ing the perturbation relax it has been verified that the results are not caused by rf rectification on the probe sheath.

The above described experimental observations may support the following qualitative physical picture of the nonlinear interactions: The large rf field in the vicinity of the antenna generates fast electrons by transit-time and collisional effects. Since tails up to 15 eV are directly observable it is possible that a small fraction ($<10^{-3}$) of fast electrons is present which resonate with the wave $[(\omega_c - \omega)/k_{||} = v_{||} \simeq 3\times 10^7$ cm/sec in Fig. 3; $\frac{1}{2} mv_{||}^2 = 27$ eV$]$. Trapped-particle effects[3] would give rise to a periodic amplitude behavior near the exciter with subsequent rapid phase mixing and formation of a constant-amplitude pattern consistent with the observation.

As the trapped particles are confined along $\vec{B}_0$ the nonlinear effects are field aligned and the resulting constant-amplitude pattern is reminiscent of whistler-wave ducting. The present nonlinear ducting mechanism due to wave-particle interactions is however different from the familiar ionospheric ducting[1] involving wave refraction in gentle field-aligned density perturbations. Although as a result of the temperature increase in the nonlinear ducts a density depression ($\delta n/n \gtrsim 10\%$) is also present, it is not believed to contribute significantly to the ducting process for two reasons: First, the ducts can be very narrow (diameter $\lesssim \lambda_{||}/2$) without observing cutoff effects; second, by producing similar density depressions with obstacles in the plasma no ducting is seen.

The observed temperature anisotropy also contributes to a decrease in wave damping but is not sufficiently high for marginal stability[5] $[(T_{e\perp}/T_{e||})_{\gamma=0} = (1-\omega/\omega_c)^{-1} \simeq 3.7]$.

It should be pointed out that the ducted whistler waves have longitudinal field components. This fact complicates the comparison with available theories on large-amplitude whistlers which to my knowledge only consider transverse modes.

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# Properties of Nonneutral Plasma*

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We describe an apparatus for producing a magnetized column of nonneutral electron plasma which is many Debye lengths in radius. The plasma exhibits the linear and nonlinear electron-wave effects observed in neutralized plasmas.

In recent years increasing research has been devoted to the equilibrium and stability properties of plasmas for which all particles have the same sign of electric charge.[1,2] It is known theoretically that such nonneutral plasmas exhibit shielding on the scale of their Debye length[3] and collective effects like plasma waves.[4] We here describe an apparatus which generates a magnetized column of electron gas which is a nonneutral plasma by the criterion that the Debye length is small compared to the radius of the column. We also report experimental verification of the properties of waves in this plasma. Except for a slow rotation, the plasma is at rest in the laboratory frame of reference. This fact distinguishes the present plasma from that obtained in electronbeam experiments; for wave experiments this distinction is crucial.[5]

The source of the electrons is a thermionic cathode. The essential feature of the apparatus is that this cathode is *not* an equipotential surface: The potential is a function of radius. It is this boundary condition which compensates for space-charge effects in the plasma and permits its radius, measured in Debye lengths, to be large.[6] The geometry of the experiment is shown in Fig. 1. Electrons are emitted from the cathode, pass through a grid, and enter a grounded cylindrical tube. The end plate is biased strongly negative (− 100 V) to reflect the electrons. The system is evacuated ($2\times10^{-6}$ Torr), and is immersed in a strong uniform axial magnetic field.

The cathode is a directly heated flat spiral of tungsten wire (six turns of 0.5-mm-diam No. 218 tungsten with a 3.5-mm gap between turns). The 13.0-A heater current for this cathode, which gives an Ohmic drop of 21.5 V, flows inward so that the center of the cathode is more negative than the outside. A separate power supply biases the center of the cathode 44.5 V negative with respect to the grounded tube.

It is not obvious *a priori* that this apparatus will produce a nonneutral plasma. It is easy to invent hypothetical effects that would prevent the apparatus from working as desired, and the real situation is too complicated to calculate all the physics from theory. The simple model which we present here should be considered a guess which was confirmed by the experimental results. Consider the number density $n(r)$ in the central section of the cylindrical tube, and the corresponding space-charge potential $\varphi_s(r)$. If the injection current is large enough, a self-consistent solution for $n(r)$ and $\varphi_s(r)$ requires that most of the entering electrons be reflected near the entrance grid. The electrons are thermally emitted from a cathode with potential $\varphi_k(r)$. Since the cathode temperature corresponds to an electron-energy spread of only 0.14 eV, small changes in $\varphi_k(r)-\varphi_s(r)$ can control the reflection coefficient, and thus $n(r)$, over a wide range. Experimentally we find that $\varphi_k(r)\simeq\varphi_s(r)$ as expected from this argument. To lowest order in radial variation the cathode potential may be written

$$\varphi_k(r)=V_c+V_f r^2/f^2, \tag{1}$$

where $V_c$ is the bias applied to the center of the cathode, $V_f$ is the voltage drop across the cathode, $f$ is the cathode radius, and $r$ is the distance from the axis of symmetry.

The functional form of Eq. (1) matches the $\varphi_s$ of a constant-density electron gas. For an infinitely long cylinder of electrons of radius $a$ and constant density $n$, surrounded by a grounded cylindrical conductor of inner radius $b$, the potential is

$$\varphi_s(r)\big|_{r\le a}=\frac{e}{4\pi\epsilon_0}(n\pi a^2)\left[2\ln\left(\frac{b}{a}\right)+1-\frac{r^2}{a^2}\right]. \tag{2}$$

Letting $\varphi_k(r)=\varphi_s(r)$ yields

$$n=(4\epsilon_0/|e|)V_f/f^2 \tag{3}$$

and

$$a^2[2\ln(b/a)+1]=f^2|V_c|/V_f. \tag{4}$$

Equation (3) states that the number density is determined by the cathode parameters. Equation (4) states that for given cathode parameters the radius of the plasma is determined by the bias voltage, $V_c$.

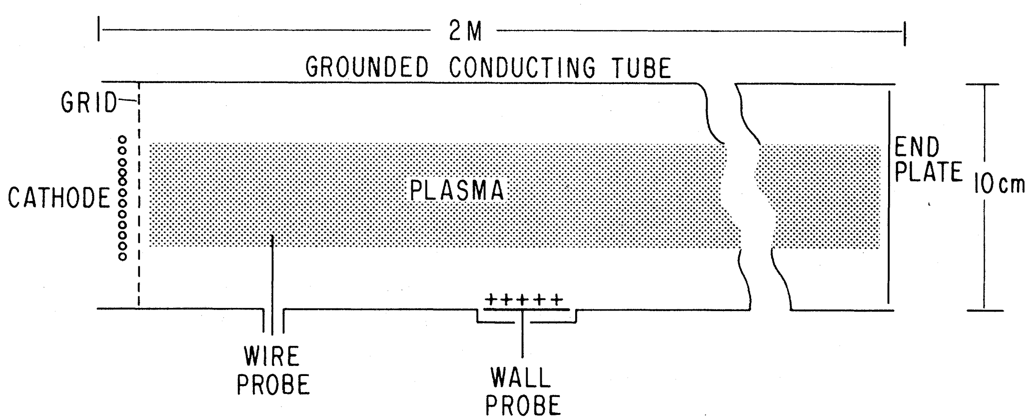


FIG. 1. Schematic of the apparatus.

As the bias $V_c$ is adjusted to be more negative, the plasma responds by increasing its radius. When the plasma has grown to the radial size of the filament no more electrons can be added. The line density at this bias is given by $N_{\max} \equiv \pi f^2 n = (-4\pi\epsilon_0/e)V_f$. An easy manipulation yields

$$f/\lambda_D = (-eV_f/kT)^{1/2} \qquad (5)$$

for the maximum plasma radius measured in Debye lengths, $\lambda_D \equiv (\epsilon_0 kT/e^2 n)^{1/2}$, where $T$ is the plasma temperature. Equation (5) states that we may make the plasma radius large (measured in Debye lengths) simply by making $V_f \gg -kT/e$.[7]

The radial density profile has been investigated experimentally by drawing a current to the wire probes indicated schematically in Fig. 1. The current to the probe as a function of radial probe insertion gives a qualitative measure of the plasma profile. As expected from Eqs. (3) and (4), making the cathode bias more negative increases the radius of the plasma, not its number density. Quantitative measurements of the line density, $N$, are made by use of wall probes.[8] Since there is a net electric charge in the tube, an equal and opposite image charge resides on the wall. When the plasma is removed from the tube by gating the grid negative to cut off the entering electrons and gating the end plate positive to allow electrons to escape along the magnetic field lines, then this image charge appears across the distributed capacitance of the wall probe and its cabling. The resulting voltage pulse is measured with an oscilloscope and the capacitance is measured with a standard bridge providing an absolute measure of the net charge in the system to an accuracy of 6%. The measurement depends only on Gauss's law and symmetry; the latter was verified experimentally by comparison between the results for various wall probes in the system.

The system has also been investigated by use of electron plasma waves. A transmitter is connected to one wire probe and a receiver to a second wire probe which may be moved in the longitudinal direction. By measuring the phase and amplitude of the received signal as a function of receiver position we may deduce the wave number and damping coefficient of the waves at a series of frequencies.[9] Additional dispersion data are obtained by measuring peaks in the noise voltage on a probe as a function of frequency. At low frequencies Landau damping is negligible and the noise peaks correspond to standing waves. These observations, combined with a knowledge of machine length, are an additional measure of the low-frequency part of the dispersion.[10] The measured dispersion data are given in Fig. 2.

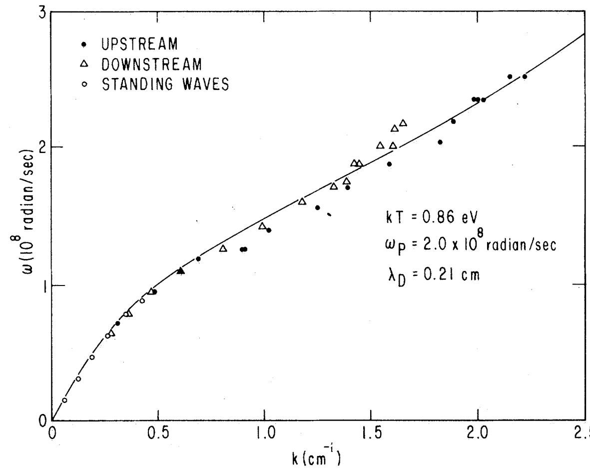


FIG. 2. Electron-plasma-wave dispersion.

Davidson and Krall[4] have shown that the dispersion of the angularly symmetric modes considered here may be calculated by use of the theory of Landau.[11,12] The curve in Fig. 2 shows the dispersion calculated numerically,[9] under the assumption that the velocity distribution is Maxwellian with temperature independent of radius. The calculation used the relative radial density profile measured with the wire probes and the net line density measured by the Gauss's-law technique with the wall probes [$1.76\times10^{10}$ (electron charges)/m]. The curve is the best eyeball[13] fit obtained by adjusting only the temperature, which primarily affects the dispersion at the larger wave numbers.

The waves would be Doppler shifted by any net drift of the plasma. The fact that the wave numbers of the upstream and downstream waves are identical demonstrates experimentally that the mean velocity in the longitudinal direction is zero.[5] The fact that the dispersion is accurately fitted with use of the line density as measured by the wall probes proves that the plasma is unneutralized. This Gauss's-law technique measures the algebraic sum of the charges in the system. The plasma-wave dispersion measures the total electron density since the ion response at these frequencies is negligible. Consideration of the precision of these measurements indicates that the ion neutralization is less than 6%.

The temperature measured by wave dispersion is 0.84 eV. Since the temperature and the density are both known, we can calculate the Debye length in the plasma. It is 0.21 cm. The wire-probe measurements give the half-maximum radius of the density profile as 2 cm and so this unneutralized plasma is about 10 Debye lengths in radius. The dispersion temperature of 0.86 eV is 6 times the 0.14-eV cathode temperature and it is interesting to inquire as to the origin of the discrepancy. It is at this point that details of the cathode geometry become important. The electrons (energy spread 24 to 44 eV with respect to ground) are almost stopped by the space charge potential in the tube. Small relative differences between the potential of a point on the imprecisely wound spiral cathode and the space-charge potential in the plasma at the corresponding radius result in a velocity spread which is large compared to that due to cathode temperature. If the spiral is deliberately distorted, we see a large increase in the plasma temperature as determined by electron plasma waves. Because the "temperature" seen by the plasma waves is mostly due to potential mismatches between the cathode and the plasma, fitting the dispersion data under the assumption that the plasma distribution function is Maxwellian is an approximation. For small wave numbers the distinction is not important. At the larger wave numbers, where the wave phase velocity is close to the velocity of some of the electrons, the details of the velocity distribution function do make some difference. The scatter of the large-wave-number data (shown in Fig. 2) is real and reflects the non-Maxwellian details of the velocity distribution function.

We have observed collisionless damping of the waves. The observed damping length is much too short to be explained by collisions and, in addition, the wave damping exhibits the spectacular dependence on phase velocity expected from the Landau[11] mechanism. The details of these damping measurements also show that the velocity distribution function is not Maxwellian.

With the transmitter amplitude sufficiently large we observe the well-known spatial oscillations of wave amplitude[14,15] due to electrons trapped in the wave. This effect has not yet been studied in detail in this plasma. As would be expected from the fact that the electrons are all reflected by the electric field due to the end plate, we see sheath echoes[16] in the apparatus. The echoes are large and their identification is completely unambiguous.

The noise level in the plasma is also of interest. Variations in plasma density can be measured by observing fluctuations in the phase velocity of a fixed-frequency electron plasma wave. At present we can only give an upper bound to these fluctuations: $\Delta n/n < 0.6\%$.

In summary, a magnetized column of unneutralized electron plasma many Debye lengths in radius is obtained when an appropriate radial potential is applied to the cathode. This plasma exhibits the classical linear and nonlinear electron-wave effects observed in neutralized plasmas.

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[6]A somewhat related scheme used with equipotential cathodes is to alter the magnetic field at the cathode. See, for example, L. Brillouin, Phys. Rev. 67, 260 (1945); Theiss, Ref. 5.

[7]Of course, this result depends directly on the choice of a parabolic radial potential profile on the cathode. For the equipotential-cathode case the maximum obtainable radius is $\sqrt{8}$ central Debye lengths. This result will be given elsewhere: C. F. Driscoll and J. H. Malmberg, to be published.

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[9]These are standard techniques. For further details see, for example, J. H. Malmberg and C. B. Wharton, Phys. Rev. Lett. 17, 175 (1964).

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