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Warm-Plasma Effects on Fast-Alfvén-Wave Cavity Resonances

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Whereas experiments on the ST Tokamak demonstrated fast-Alfvén-wave cavity resonances to be sharp and resolvable and suggested highly efficient coupling could be obtained for plasma heating, warm-plasma effects appropriate to fusion plasmas suggest such resonances will not be resolvable and that the characteristic mode structure will be significantly modified through the coupling to ion-Bernstein modes unless the heating is shifted to higher harmonics of the ion-cyclotron frequency.

Experiments on the ST Tokamak employing the fast Alfvén wave for plasma heating at the first harmonic of the ion-cyclotron frequency ($\omega=2\Omega_i$) demonstrated sharp peaks in the antenna-loading impedance whenever the wave frequency matched a cavity-resonance frequency of the toroidal cavity.[1] The high potential efficiency of such cavity-resonance rf heating makes the scheme attractive for fusion-plasma heating. As experiments progress toward fusion parameters, however, the resonance widths will become broader and the mode spacing smaller so the modes may overlap. We also find mode conversion and tunneling effects due to coupling with ion-Bernstein modes which may completely change the mode structure. These effects are investigated for the first few harmonics of the ion-cyclotron frequency.

Whereas toroidal effects in a cold plasma have been shown to be important in the propagation of the ion-cyclotron wave,[2-5] they have at the same time been shown to be relatively unimportant in the propagation of the fast Alfvén wave except for a slight cavity-resonance shift to lower frequency. The damping is also affected by toroidal effects since only a small fraction of the cross section of the torus is resonant at the fundamental or harmonic of the ion-cyclotron frequency. Therefore, except for the damping modification, we shall neglect the toroidal effects and concentrate on temperature effects through finite-Larmor-orbit corrections in order to calculate the cavity-resonance widths. Perkins *et al.*[5] have estimated the damping decrement for the fundamental and first harmonic, the latter of which has the same form as our result but differs by a factor of about 3.

Employing for the most part our previous notation and formalism[6] for the warm plasma in a cylindrical wave guide, we restrict our problem to that of determining the wave-guide cutoff, defined by $k_\parallel=0$ (or no azimuthal variation about the major axis in a torus). In this case, the plasma solutions decouple into transverse-electric and transverse-magnetic modes so that the fast-Alfvén-wave cutoff condition becomes

$$k_\perp^2 K_1' = K_1'(K_1'+2K_0')+K_2'^2, \tag{1}$$

where for $T_{\perp i}=T_{\parallel i}$, with no drift velocity and cold electrons ($\lambda_e=0$, $|\omega\pm n\Omega_e|/k_\parallel V_e \gg 1$),

$$\begin{aligned}
K_1' &= \frac{\omega^2}{C^2}\left[1+\frac{\omega_{pi}^2\exp(-\lambda_i)}{\omega k_\parallel V_i}\sum_{n=-\infty}^{\infty}\frac{n^2 I_n(\lambda_i)}{\lambda_i}Z\left(\frac{\omega+n\Omega_i}{k_\parallel V_i}\right)\right],\\
K_2' &= \frac{i\omega\omega_{pi}^2\exp(-\lambda_i)}{C^2 k_\parallel V_i}\sum_{n=-\infty}^{\infty} n[I_n(\lambda_i)-I_n'(\lambda_i)]Z\left(\frac{\omega+n\Omega_i}{k_\parallel V_i}\right)-\frac{i\omega\omega_{pi}^2}{C^2\Omega_i},\\
K_0' &= \frac{\omega\omega_{pi}^2\exp(-\lambda_i)}{C^2 k_\parallel V_i}\sum_{n=-\infty}^{\infty}\lambda_i[I_n(\lambda_i)-I_n'(\lambda_i)]Z\left(\frac{\omega+n\Omega_i}{k_\parallel V_i}\right),
\end{aligned} \tag{2}$$

where $\lambda_i=k_\perp^2 V_i^2/2\Omega_i^2$ and we have not yet let $k_\parallel\to 0$. In an actual heating experiment, there will be at least a few wavelengths around the torus so cyclotron damping will occur, but as long as $k_\perp \gg m/R_0$, where m is the azimuthal mode number the long way around and R_0 is the major radius, the cutoff frequency will depend only weakly on $k_\parallel \approx m/R_0$. We will assume $k_\parallel$ to be small, however, so that we may use the large-argument expansion for the plasma dispersion function for all but the resonant term.

Since we are interested in harmonic effects only as they influence the damping, we will keep only zero-order terms in λ_i plus the resonant term at the harmonic. In this case, assuming $\lambda_i \ll 1$, the dielectric tensor elements may be approximated by (where V_A is the Alfvén-wave speed)

$$K_1' = \frac{\omega^2}{V_A^2}\left[\frac{1}{1-\omega^2/\Omega_i^2} + \frac{\lambda_i^{n-1}}{2^n n!}\frac{\omega}{k_\parallel V_i} Z\left(\frac{\omega - n\Omega_i}{k_\parallel V_i}\right)\right],$$

$$K_2' = \frac{i\omega^2}{V_A^2}\left[\frac{\omega}{\Omega_i(1-\omega^2/\Omega_i^2)} + \frac{\lambda_i^{n-1}}{2^n n!}\frac{\omega}{k_\parallel V_i} Z\left(\frac{\omega - n\Omega_i}{k_\parallel V_i}\right)\right],$$

and $K_0' \approx 0$ since the resonant term is of order λ_i^n and can be neglected.

In a torus, both $k_\parallel$ and Ω_i vary with radius as $k_\parallel = m/R$ and $\Omega_i = \Omega_{i0}R_0/R$, where R is the distance from the major axis and Ω_{i0} is the ion-cyclotron frequency at the major radius R_0. If we average across the plasma cross section, we can replace R by R_0 in the nonresonant terms and average the resonant term by calculating for a circular cross section,

$$\left\langle \frac{\omega}{k_\parallel V_i} Z\left(\frac{\omega - n\Omega_i}{k_\parallel V_i}\right)\right\rangle = \frac{\omega\pi}{4mV_i r_0}\int_{R_0-r_0}^{R_0+r_0} Z\left(\frac{\omega R - n\Omega_{i0}R_0}{mV_i}\right)\cos\left[\frac{\pi(R-R_0)}{2r_0}\right] R\,dR,$$

where r_0 is the plasma radius. Using the two-pole approximation for the plasma dispersion function,[7] we find for $\omega = n\Omega_{i0}$ that

$$\left\langle \frac{\omega}{k_\parallel V_i} Z\left(\frac{\omega - n\Omega_i}{k_\parallel V_i}\right)\right\rangle = \frac{i\pi^2 R_0}{4r_0} - 1 + O\left(\frac{mV_i}{r_0\omega}\right)^2.$$

We may now let $k_\parallel \to 0$ and use the dispersion relation (1) to obtain

$$\omega \approx k_\perp V_A\left[1 - \frac{i\lambda_i^{n-1}(n-1)^2\pi^2 R_0}{2^{n+3}n!\,r_0}\right],$$

and since $\mathrm{Re}(\omega) = n\Omega_{i0}$ we find

$$\mathrm{Im}(\omega) \approx -\frac{\lambda_i^{n-1}(n-1)^2\pi^2 R_0\Omega_{i0}}{2^n(n-1)!\,8r_0}.$$

For the spacing between resonances, we assume that the external antenna configuration can determine all but the radial mode number, and that there is an integral number of half wavelengths across the plasma, so the spacing between resonances is given by $\Delta\omega = \pi V_A/2r_0$. The ratio of the width to the spacing is then

$$\gamma_n = \left|\frac{\mathrm{Im}(\omega)}{\Delta\omega}\right| = \frac{\lambda_i^{n-1}(n-1)}{2^{n-1}(n-2)!}\frac{\pi R_0\Omega_{i0}}{8V_A}.$$

If we define $\beta_i = V_i^2/V_A^2 = 2\mu_0 n_i \kappa T_i/B_0^2$, we note $2\lambda_i = n^2\beta_i$, so that

$$\gamma_n = [(n-1)\beta_i^{n-2}/(n-2)!]\,(n/2)^{2n-2}\gamma_2, \quad n \geq 2,$$

where

$$\gamma_2 = \pi R_0\Omega_{i0}\beta_i/8V_A.$$

For the ST Tokamak parameters and the probable parameters of the PLT device, $\gamma_2 \ll 1$, so the modes are resolvable, but for probable fusion parameters, $\gamma_2 > 1$, so the modes will be unresolvable at the first harmonic. At higher harmonics the prospect of resolving the modes is improved, since $\gamma_3 \approx (10\beta_i)\gamma_2$ and $\gamma_4 \approx (10\beta_i)^2\gamma_2$ and $\beta_i \ll 1$.

There is an upper limit on the harmonic number in a torus if we demand that only one harmonic be resonant in the plasma and that it should occur at R_0, since this requires $n < R_0/r_0$. Some designs do not lend themselves to cavity-mode-resonance heating; for UWMAK,[8] $\gamma_2 > 1$ and $R_0/r_0 < 3$ while for FINTOR,[9] $\gamma_3 \ll 1$ and $R_0/r_0 = 5$ so that design could conceivably use cavity-resonance heating.

Near each harmonic, $k_\perp$ for the fast wave and $k_\perp$ for the ion-Bernstein mode cross each other, indicating mode conversion may occur, and Weynants[10] has shown that complex modes occur, which implies some tunneling may occur. We estimate here the tunneling factor to determine whether the fast Alfvén wave will be coupled across the layer of complex modes. Again keeping zero-order terms in λ_i and the resonant terms, except this time we assume $k_\parallel \to 0$ and ignore ion-cyclotron damping and let $\omega = n\Omega_i$ in the nonresonant terms, we find from Eqs. (2)

$$K_1' = \frac{\omega^2}{V_A^2}\left[\frac{1}{1-n^2} + \frac{n\lambda_i^{n-1}}{(n-1)!2^{n-1}x}\right], \quad K_2' = i\frac{\omega^2}{V_A^2}\left[\frac{n}{1-n^2} + \frac{n\lambda_i^{n-1}}{(n-1)!2^{n-1}x}\right],$$

where $x = n^2 - \omega^2/\Omega_i^2$. Again $K_0' \approx 0$ since the resonant term is of higher order in λ_i. Using these expressions in (1) and the relation $k_\perp^2 V_A^2/\omega^2 = 2\lambda_i/n^2\beta_i$ we obtain after rearranging

$$n(n+1)\lambda_i^n - n^3\beta_i\lambda_i^{n-1} - 2^{n-1}(n-2)!x\lambda_i + 2^{n-2}n^2(n-2)!\beta x = 0. \quad (3)$$

For the first harmonic, $n=2$, and we obtain

$$3\lambda_i^2 - (x+4\beta_i)\lambda_i + 2\beta_i x = 0$$

with solutions

$$\lambda_{i\pm} = \tfrac{1}{6}[x + 4\beta_i \pm (x^2 - 16\beta_i + 16\beta_i^2)^{1/2}]. \quad (4)$$

The solutions coincide when the discriminant vanishes, or when

$$x_\pm = 4\beta_i(2 \pm \sqrt{3}),$$

and the solutions are complex conjugate pairs between x_+ and x_-. The average value of λ_i over this complex region is

$$\langle\lambda_{i\pm}\rangle = (x_+ - x_-)^{-1}\int_{x_-}^{x_+} \lambda_{i\pm}(x)\,dx = (2 \pm i\pi/2\sqrt{3})\beta_i.$$

If we now take the average value of λ_i to determine the magnitude of $\mathrm{Im}(k_\perp)$ and use $\Delta x = 8\sqrt{3}\beta_i = 8\Delta R/R_0$ to determine the range, the tunneling factor is given by

$$\eta_2 = |\mathrm{Im}\langle k_\perp\rangle \Delta R| = \pi R_0\Omega_i\beta_i/4V_A = 2\gamma_2.$$

Analysis of the $n=3$ and $n=4$ cases from Eq. (3), taking the average numerically, seems to support a general relation $\eta_n \cong 2\gamma_n$. Thus if the modes are not resolvable, the coupling across the complex layer is small and the wave from the high-magnetic-field side will partially mode convert to an ion-Bernstein mode and partially reflect and a wave from the low-magnetic-field side will be absorbed at the cyclotron layer by ion-cyclotron damping or reflect. In either case, the mode structure will be greatly modified and the raising of the ion temperature by the waves may saturate as the mode structure changes near $\eta \approx 1$. As we noted previously, γ_2 is greater than 1 for fusion plasmas but the higher harmonics are encouraging, so for some harmonic where $\eta_n \ll 1$, the coupling across the layer should be high and the usual cavity-mode behavior should be observed and the individual cavity modes should be resolvable.

The effects of including finite $k_\parallel$, which would be necessary to obtain cyclotron damping, do not affect the basic results above except in the immediate region of the resonance. This can be seen by examining the solutions of the full warm-plasma dispersion relation for general $k_\perp$ and $k_\parallel$. The behavior of the two solutions $\lambda_{i\pm}$ indicated by Eq. (4) is demonstrated in Fig. 1 where we plot numerical solutions of the full warm-plasma dispersion relation for $k_\parallel = 3\ \mathrm{m}^{-1}$, and include terms through $O(\lambda_i^5)$ for an electron density of $5\times10^{14}\ \mathrm{cm}^{-3}$, $B_0 = 100$ kG, and $T_e = T_i = 10$ keV.

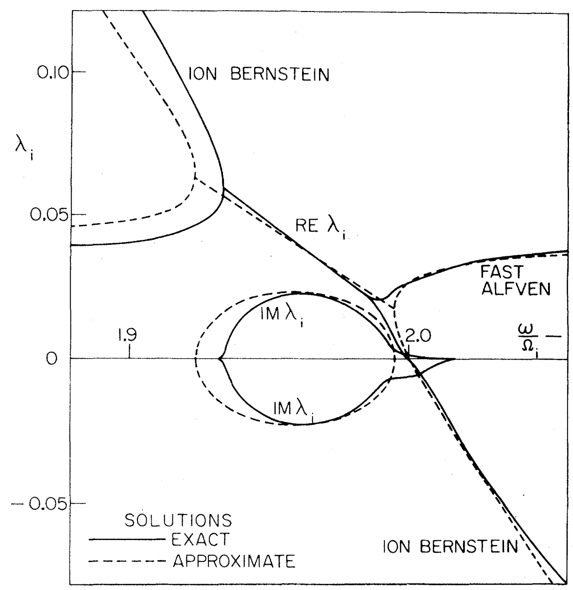

FIG. 1. Exact and approximate solutions of the warm-plasma dispersion relation near the first harmonic ($\omega = 2\Omega_{i0}$). Plasma parameters are $n_e = 5\times10^{14}\ \mathrm{cm}^{-3}$, $B_0 = 10$ T, $T_i = T_e = 10$ keV, $k_\parallel = 3\ \mathrm{m}^{-1}$ in a deuterium plasma.

The behavior near resonance is modified due to cyclotron damping, but otherwise the exact solutions follow the approximate solutions closely.

For heating of plasmas using the fast-wave cavity resonance at the first harmonic of the ion-cyclotron frequency, we find for fusion plasmas that the cavity modes will probably not be resolvable and that mode conversion and tunneling will modify greatly the cavity-mode structure. These effects should be small in present machines unless high-density operation is investigated, but large in fusion devices. We find both effects to have exactly the same parameter dependence, so that if the modes are resolvable, tunneling will presumably be highly efficient and the usual cavity-mode structure will be observed, whereas if the modes begin to overlap, the mode structure changes, the modes become untrackable, and the antenna coupling changes.

For higher harmonics, the situation improves, so that at the second or third harmonic the modes may be resolvable because they are narrowed by a factor of the order of $(10\beta_i)$ and the tunneling distance is reduced by the same factor; the reduction factor is proportional to $(10\beta_i)^2$ for the third harmonic. It thus appears we must move to higher harmonics for cavity-resonance heating of fusion plasmas.

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Preheat Effects on Microballoon Laser-Fusion Implosions*

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Nonequilibrium hydro-burn simulations of early laser-driven-compression experiments indicate that low-energy photons from the vicinity of the ablation surface are preheating the microballoon pushers, thereby severely limiting the compression achieved (similar degradation may result from 1–4% energy deposition by superthermal electrons). This implies an 8- to 27-fold increase in the energy requirements for breakeven, unless radiative preheat can be drastically reduced by, say, the use of composite ablator pushers.

Theory[1] predicts that shells can be compressed to many times solid density by laser-driven ablative implosions. In an ablative implosion only the exterior of the shell is heated. The shell is shocked, compressed, and driven towards the origin by the reaction force to material streaming off. DT fuel inside the shell can thus be brought to densities and temperatures favoring thermonuclear burn.[2] When a low level of preheat is introduced, the shocks are weakened, the back-pressure is increased, and the degree of shell and fuel convergence is reduced. Under extreme preheat, the entire shell is raised to high temperatures and pressures before any shocks can cross. It simply expands at both its surfaces, compressing the fuel within, but only to minimal densities.

Laser-fusion experiments have been reported[3-5] which have produced x-ray pinhole pictures as proof of compression, and from 10^4 to 10^7 neutrons.[3,5] In this paper we report the results of calculations which indicate that the phenomenol-