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Giant Quadrupole Resonance in Deformed Nuclei*

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With use of the ordinary quadrupole-quadrupole (Q-Q) interaction, a total splitting of the $K=0,1,2$ components in the giant quadrupole resonance (GQR) of about 6 MeV is predicted for deformed ^{154}Sm. A 0.8 ± 0.3-MeV broadening of the GQR is observed for ^{154}Sm relative to spherical ^{144}Sm with inelastic α scattering. A model requiring rigorous self-consistency is proposed which modifies the Q-Q interaction, reduces the predicted splitting to about 2 MeV, and removes inconsistencies in the strength of the interaction for $K=0$ and $K=2$ components in low-lying β and γ bands.

The splitting of the giant dipole resonance in deformed nuclei is a well-established phenomenon.[1] Qualitatively it may be understood in terms of either a macroscopic or a single-particle (Nilsson model) view of the nucleus and more sophisticated models have been proposed in order to quantitatively describe the numerous experimental results. There has been little reported theoretical work on the effects of deformation on the giant quadrupole resonance (GQR) however. In the harmonic-oscillator model, the high-frequency isoscalar quadrupole mode corresponds to $\Delta N=2$ particle-hole excitations. The existence of this mode has been implied by effective-charge phenomena. Following the development of Mottelson,[2] when a particle moves outside a closed-shell core it distorts the core and induces a quadrupole moment as large as that of the valence particle. When a second particle is added, an effective interaction between the particles is induced because of the distortion of the core. This effective interaction has the form of a quadrupole-quadrupole (Q-Q) interaction and is given by $V(1,2)=-\chi Q(1)\cdot Q(2)$. The strength of the interaction is

$$\chi=\chi_{\rm self}=(4\pi/5)m\bar{\omega}_0^{\,2}/\langle\textstyle\sum_i r_i^{\,2}\rangle_{\rm sph}$$

and is determined by requiring the shapes of the potential and density distributions to be the same (nuclear self-consistency). This must be the case as the average potential is generated by the nucleons themselves moving more or less independently in this potential and interacting through a short-range force. The Q-Q interaction thus derived has been applied quite successfully to low-lying quadrupole collective motion in spherical nuclei.[3] In well-deformed nuclei on the other hand different strengths are required for $K=0$ and $K=2$ to fit the data.[4] For example, $\chi_{K=0,\beta}\cong0.85\chi_{\rm self}$ and $\chi_{K=2,\gamma}=(1.4-1.5)\chi_{\rm self}$, which is indicative of some problem with the model.

Random-phase-approximation (RPA) calculations for ^{144}Sm utilizing the Q-Q interaction give both eigenfrequencies and transition probabilities for the low-lying collective 2^+ states and the GQR[5] in reasonable agreement with the experimental values obtained. In a deformed nucleus such as ^{154}Sm or ^{150}Nd one would expect three GQR components ($K=0$, 1, and 2) due to coupling with the low-lying quadrupole rotational motions. In an axially symmetric system with $\chi=\chi_{\rm self}$ the total splitting of the three peaks would be $2\sqrt{2}\,\epsilon\hbar\omega_0$, implying for ^{154}Sm a total splitting of about 6 MeV [Fig. 1(a)].

The first experimental attempts to observe splitting of the GQR revealed little if any difference[5,6] in the width or shape of the resonance between spherical and deformed Sm isotopes. More recent (e,e') experiments[7] have shown a broader GQR in ^{150}Nd than in ^{142}Nd, but only by ~2 MeV. We have undertaken further investigation of the GQR in ^{144,148,154}Sm by inelastic α scattering. A beam of 115-MeV α particles was used to bombard self-supporting metal foils enriched to >99% in the appropriate isotopes. Spectra from $E+E$ solid-state detector telescopes were recorded in

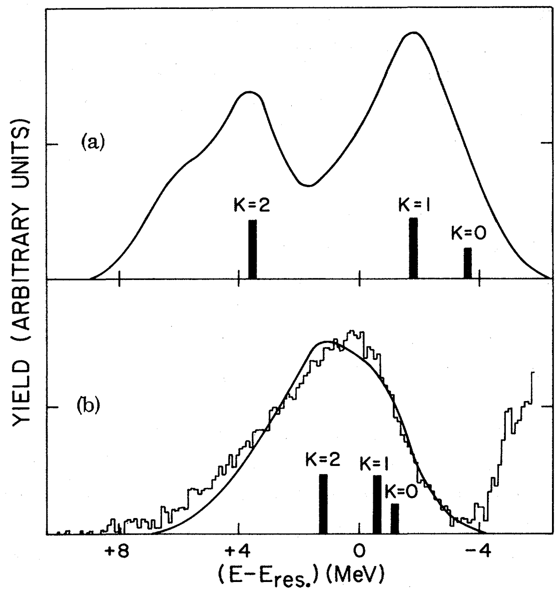

FIG. 1. The vertical lines indicate the predicted strength distribution of the GQR for ^{154}Sm using (a) the Q-Q and (b) the Q″-Q″ interaction. The solid line shows the line shape for ^{144}Sm folded into the predictions for ^{154}Sm. The histogram is the ^{154}Sm experimental data after background subtraction.

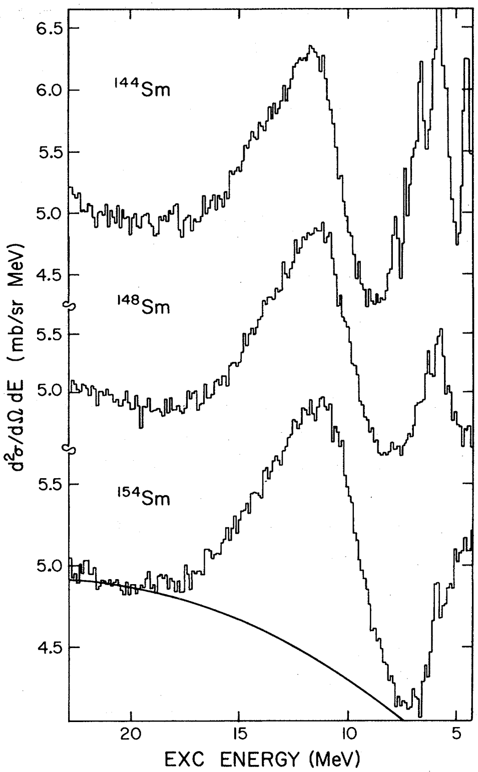

FIG. 2. Inelastic α spectra in the giant resonance region for ^{144,148,154}Sm taken at $\theta_{lab}=14°$. Oxygen contaminants have been subtracted. The background chosen for analysis is indicated by the solid line.

1° steps for $13° \leq \theta_{lab} \leq 24°$. The GQR's observed are shown in Fig. 2. It is apparent that the width systematically increases as the nucleus becomes soft and then permanently deformed. The background was determined by a least-squares fit of a fourth-order polynomial to the continuum above and below the GQR at angles where the peak is a minimum. This function was renormalized and subtracted from the data at other angles. The angular distributions are fitted reasonably well by $L=2$ distorted-wave Born-approximation calculations but small components of other even multipoles ($L=0$ or 4) cannot be ruled out. While peak yields are quite sensitive to the background chosen, significant changes in width can be generated only by assuming rather unreasonable background shapes. The widths of these asymmetric peaks were taken to be the rms deviations from the centroid (× 2.35) after background subtraction and are equivalent to the full width at half-maximum (FWHM) of the peak for a Gaussian shape. Widths and centroids determined by averaging results from at least eight spectra for each isotope are given in Table I. The errors given are the rms deviations from the mean obtained for all the data taken. The width observed for ^{154}Sm (4.7± ± 0.3 MeV) is in good agreement with the electron-scattering result for ^{150}Nd (5.0 ± 0.2 MeV). The width obtained for ^{144}Sm (3.9 ± 0.2 MeV) is consistent with the width of the ^{142}Nd GQR peak shown in Fig. 1 of Ref. 7 (3.8 ± 0.4 MeV for FWHM), but disagrees with the 2.8 ± 0.2-MeV value given by the authors. The 2.8-MeV width could be obtained for the ^{142}Nd(e,e') peak by fitting only the upper $\frac{1}{3}$ of the peak or possibly by assuming that the peak observed is actually several peaks, but the authors make no such comments. Thus for both Sm and Nd the broadening of the GQR in deformed nu-

TABLE I. Energies, widths, and energy-weighted-sum-rule (EWSR) fractions observed for the GQR in the Sm isotopes. The relative errors on the EWSR percentages are less than 10%.

Nucleus	E_x (MeV)	No. of angles	FWHM (MeV)	% $E2$ EWSR
^{144}Sm	13.03 ± 0.30	8	3.89 ± 0.19	91
^{148}Sm	12.54 ± 0.20	12	4.25 ± 0.16	104
^{154}Sm	12.36 ± 0.30	12	4.72 ± 0.25	102

clei ($\Gamma_{\rm def}-\Gamma_{\rm sph}$) appears to be ~1 MeV, considerably less than expected from the predicted 6-MeV splitting [Fig. 1(a)].

One must then investigate more carefully the nature of the effective interaction. With use of the anisotropic harmonic-oscillator model,[2] the energy of the system E is defined as

$$E=\sum_\alpha\langle\alpha|(2m)^{-1}p^2+\tfrac{1}{2}m(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)|\alpha\rangle\rho_\alpha,$$

where $|\alpha\rangle=|n_xn_yn_z\rangle_\alpha$ and $\rho_\alpha=1$ and 0 for the states below and above the Fermi energy, respectively. Using the notation $\Sigma_x\equiv\Sigma_\alpha\langle\alpha|n_x+\frac{1}{2}\alpha\rangle\rho_\alpha$, etc., one gets

$$E=\hbar\omega_x\Sigma_x+\hbar\omega_y\Sigma_y+\hbar\omega_z\Sigma_z.$$

With imposition of the saturation condition which is equivalent to volume conservation, $\omega_x\omega_y\omega_z=\overline{\omega}_0^{\,3}$, the minimization of E yields $\omega_x\Sigma_x=\omega_y\Sigma_y=\omega_z\Sigma_z$. This is the condition required for self-consistency between the density and the potential. Thus we can write

$$E=3\hbar\overline{\omega}_0(\Sigma_x\Sigma_y\Sigma_z)^{1/3}.$$

If one now employs the Landau theory[8] and uses for the single-particle energy $\epsilon_\alpha=\partial E/\partial\rho_\alpha$ and for the two-body effective interaction $\langle\alpha\beta|V|\alpha\beta\rangle=\partial^2E/\partial\rho_\alpha\partial\rho_\beta$, then one gets $V(1,2)=-\chi Q''(1)\cdot Q''(2)$, where Q'' is a quadrupole operator expressed in doubly stretched coordinates. The original coordinate x, the stretched coordinate x', and the doubly stretched coordinate x'' are related by $x'=(\omega_x/\omega_0)^{1/2}x$ and $x''=(\omega_x/\omega_0)^{1/2}x'$. In an axially symmetric case, the deformation parameter ϵ is given by $\omega_x=\omega_y=\omega_0(1+\frac{1}{3}\epsilon)$, and $\omega_z=\omega_0(1-\frac{2}{3}\epsilon)$. The strength χ is $\chi_{\rm self}$ if all configurations of $\Delta N=0$ and $\Delta N=2$ are included in the calculation. The Hamiltonian for the system then becomes $H=H_{\rm def}-\frac{1}{2}\chi Q''\cdot Q''$.

The Q″-Q″ interaction model has significant features which favor it over the Q-Q interaction model in a deformed system.

(1) The Q-Q interaction is derived in a spherical system and its deformed Hartree field fails to satisfy the saturation condition and nuclear self-consistency in higher-order terms of the potential parameters while the Q″-Q″ interaction is derived in a deformed system and hence satisfies these conditions.

(2) It has the natural form for the effective interaction responsible for the fluctuation mode about a deformed equilibrium. That is, $\langle\psi_{\rm def}|Q_{20}''\times|\psi_{\rm def}\rangle$ is proportional to $2\omega_z\Sigma_z-\omega_x\Sigma_x-\omega_y\Sigma_y$ which vanishes because of nuclear self-consistency, while $\langle\psi_{\rm def}|Q_{20}|\psi_{\rm def}\rangle\neq 0$. Here

$$\psi_{\rm def}=\prod_{\alpha;\rho_\alpha=1}|\alpha\rangle$$

and is an eigenfunction of $H_{\rm def}$.

(3) By use of the Q″-Q″ interaction the $K=1$ spurious state is removed automatically in the RPA when $\chi=\chi_{\rm self}$; i.e., the lowest collective state for $K=1$ has zero energy.

(4) The Q″-Q″ interaction with the same strength for each K component can be approximately converted into the form of the Q-Q interaction with different strengths for each K. With $\epsilon=0.3$ they are $0.75\chi_{\rm self}$ and $1.5\chi_{\rm self}$ for $K=0$ and $K=2$, respectively, in agreement with empirical findings.[4] Similar explanations for the different strengths have been attempted[9] in less systematic ways.

(5) An RPA method in an axially symmetric deformed system with $\chi=\chi_{\rm self}$ gives a GQR with a total splitting of $2(\sqrt{2}/3)\epsilon\hbar\omega_0$ (i.e., $\frac{1}{3}$ of the value with the Q-Q interaction). For ^{154}Sm the predicted splitting of 2 MeV leads to a broadening of the GQR in agreement with experiment [Fig. 1(b)].

In conclusion, use of the ordinary Q-Q interaction results in a predicted splitting of the GQR in deformed ^{150}Nd and ^{154}Sm much larger than observed. A rigorous application of self-consistency to the coupling of quadrupole modes leads to a Q″-Q″ interaction (Q-Q expressed in doubly stretched coordinates) which predicts a splitting consistent with that observed and provides a natural explanation of the different strengths for the $K=0$ and $K=2$ contributions in the low-lying β and γ bands.

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Deformed $\frac{9}{2}^+$ States and $\Delta J = 1$ Rotational Bands in $^{113, 115, 117, 119}$Sb Nuclei*

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A set of $\Delta J = 1$ rotational bands has been observed on deformed (prolate) $\frac{9}{2}^+$ states in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb via (^{6}Li, $3n\gamma$) reactions on stable even Cd targets. The pattern of band spacings is suggestive of a $j = \frac{9}{2}^+$ state coupled to a triaxial rotor. The energy systematics of the deformed states relate to the deformation-energy surfaces in this $Z = 50$ closed-shell region.

The large stable deformations occurring for nuclei in the $150 < A < 190$ and $A > 220$ regions which are far from closed shells have been understood by minimizing the sum of Nilsson deformed-single-particle energies as a function of deformation[1] together with Coulomb and pairing corrections.[2] Recently there has been considerable theoretical work[3,4] aimed at mapping these deformation-energy surfaces into other nuclear regions. The most recent calculations[4] employ the Strutinsky prescription[5] to renormalize the deformation-energy trends of deformed Woods-Saxon single-particle states to that of a liquid drop. For nuclear regions such as $A \approx 100$ and $A \approx 140$ away from closed shells, energy minima at both prolate and oblate deformations including asymmetric shapes are predicted.[4] The closed-shell regions, however, where nuclei are expected to be spherical have not been theoretically explored with care in regard to the coexistence of deformations. In this Letter, we report the observation, via (^{6}Li, $3n\gamma$) reactions on stable even Cd isotopes, of a set of $\Delta J = 1$ rotational bands built on deformed $\frac{9}{2}^+$ states in ^{113}Sb, ^{115}Sb, ^{117}Sb, and ^{119}Sb which are single-proton nuclei relative to the $Z = 50$ closed proton shell. These rotational bands, which for ^{115}Sb include seven members up to $J = \frac{21}{2}^+$, contain the features of a $1g_{9/2}$ proton hole strongly coupled to prolate core deformation.[6,7] The energy spacings in the bands are suggestive of axially asymmetric rotors of triaxial shapes.[7] Results from a previous ^{115}In(α, $2n$)^{117}Sb experiment showed part of the band in ^{117}Sb for which a symmetric rotational interpretation was given.[8] The systematic behavior of these deformed states and the related rotational-band spacings gives a very sensitive probe of the core-deformation-energy surfaces and the nuclear shapes as a function of neutron number. A theoretical understanding of the coexistence of these deformed properties with the known spherical properties is important. Deformation properties including possible triaxial shapes for the $Z = 82$ closed-shell region have been discussed from experimental band information in proton-hole Tl nuclei.[7,9]

To study the excited states of the odd Sb isotopes, (^{6}Li, $3n$) reactions on isotopically enriched even Cd targets were employed. These fusion-evaporation reactions[10] populate high-spin states in $\Delta Z \leq 3$ nuclei with large alignment. The γ-ray decay modes are predominantly stretched cascades $J \to J - L$ whose angular distributions are characteristic of the multipolarity L and therefore sensitive to the angular momentum J of the states involved. Measurements on these decay γ rays are thus expected to reveal the properties of the Sb high-spin states.

In order to determine the level scheme and decay properties of the excited states in the odd Sb isotopes, the following set of experiments using Ge(Li) detectors were performed: γ excitation, γ-γ coincidence, γ angular distribution, and pulsed-beam-γ timing measurements. At energies from the Coulomb barrier to 35 MeV, the main outgoing channels of the ^{6}Li-induced reac-