. Our instrument was flown twice on high-altitude balloons from Palestine, Texas, obtaining an exposure factor of 20 m$^2$ sr h. We measure the fraction of $^{15}N$ to be $0.45\pm0.07$ at 1.5 GeV/amu consistent with an origin for nitrogen primarily due to spallation in the interstellar medium.

Knowledge of the isotopic composition of cosmic-ray nuclei is of fundamental importance in understanding their origin and history. To date much has been learned about elemental abundances but information on specific isotopic abundances has been limited to a few elements at low energies [up to a few hundred MeV per amu). It is isotopic, rather than elemental, abundance measurements that place the strongest constraints on the models of galactic confinement and propagation and on the characteristics of the sources. Also, isotope results at higher energies ($\gtrsim$ 1 GeV/amu) are little influenced by extrapolation out of the solar-modulation region and benefit from the energy independence of the spallation cross sections and negligible interstellar-ionization loss of the particles. A method applicable for higher energies and using the geomagnetic field has been proposed and extensively developed by Peters[1] and his co-workers. At a given location, the geomagnetic field transmits all particles above a certain rigidity $R_{max}$ and none below another rigidity $R_{min}$. Between these two values lies the rigidity range called the penumbra. This method involves comparing velocity spectra in the penumbra of two elements, one considered a