Coherence Brightening and Laser Lethargy in X-Ray Laser Amplifiers*

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The lethargic response of the lasing medium produced by the combined effects of finite bandwidth and rapid decay of the population inversion leads to a reduction of the linear gain expected in certain x-ray laser amplifiers. However, the effects of coherence brightening are also important, and in the superradiant regime, the laser output approaches that predicted by the usual calculation.

There has been considerable recent interest in investigating the possibilities of a laser operating in the hard-uv and soft-x-ray regime.[1] Because the spontaneous lifetime of the transition involved is usually very short, one typically sweeps the excitation in the direction of lasing so that the atoms will be prepared in the excited state just as the radiation from the previously excited atom reaches them. In particular, two recent theoretical papers have considered the problem of gain in a medium with swept excitation.[2,3] In those analyses the gain is calculated by using the usual "steady-state" amplification calculation.

In the present work we show that such an approach may be misleading. One must take into explicit account the lethargic response of the laser due to its finite amplifier bandwidth, together with the rapid atomic decay. These combined effects, which we call laser lethargy, decrease the gain below that given by the usual treatment.[4] Such considerations are order-of-magnitude effects when one tries to reduce the inhomogeneous broadening in order to increase the gain.

We note further that the effects of coherence brightening (i.e., a peak power which increases as N^2 and a pulse width which goes as $\sim 1/N$),[5-7] together with its attendant power broadening, tend to counter the effects of laser lethargy.[8] However, in view of the present motives, namely, assessing the effects of coherence properties on x-ray laser feasibility, we concern ourselves here not with the fully quantized theory, but rather with the coupled Schrödinger-Maxwell equations containing appropriately added noise terms which are taken to simulate the effects of spontaneous emission in the present analysis.

We describe the amplifier medium as an ensemble of two-level systems with states $\alpha = a, b$ prepared by an excitation swept at the speed of light. We present computational results obtained in the frame of a hybrid model consisting in a three-level amplifying medium (i.e., the decays go to distant ground states), with $T_2 = 2T_1$. This model only partially represents the x-ray laser proposed in Ref. 2, which lases directly to the ground state ($\gamma_b = 0$). However, one expects on physical grounds that this calculation contains results that are useful and interesting in the present context.

For the calculation presented in this paper, we take $T_2^* = T_1$, where $T_2^* \equiv 3.33/\Delta\omega_D$, and $\Delta\omega_D$ is the full Doppler width at half-maximum. We take the gain g such that $gL = 95$, where L is the length of amplifier; this value is chosen so as to take the amplifier into the nonlinear regime. We also include a small loss κ ($\kappa/g \ll 1$). The results of our analysis are summarized in Fig. 1. In this figure we show the energy of the x-ray pulse as a function of the length z of amplifying medium (lower curve). This curve is the result of the ensemble average over an appropriate set of calculations, each of which involves a different sequence of random numbers for the noise. The standard deviation of these calculations is given by the vertical bars. The upper curve shows the way the pulse would have grown if the usual estimate for the gain were correct. A comparison of the two cases shows that for small z (linear regime), the actual gain of the laser amplifier is much smaller than predicted by the simple theory. However, as discussed in this note, superradiant considerations tend to compensate for this effect in the nonlinear regime.

Assuming that the envelope $\mathcal{E}(\mu,z)$ of the electric field varies slowly compared to the smallest of the atomic time constants, its equation of motion can be written in the form[2]

$$\partial\mathcal{E}^2(\mu,z)/\partial z + 2\kappa\mathcal{E}^2(\mu,z) = \mathcal{E}^2(\mu,z)g(\mu), \tag{1}$$

with

$$g(\mu) = \frac{2\nu N\mathcal{P}^2}{\hbar c\epsilon_0}\int_0^{\mu} d\mu'[r_a \exp(-\gamma_a\mu') - r_b\exp(-\gamma_b\mu')]\exp[-\gamma_{ab}(\mu-\mu')]D(\mu-\mu'). \tag{2}$$

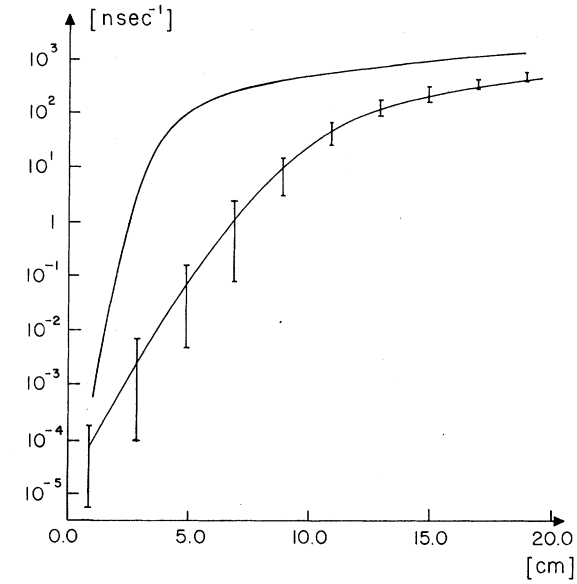

FIG. 1. Energy of the pulse as a function of the length of amplifier. The lower curve represents the actual growth of the pulse, while in the upper curve, the time decay of the gain is not taken into account.

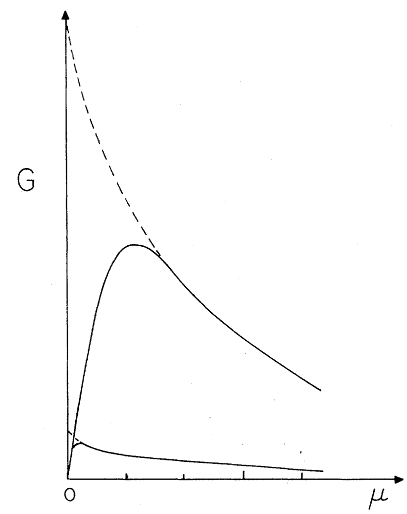

FIG. 2. Temporal behavior of $g(\mu)$. The upper curve corresponds to a Doppler width 10 times less than the lower one. The dashed curves represent the evolution of the gain if the finite rise time of the gain is not taken into account.

Here μ is the retarded time $\mu \equiv t - z/c$; $D(\mu)$ is the cosine Fourier transform of the Doppler distribution $\sigma(\omega)$ and is equal to $\exp[-\mu^2/T_2^{*2}]$; r_a (r_b) is the fraction of "atoms" pumped into the upper (lower) level; $\mathcal{P}$ is the dipole matrix element between the states a and b; ν is the central frequency of the electric field; and N is the number of "atoms" per unit volume.

We note that the assumption that $\mathcal{E}(\mu)$ varies slowly compared to the relevant timescales of the amplifier is not generally valid in the x-ray problem, and the result is a reduction of the gain by a factor of 2 to 3 below that indicated by $g(\mu)$. The other reductions follow from a detailed examination of the function $g(\mu)$ itself.

In Fig. 2 we plot the value of $g(\mu)$ for different values of the Doppler width $\Delta\omega_D$. In that figure the solid curves are the function $g(\mu)$, whereas the dashed curves are obtained from Eq. (2) by taking $D(\mu-\mu')$ to be a δ function with respect to the decay terms γ_a and γ_b (but not necessarily with respect to γ_{ab}). The conventional gain estimate, which we denote as g, is obtained from the dashed curve at $\mu = 0^+$, where 0^+ is a time which is long compared to T_2^* but short compared to γ_a^{-1} or γ_b^{-1}. We find that a better estimate of the gain is given by the maximum value of $g(\mu)$ which we denote as an effective-gain parameter g_{eff}. The two estimates (g versus g_{eff}) approach each other as $T_2^* \to 0$ (large $\Delta\omega_D$).

In the homogeneously broadened limit, i.e., for $T_2^* \to \infty$, the gain reductions can be obtained in analytic form. Here, the Fourier transform $D(\mu-\mu')$ of the inhomogeneous broadening is independent of $\mu-\mu'$, and Eq. (2) can be integrated in a straightforward manner. For a three-level system with $T_1 = T_2$ and $r_b = 0$, $g(\mu)$ is given by

$$g(\mu) = g\mu \exp(-\mu/T_1). \tag{3}$$

The maximum value of $g(\mu)$ is $g_{\text{eff}} = g/e$. Taking into account the factor of 3 due to the *Ansatz* that $\mathcal{E}(\mu)$ varies slowly compared to the medium characteristic times, the actual effective gain in this case is about 7 times smaller than the value given by the simple theory.

In the case of a two-level amplifier (lasing directly to the ground state), the gain can be expressed as a function of the initial fractional inversion $F = (\rho_{aa} - \rho_{bb})/(\rho_{aa} + \rho_{bb})$ in the form

$$g(\mu) = \int_0^\mu d\mu' \exp\left[-\frac{(\mu-\mu')}{T_2}\right]\left[\frac{F+1}{F}\exp\left(\frac{-\mu'}{T_1}\right) - \frac{1}{F}\right]. \tag{4}$$

For $T_2 = 2T_1$, the maximum gain $g_{\rm eff}$ is then

$$g_{\rm eff} = \tfrac{1}{4}[F/(F+1)]g. \tag{5}$$

In the best case ($F=1$ or total inversion), $g_{\rm eff}$ is smaller than g by a factor of 8. Then taking into account the factor of 2–3 arising from the fact that $\mathcal{E}(\mu', z)$ has been taken out of the integral, we thus find that the gain parameter in the two-level case is smaller than that of the simple theory by about an order of magnitude. Luckily, the story does not end here, and the coherence-brightening effects increase the gain in the nonlinear regime.

Let us now return to the calculation in Fig. 1 and consider the behavior of the amplifier for large z when the gain saturates. In this regime the results of the simple theory and the present analysis converge on each other. This convergence shows that the x-ray laser is not saturating in the same fashion as happens in the conventional model. In that case the energy increases as a linear function of z in the large-signal regime, whereas in the present case, the x-ray laser energy increases more rapidly. The reason for this lies in the coherence brightening, i.e., superradiance, that occurs in the nonlinear regime.

In the nonlinear regime, in addition to the normal broadening, there is also a power broadening. The rise time of the gain is then decreased over that of the linear regime by an amount $\sim (\mathcal{P}\mathcal{E}/\hbar)^{-1}$. As this rise time gets shorter, the gain function $g(\mu)$ is no longer appropriate, but rather something intermediate between the solid and dashed curves in Fig. 2. In the limit of very large z, $g_{\rm eff} \rightarrow g$, and the x-ray laser begins to behave in a manner more like "ordinary" laser-pulse amplification.

In conclusion we note that any increase which one expects to realize by reducing the inhomogeneous broadening inherent in the x-ray-laser scheme in question must be reevaluated in view of the present calculations. It is, however, noteworthy that when one truly reaches the nonlinear (coherence-brightened) regime in which the atomic transition time is no longer $\tau_{\rm spontaneous}$, then the laser output begins to approach that which one would expect on the basis of the usual gain calculation.

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