65, 1 (1965); D. R. Bès and C. Yi-Chang, Nucl. Phys. 86, 581 (1966).
[5]J. M. Moss, C. M. Rozsa, J. D. Bronson, and D. H. Youngblood, Phys. Lett. 53B, 51 (1974).
[6]D. J. Horen, F. E. Bertrand, and M. B. Lewis, Phys. Rev. C 9, 1607 (1974); D. J. Horen, J. Arvieux, M. Buenerd, J. Cole, G. Perrin, and P. de Saintignon, Phys. Rev. C 11, 1247 (1975).
[7]A. Schwierczinski, R. Frey, E. Spamer, H. Theissson, and Th. Walcher, Phys. Lett. 55B, 171 (1975).
[8]G. E. Brown, *Many-Body Problems* (North-Holland, Amsterdam, The Netherlands, 1972).
[9]T. Udagawa and R. K. Sheline, Phys. Rev. 147, 671 (1966).

Deformed $\frac{9}{2}^+$ States and $\Delta J = 1$ Rotational Bands in $^{113, 115, 117, 119}$Sb Nuclei*

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A set of $\Delta J = 1$ rotational bands has been observed on deformed (prolate) $\frac{9}{2}^+$ states in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb via (^{6}Li, $3n\gamma$) reactions on stable even Cd targets. The pattern of band spacings is suggestive of a $j = \frac{9}{2}^+$ state coupled to a triaxial rotor. The energy systematics of the deformed states relate to the deformation-energy surfaces in this $Z = 50$ closed-shell region.

The large stable deformations occurring for nuclei in the $150 < A < 190$ and $A > 220$ regions which are far from closed shells have been understood by minimizing the sum of Nilsson deformed-single-particle energies as a function of deformation[1] together with Coulomb and pairing corrections.[2] Recently there has been considerable theoretical work[3,4] aimed at mapping these deformation-energy surfaces into other nuclear regions. The most recent calculations[4] employ the Strutinsky prescription[5] to renormalize the deformation-energy trends of deformed Woods-Saxon single-particle states to that of a liquid drop. For nuclear regions such as $A \approx 100$ and $A \approx 140$ away from closed shells, energy minima at both prolate and oblate deformations including asymmetric shapes are predicted.[4] The closed-shell regions, however, where nuclei are expected to be spherical have not been theoretically explored with care in regard to the coexistence of deformations. In this Letter, we report the observation, via (^{6}Li, $3n\gamma$) reactions on stable even Cd isotopes, of a set of $\Delta J = 1$ rotational bands built on deformed $\frac{9}{2}^+$ states in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb which are single-proton nuclei relative to the $Z = 50$ closed proton shell. These rotational bands, which for ^{115}Sb include seven members up to $J = \frac{21}{2}^+$, contain the features of a $1g_{9/2}$ proton hole strongly coupled to prolate core deformation.[6,7] The energy spacings in the bands are suggestive of axially asymmetric rotors of triaxial shapes.[7] Results from a previous ^{115}In(α, $2n$)^{117}Sb experiment showed part of the band in ^{117}Sb for which a symmetric rotational interpretation was given.[8] The systematic behavior of these deformed states and the related rotational-band spacings gives a very sensitive probe of the core-deformation-energy surfaces and the nuclear shapes as a function of neutron number. A theoretical understanding of the coexistence of these deformed properties with the known spherical properties is important. Deformation properties including possible triaxial shapes for the $Z = 82$ closed-shell region have been discussed from experimental band information in proton-hole Tl nuclei.[7,9]

To study the excited states of the odd Sb isotopes, (^{6}Li, $3n$) reactions on isotopically enriched even Cd targets were employed. These fusion-evaporation reactions[10] populate high-spin states in $\Delta Z \leq 3$ nuclei with large alignment. The γ-ray decay modes are predominantly stretched cascades $J \to J - L$ whose angular distributions are characteristic of the multipolarity L and therefore sensitive to the angular momentum J of the states involved. Measurements on these decay γ rays are thus expected to reveal the properties of the Sb high-spin states.

In order to determine the level scheme and decay properties of the excited states in the odd Sb isotopes, the following set of experiments using Ge(Li) detectors were performed: γ excitation, γ-γ coincidence, γ angular distribution, and pulsed-beam-γ timing measurements. At energies from the Coulomb barrier to 35 MeV, the main outgoing channels of the ^{6}Li-induced reactions are $2n$, $3n$, $3np$, and $4n$. To help distinguish the γ rays associated with the odd Sb isotopes ($3n$ channel), γ-excitation measurements were made over this energy range; the γ-excitation yields contain a dependence on the exit channel. Because of the complex nature of the γ-ray spectra from these reactions, γ-γ coincidence measurements with a Ge(Li)–Ge(Li) detector combination were required to identify the γ-ray cascades; the two γ-ray energy spectra and the relative timing spectra were collected as triple events on magnetic tape. The final γ-γ coincidence data were obtained from scans of the magnetic tapes that included appropriate Compton and random subtraction. The γ-γ coincidence results also aid in the identification of the residual nuclei by establishing a connection between unknown cascade γ rays with those known from β decay or light-ion reactions. To obtain information on the spins of the levels and the γ-ray multipolarities as well as the γ-ray intensities I_λ, the γ-ray angular distributions were measured in singles at four or five angles. The γ-ray photopeak areas were extracted and fitted by $W(\theta)=I_\gamma(1+A_2P_2+A_4P_4)$. In this way the A_2 and A_4 coefficients of the Legendre polynomials and the intensities I_γ corrected for efficiency were obtained for the observed γ rays. Information on J values is obtained from the $W(\theta)$ on the assumption that the high-spin levels are populated in low-m substates and that they decay via stretched $J \rightarrow J-L$ γ transitions.[10] The intensities I_γ are important for ordering the cascade γ rays. The pulsed-beam–γ timing measurements made with time resolutions of ~5 nsec (full width at half-maximum) yield information on lifetimes of the levels; lifetime limits are useful in considering the higher γ-ray multipolarities.

The γ-ray decay schemes deduced for the four odd Sb isotopes from the above (^{6}Li, $3n$) measurements contain γ cascades within two separate groups of levels.[11] One group appears to be made up of states formed by coupling single-proton states to the appropriate Sn core excitations while the other group is a systematic (bandlike) set of levels with a high degree of similarity for each of the four Sb isotopes. The decay schemes for the latter group are collected in Fig. 1 and represent the focus of the present Letter. The band heads decrease in energy in going from ^{113}Sb to ^{119}Sb while the pattern of level spacings in the bands is essentially identical for ^{115}Sb, ^{117}Sb, and ^{119}Sb with small deviations occurring in ^{113}Sb.

The lowest member of each of these bands is assigned $J=(\frac{9}{2}^+)$ on the basis of the angular distribution measurements which indicated stretched quadrupole transitions ($A_2\approx 0.25$, $A_4\approx -0.10$) to the $\frac{5}{2}^+$ ground states and in three cases, transitions containing large dipole components ($A_2 \approx -0.20$) to the $\frac{7}{2}^+$ first-excited states. The angular distributions of the transitions connecting the band members are consistent with $J\rightarrow J-1$ $M1$-$E2$ cascades which are corroborated by the systematic occurrence of $J\rightarrow J-2$ crossover transitions for which only an $E2$ can effectively

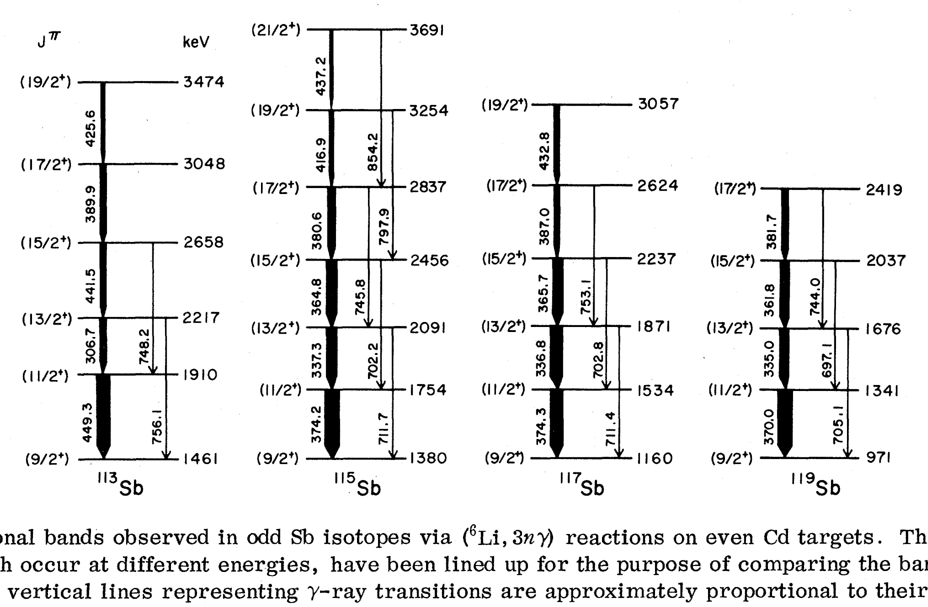

FIG. 1. Rotational bands observed in odd Sb isotopes via (^{6}Li, $3n\gamma$) reactions on even Cd targets. The $\frac{9}{2}^+$-state band heads, which occur at different energies, have been lined up for the purpose of comparing the band spacings. The widths of the vertical lines representing γ-ray transitions are approximately proportional to their intensities.

compete. Lifetime limits of $\tau \lesssim 5$ nsec for these states obtained from the pulsed-beam–γ results rule out the importance of higher multipolarities. On the basis of the above, the other band members are given strong, although not rigorous, spin assignments which increase by $\Delta J = 1$ with increasing energy as shown in Fig. 1. These spin assignments are supported by the excitation studies; in all cases, the transitions between higher members of the bands exhibited excitation functions whose maxima were shifted towards higher energy relative to those for the decay transitions of the band heads. In the ^{115}In(α, $2n$)^{117}Sb study, γ-ray and conversion-electron measurements yielded the same J assignments for the five lowest members of the ^{117}Sb band.[8]

The observed $\Delta J = 1$ bandlike levels in the odd Sb isotopes shown in Fig. 1 are characteristic of the rotational spectrum for a $\pi g_{9/2}$ hole orbital (two-particle, one-hole state) strongly coupled to a prolate core deformation. The level energies in the bands show deviations from the simple symmetric-rotor equation $E_J = (\hbar^2/2\mathcal{J})J(J+1)$; however, this is not surprising since Coriolis effects on the band energies can be significant. An axially asymmetric rotor involving triaxial shapes, which are theoretically the most natural deformations for this region, is another possible interpretation of these rotational bands. The energies of the bands can, of course, be influenced by small admixtures of nearby states[11] made up of proton single-particle orbitals coupled to the phonon excitations of the Sn cores. Using the above rotational equation, the values of $\hbar^2/2\mathcal{J}$ vary from ~35 keV at the bottom of the bands down to ~20 keV at the top for ^{115}Sb, ^{117}Sb, and ^{119}Sb while in ^{113}Sb the values are ~15% larger. For a symmetric deformation, the band head would be described by the [404] $\frac{9}{2}^+$ Nilsson wave function; this state for protons rises above the Fermi level at a deformation of $\beta \approx 0.2$.[1] Fromm *et al.*[8] have interpreted their ^{117}Sb band information in this way.

A rotational interpretation is also implied by the intensity ratios of the $J \rightarrow J-1$ γ-ray cascade to $J \rightarrow J-2$ crossover transitions observed in the four bands in that the values of $|(g_{sp} - g_R)/Q_0|$ deduced from these ratios are approximately equal. This is expected for a high-j state plus rotor for both symmetric and asymmetric shapes.[1,7] In addition, the observed $J \rightarrow J-1$ angular distribution coefficients are consistent with the $M1$-$E2$ mixing ratios predicted by the rotational interpretation; a positive value of δ is obtained for all transitions which implies positive $(g_{sp} - g_R)/Q_0$ and prolate deformations. Evaluating g_{sp} for the [404] $\frac{9}{2}^+$ symmetric state (namely, g_K), an average $Q_0 = +3.0$ b is deduced from the above γ-ray results which yields a prolate deformation of $\beta \approx 0.2$.

Although the results discussed above document rotational spectra, they do not clearly differentiate between symmetric and asymmetric rotors. The systematics observed in the present experiment for the band spacings, however, are suggestive of axially asymmetric rotors of triaxial shapes. The $j+1$ and $j+2$ levels, namely the $\frac{11}{2}^+$ and $\frac{13}{2}^+$ members, are nearer each other than the other members for all four Sb isotopes; the precision with which this pattern repeats itself for ^{115}Sb, ^{117}Sb, and ^{119}Sb is phenomenal. This approach of the $j+1$ and $j+2$ levels is a characteristic feature of a triaxial rotor with an asymmetry parameter of $\gamma \approx 25°$.[7] Admixtures of the particle-plus-phonon states appear to be rather unimportant for this feature since their energy relative to the rotational band varies considerably over the four isotopes.[11] Since the closed-shell Sn cores are known to be soft vibrators which are theoretically equivalent to triaxial rotors with broad distributions in γ centered at $\gamma = 30°$, the asymmetric triaxial shape is not an unreasonable description of the deformations; a symmetric shape is not expected theoretically.[7]

A calculation of these odd Sb bands has been made by Meyer ter Vehn[12] with a static triaxial-rotor–plus–particle model which includes an unattenuated Coriolis interaction. A good fit of the band energies in ^{115}Sb, ^{117}Sb, and ^{119}Sb including the $\frac{11}{2}^+$ and $\frac{13}{2}^+$ members was obtained for a static asymmetric parameter $\gamma = 20°$, but not for $\gamma = 0°$, the symmetric rotor. The value of the deformation parameter required by this best fit was $\beta = 0.317$, an unexpectedly large value. These triaxial-rotor parameters are consistent with rotational 2^+ states in the even Te core nuclei which would have energies in this rotor model of less than half the experimentally observed energies. A summary of experimental and theoretical work on time-averaged quadrupole shapes of even isotopes in this region[13] gives $\beta_{\rm rms}$ parameters for the Te nuclei that are less than 0.20 and about 0.11 for the Sn nuclei; the averaged γ parameters given for these nuclei are between 20° and 30°. For the even nuclei, however, the quadrupole-shape contributions include vibrations (broad distributions in γ with an average of 30°) and admixtures of rotations of a static deformation; thus these averaged dynamic parameters are not necessarily comparable in a direct way to intrinsic static parameters in odd nuclei. The static core deformations (β) deduced for the present Sb rotational bands, which are larger than the β_{rms} of the even-Te core nuclei, are strongly influenced by the $1g_{9/2}$ proton hole (the Sb single-particle states show no rotational bands). Dynamic aspects of the triaxial-rotor model, namely soft vibrations in the β and γ parameters about their static values, may also be important for the present results. Further theoretical investigations of the band-spacing systematics in terms of the deformation shapes including dynamic solutions are needed for this closed-shell region.

The energies of the deformed $\frac{9}{2}^+$ states (band heads) determined in the present experiment are 1461, 1380, 1160, and 971 keV in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb, respectively. These deformed-state energies involve the potential energy of the core and of the $j=\frac{9}{2}^+$ state both of which depend sensitively on the deformation shape, the vibrational zero-point contributions, and pairing corrections. An understanding of the energy behavior of these states in terms of the deformation-energy surfaces of the core as a function of neutron number represents an interesting theoretical challenge for this $Z=50$ closed-shell region. An extension of the Strutinsky-type calculation[4] to these nuclei would be an interesting first step.

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[1]B. R. Mottelson and S. G. Nilsson, K. Dan. Vidensk. Selsk., Mat.-Fys. Skr. 1, No. 8 (1959).

[2]D. Bes and Z. Szymanski, Nucl. Phys. 28, 42 (1961).

[3]D. A. Arseniev, A. Sobiczewski, and V. G. Soloviev, Nucl. Phys. A126, 15 (1969), and A139, 269 (1969).

[4]A. Faessler, J. E. Galonska, U. Gotz, and H. C. Pauli, Nucl. Phys. A230, 302 (1974).

[5]V. M. Strutinsky, Nucl. Phys. A122, 1 (1968); M. Brack, J. Damgaard, H. C. Pauli, A. S. Jensen, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. 44, 320 (1972).

[6]F. S. Stephens, Rev. Mod. Phys. 47, 43 (1975).

[7]J. Meyer ter Vehn, F. S. Stephens, and R. M. Diamond, Phys. Rev. Lett. 32, 1383 (1974); J. Meyer ter Vehn, to be published.

[8]C. Heiser, Nucl. Phys. A145, 81 (1970); W. D. Fromm, H. F. Brinkmann, F. Donau, C. Heiser, F. R. May, V. V. Paschkevich, and H. Rotter, Nucl. Phys. A243, 9 (1975).

[9]J. O. Newton, S. D. Ciriolov, F. S. Stephens, and R. M. Diamond, Nucl. Phys. A148, 593 (1970).

[10]J. O. Newton, in *Nuclear Spectroscopy and Reactions*, edited by J. Cerny (Academic, New York, 1974), p. 185; B. A. Brown, P. M. S. Lesser, and D. B. Fossan, Phys. Rev. Lett. 34, 161 (1975).

[11]R. E. Shroy, A. K. Gaigalas, G. Schatz, and D. B. Fossan, to be published.

[12]J. Meyer ter Vehn, private communication.

[13]D. Cline, University of Rochester Report No. NSRL-40A, 1971 (unpublished), and J. Phys. Soc. Jpn., Suppl. 34, 377 (1973).

Probing the Deuteron Wave Function with Sub-Coulomb (d,p) Reactions

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A new method is proposed to obtain information about η, the asymptotic D-state–to–S-state ratio for the deuteron. The method is based on the fact that polarized-beam measurements for sub-Coulomb (d,p) reactions are sensitive to the deuteron D state.

In this Letter we wish to point out that one can obtain new quantitative information about the internal wave function of the deuteron from polarized-beam studies of (d,p) stripping reactions on heavy nuclei. We shall primarily be concerned with measurements of the three tensor analyzing powers,[1] T_{20}, T_{21}, and T_{22}. Recent papers[2,3] have shown that these quantities are sensitive to the D state of the deuteron. We propose that such measurements can be used to obtain quantitative information about the D-state wave function.

The reliability of this information will, of course, be limited by the accuracy of the theory used to relate the measured analyzing powers to the deuteron wave function. In general, nuclear-reaction calculations are not accurate in a quantitative sense. However, the predictions of exist-