tion; thus these averaged dynamic parameters are not necessarily comparable in a direct way to intrinsic static parameters in odd nuclei. The static core deformations (β) deduced for the present Sb rotational bands, which are larger than the β_{rms} of the even-Te core nuclei, are strongly influenced by the $1g_{9/2}$ proton hole (the Sb single-particle states show no rotational bands). Dynamic aspects of the triaxial-rotor model, namely soft vibrations in the β and γ parameters about their static values, may also be important for the present results. Further theoretical investigations of the band-spacing systematics in terms of the deformation shapes including dynamic solutions are needed for this closed-shell region.

The energies of the deformed $\frac{9}{2}^+$ states (band heads) determined in the present experiment are 1461, 1380, 1160, and 971 keV in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb, respectively. These deformed-state energies involve the potential energy of the core and of the $j=\frac{9}{2}^+$ state both of which depend sensitively on the deformation shape, the vibrational zero-point contributions, and pairing corrections. An understanding of the energy behavior of these states in terms of the deformation-energy surfaces of the core as a function of neutron number represents an interesting theoretical challenge for this $Z=50$ closed-shell region. An extension of the Strutinsky-type calculation[4] to these nuclei would be an interesting first step.

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†Permanent address: State University of New York at Binghamton, Binghamton, N. Y. 13901.

‡Max Kade Fellow—University of Erlangen, Erlangen, Germany.

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Probing the Deuteron Wave Function with Sub-Coulomb (d,p) Reactions

L. D. Knutson* and W. Haeberli
University of Wisconsin,† Madison, Wisconsin 53706
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A new method is proposed to obtain information about η, the asymptotic D-state–to–S-state ratio for the deuteron. The method is based on the fact that polarized-beam measurements for sub-Coulomb (d,p) reactions are sensitive to the deuteron D state.

In this Letter we wish to point out that one can obtain new quantitative information about the internal wave function of the deuteron from polarized-beam studies of (d,p) stripping reactions on heavy nuclei. We shall primarily be concerned with measurements of the three tensor analyzing powers,[1] T_{20}, T_{21}, and T_{22}. Recent papers[2,3] have shown that these quantities are sensitive to the D state of the deuteron. We propose that such measurements can be used to obtain quantitative information about the D-state wave function.

The reliability of this information will, of course, be limited by the accuracy of the theory used to relate the measured analyzing powers to the deuteron wave function. In general, nuclear-reaction calculations are not accurate in a quantitative sense. However, the predictions of existing direct-reaction theories can be quite accurate provided that the reactions are carried out at energies below the Coulomb barrier. Calculations are reliable in this special case because the Coulomb repulsion causes the reactions to take place outside the nucleus.[4]

According to Johnson and Santos,[5] the magnitude of the deuteron D-state effect in a (d,p) reaction is determined by a single parameter (D_2) whose value depends on the nature of the deuteron internal wave function.[6] It is this parameter which can be determined from the tensor-analyzing-power measurements.

We shall begin by discussing the relationship between D_2 and the deuteron wave function. To illustrate how one can determine D_2 from the tensor analyzing powers, we analyze existing measurements for the reaction $^{208}\mathrm{Pb}(d,p)^{209}\mathrm{Pb}$ at 9 MeV.[7] Finally we shall discuss how the measurements and analysis must be improved if one is to determine the value of D_2 with high precision.

Using the notation of Blatt and Weisskopf[3] the deuteron wave function can be written in the form

$$\varphi_d^{\,m}(\vec{\rho}) = \rho^{-1}[u(\rho)Y_{101}{}^m + w(\rho)Y_{121}{}^m]. \tag{1}$$

Here the functions u and w are the radial wave functions for the S state and D state, respectively, and $\vec{\rho}$ is the neutron-proton separation. By using Eqs. (18), (36), and (41) of Ref. 5, one can show that the parameter D_2 is related to the functions u and w by

$$D_2 = \tfrac{1}{15}\int_0^\infty \rho^3 w(\rho)\,d\rho \Big/ \int_0^\infty \rho\, u(\rho)\,d\rho. \tag{2}$$

Figure 1 shows the radial dependence of the function $\rho^3 w$ which appears in the integral for D_2. Also shown are the functions w^2 and $\rho^2[u(\rho)w(\rho) - w^2(\rho)/\sqrt{8}]$ whose integrals are proportional to the deuteron D-state probability (P_D) and quadrupole moment (Q), respectively. One sees that the values of Q and P_D are sensitive to the magnitude of w at relatively small values of ρ whereas D_2 is sensitive to w at large ρ. Thus D_2 is not simply related to either P_D or Q.

Since the D_2 integrand is small for small values of ρ one can estimate D_2 by using asymptotic expressions[8] for u and w. Outside the range of the n-p potential

$$\begin{aligned} &u(\rho) \to Ne^{-\alpha\rho}, \\ &w(\rho) \to \eta N e^{-\alpha\rho}[1 + 3/(\alpha\rho) + 3/(\alpha\rho)^2], \end{aligned} \tag{3}$$

where η and N are normalization constants, and where $\hbar^2\alpha^2/M$ is the deuteron binding energy. Substitution of Eq. (3) into Eq. (2) gives

$$D_2 \simeq \eta/\alpha^2. \tag{4}$$

Thus D_2 is closely related to η, the asymptotic D-state-to-S-state ratio. The approximation given in Eq. (4) is very accurate, typically overestimating the exact values of D_2 [as calculated from Eq. (2)] by about 1%. This emphasizes that D_2 is sensitive only to the tail of the wave function.

In Table I we present values of D_2 calculated from Eq. (2) for a number of deuteron wave functions.[9-13] The first three entries in the table are wave functions derived from nucleon-nucleon potentials which reproduce the n-p elastic scattering data. The remaining wave functions reproduce the triplet effective range and scattering length, the deuteron binding energy, and the quadrupole moment.

The calculations presented in this Letter are based on the distorted-wave Born approximation (DWBA). A brief comment on the accuracy of this approximation is appropriate. In order to obtain the usual DWBA expressions it is necessary

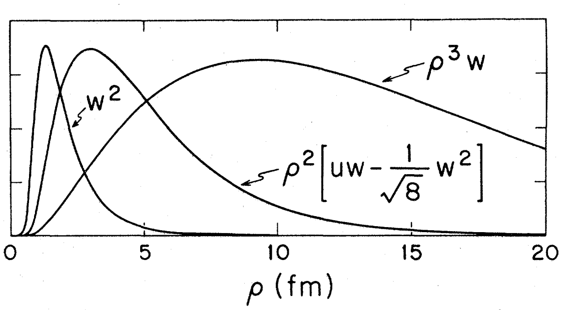

FIG. 1. Radial dependence of the functions w^2, $\rho^3 w$, and $\rho^2(uw - w^2/\sqrt{8})$. The vertical scale is arbitrary. The functions shown were calculated from the deuteron wave function of Reid (Ref. 11).

TABLE I. Values of D_2 and P_D calculated from various deuteron wave functions. The first three entries are calculated from nucleon-nucleon potentials which reproduce the n-p elastic scattering data.

Wave function	Ref.	P_D (%)	D_2 (fm^2)
Hamada-Johnston	9	6.9	0.487
Holinde *et al.*	10	5.7	0.456
Reid soft-core	11	6.5	0.484
Hulthen-Sugawara	12	3.0	0.532
Hulthen-Sugawara	12	5.0	0.466
Yamaguchi	13	4.0	0.524

to make two approximations to the exact quantum mechanical theory.[14] In the first approximation one replaces the exact quantum mechanical wave function for the $d+{}^{208}\mathrm{Pb}$ system (Ψ_{dA} in the notation of Ref. 14) by a simple optical-model wave function. In doing so, one has neglected the inelastic scattering and reaction channels in Ψ_{dA}.[15] Secondly, one sets to zero the nuclear potential term

$$\sum_{i=1}^{A} v_{ip} - U_p.$$

Here U_p is the proton optical-model potential and v_{ip} is the potential between the outgoing proton and an individual nucleon in the target.

We argue that for sub-Coulomb reactions, DWBA is a good approximation to the exact theory. Neglecting the reaction channels introduces little error since the elastic scattering term in Ψ_{dA} dominates the reaction terms. It is also reasonable to neglect the nuclear-potential term since, for sub-Coulomb energies, the scattering wave functions are extremely small in magnitude inside the nucleus where the potentials differ from zero. Goldfarb has demonstrated[4] that for sub-Coulomb (d,p) reactions with Q values near zero, the neutron transfer takes place outside the nuclear surface. In this region, the deuteron and proton optical-model wave functions and the wave function of the bound neutron are known accurately, even if the corresponding nuclear potentials are not well known. Thus, one expects the DWBA predictions to be accurate for sub-Coulomb reactions with $Q \simeq 0$.

In the DWBA calculations we make use of the local-energy approximation[5] (since no suitable finite-range code was available). By extending the calculations of Goldfarb and Parry[16] to include the deuteron D state, one can show that use of the local-energy approximation changes the calculated tensor analyzing powers by at most a few percent for sub-Coulomb reactions with Q values near zero.

Measured tensor analyzing powers[2] for two $Q \simeq 0$ transitions in ${}^{208}\mathrm{Pb}(d,p){}^{209}\mathrm{Pb}$ at a deuteron energy of 9 MeV are shown in Fig. 2. The solid curves in the figure are DWBA calculations[17] obtained with $D_2 = 0.484$ fm^2, which is the value calculated from the Reid[11] wave function. These calculations are consistently larger in magnitude than the measurements. The best fit to the measurements is obtained with $D_2 = 0.432$ fm^2. The calculations for this value of D_2 are given by the dotted curves. We estimate that the uncertainty

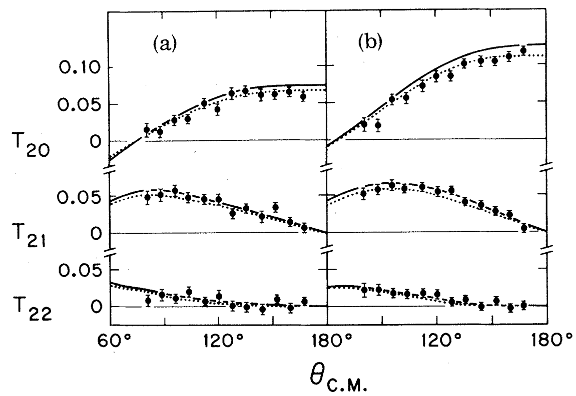

FIG. 2. Angular distributions of the tensor analyzing powers T_{20}, T_{21}, and T_{22} for the reaction ${}^{208}\mathrm{Pb}(d,p){}^{209}\mathrm{Pb}$ at 9 MeV leading to states with (a) $J^\pi = \frac{1}{2}^+$, $E_x = 2.03$ MeV and (b) $J^\pi = \frac{5}{2}^+$, $E_x = 1.57$ MeV. The displayed errors are statistical only. The curves show the result of DWBA calculations which include the effects of the deuteron D state. The solid curves were obtained with $D_2 = 0.484$ fm^2. The dotted curves show the best fit to the measurements, which was obtained with $D_2 = 0.432$ fm^2.

in the deduced value of D_2 is ± 0.032 fm^2. This estimate includes contributions from the statistical errors, from an overall normalization uncertainty in the measurements, and from uncertainties in the nuclear optical-model potentials.

The value of D_2 obtained from the measurements is significantly smaller than the values calculated from existing deuteron wave functions[18] (see Table I). While the results of the present analysis are not entirely conclusive, it would appear that experiments similar to the one described here can provide important new information about the deuteron wave function. We believe that with certain refinements in the method, it should be possible to reduce the relative uncertainty in D_2 to 3–4%.

If this level of accuracy is to be achieved several improvements in the measurements and calculations will be required. First of all, it will be necessary to improve the accuracy of the tensor-analyzing-power measurements. Here it will be particularly important to eliminate overall normalization errors in the measurements. Secondly, it is important that the measurements be obtained at a lower deuteron energy. At $E_d = 7$ MeV, for example, the differential cross sections are sufficiently large (about 1 mb/sr at back angles) that the measurements can be obtained without great difficulty, and, in addition, the sensitivity of the calculated tensor analyzing powers to the nuclear interactions is greatly reduced.

In a more detailed analysis of the measurements, one would presumably carry out a full finite-range DWBA calculation. In addition, it will be important to take into account the effects of the approximations which are used in the DWBA calculations. In particular, it will be important to show that the two-step process in which the deuteron is distorted or broken up and the neutron is subsequently captured has little effect on the tensor analyzing powers. Of course, such questions are quite difficult to answer, even for reactions which are essentially free from nuclear interactions. However, one is encouraged by the fact that the simple DWBA calculations correctly predict the angular dependence of all three tensor analyzing powers. This suggests that the corrections to the DWBA results will not be large.

*Present address: Nuclear Physics Laboratory, University of Washington, Seattle, Wash. 98195.

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[1]The tensor analyzing powers are a measure of the change in cross section which results when the incident deuteron beam is aligned or tensor polarized (i.e., when the population of the $m=0$ magnetic substate is different from $\frac{1}{3}$). The analyzing powers are defined according to the Madison Convention as given in *Polarization Phenomena in Nuclear Reactions,* edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1971), p. xxv.

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[17]The optical-model potentials used in the calculations are given in Ref. 2.

[18]Using the method of Ref. 16, we have estimated the change in the tensor analyzing powers which results from the use of the local-energy approximation. The sign of this change is such that the discrepancy between the measurements and the predictions obtained from the existing deuteron wave functions would be even greater if one carried out a full finite-range calculation.