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Experimental Signature of Scaling Violation Implied by Field Theories

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Renormalizable field theories are found to predict a surprisingly specific pattern of scaling violation in deep inelastic scattering. Comparison with experiments is discussed. The feasibility of distinguishing asymptotically free field theories from conventional field theories is evaluated.

A problem of central importance in particle physics today is to understand the gross scaling behavior of the structure functions in deep inelastic lepton-hadron scattering and to pin down the pattern of violation of strict scaling, if any. Much effort has been spent in studying this problem in the general framework of renormalizable field theories.[1] This raised the exciting possibility of gaining crucial evidence toward answering the long-standing question of *whether renormalizable field theories are viable as the underlying theory for hadron physics*. The bases for this hope are as follows: (i) Strict Bjorken scaling is incompatible with renormalizable field theories (except in the trivial case of free particles)[2]; (ii) techniques have now been developed to extract from these theories[3,4] the expected pattern of scaling violation which can be confronted with experiment.[5]

Assuming the answer to the question posed above turns out to be yes, a second interesting question to ask is: *Can the data further distinguish which class of field theories is the relevant one for hadron physics*? Here I have in mind, in particular, the conventional type (CT) versus the much publicized asymptotically free (AF) field theories.[5]

An evaluation of the prospect for resolving these important issues by forthcoming experiments cannot be made until the expected patterns of scaling violation are systematically analyzed. Here I carry out such an analysis and compare the results obtained for the two types of theories with available data as well as with each other. These studies indicate great promise for gaining insight into the questions raised; they also identify crucial features to be looked for in the next generation of experiments.

For definiteness, throughout this paper we shall be concerned with the well-known structure function νW_2 which we simply denote by $F(x,q^2)$, where $x=q^2/2M\nu$. It is one of the two independent spin-averaged current correlation functions $\langle N|J^\mu(z)J^\nu(0)|N\rangle$, where J^μ is the electromagnetic current operator.[1] The scaling-limit behavior of $F(x,q^2)$ can be studied in renormalizable field theories by the technique of Wilson expansion[1] of the operator product $J^\mu(z)J^\nu(0)$ and the application of the Callan-Symanzik equation to the coefficient functions in this expansion.[3] Contact with experiment is made through the moment integrals

$$\int_0^1 dx\, x^{n-2}F(x,q^2)=M(n,q^2). \tag{1}$$

Typically, CT theories imply the following high-energy behavior[1]:

$$M(n,q^2)=C(n)(q^2)^{-\lambda(n)} \text{ (large } q^2), \tag{2}$$

where $\lambda(n)$ are the, in principle, calculable anomalous dimensions of the tensor operators which enter the Wilson expansion. On the other hand, AF theories imply[5]

$$M(n,q^2)=C(n)(\ln q^2)^{-\lambda(n)} \text{ (large } q^2). \tag{3}$$

These two equations can be cast in the same form provided we choose a new variable k in place of q^2:

$$k=\begin{cases}\ln(q^2/q_0^2) \text{ (CT)},\\ \ln(\ln q^2/\ln q_0^2) \text{ (AF)};\end{cases} \tag{4}$$

then

$$M(n,k)=M(n,0)e^{-\lambda(n)k}, \tag{5}$$

where q_0^2 is some reference value of q^2. Since $\lambda(n)$ is calculable from theory, Eq. (5) says that the moment functions at arbitrary k (q^2) are completely specified by their values at $k=0$ ($q^2=q_0^2$). Furthermore, because Eq. (1) is invertible, we arrive at the conclusion that[4,6] *$F(x,q^2)$ can be calculated for arbitrary x and q^2 (in the scaling region) provided theory supplies $\lambda(n)$ and experiment supplies the initial values of $F(x,q_0^2)$ at some q_0^2.*

I have systematically calculated $F(x,q^2)$ for various input $\lambda(n)$ suggested by both CT and AF theories. I first show results from two typical examples, one for each type of theory. Then I spell out explicitly the pattern of violation of scaling implied by these calculations. (Only those aspects which are independent of the particular examples chosen will be stated.) These features are then compared with existing data. Comments on future experiments are also made. Details of the calculations and more extensive discussion of results will be published elsewhere.

We need, as initial experimental input, $F(x,q_0^2)$ with q_0^2 in the scaling region. Published data[7] of this kind only exist for proton targets. Hence we shall confine ourselves to this case. I choose $q_0^2=4$ GeV^2 in these calculations because relatively extensive data at $q^2=4$ exist.[8]

Figure 1 shows the predicted behavior of $F(x,q^2)$ as a function of q^2 for various values of x in the two model theories. In the CT case [Fig. 1(a)]

$$\lambda(n)=A[1-6/n(n+1)]$$

was used,[9] while in the AF case [Fig. 1(b)]

$$\lambda(n)=A[-3-18/n(n+1)+4\sum_{m=1}^{n}(1/m)]$$

was used.[10] The constant A in these expressions is model dependent; it is best left as a parameter to be determined by experiment. In the present calculations, A is absorbed into the definition of the new variable k. This is reflected in the specification of the horizontal scales in Fig. 1. Notice also, in the AF case, that k actually depends on a second parameter μ which sets the scale for q^2 inside the logarithm. Theory has little to say about the magnitude of μ^2. However, we note that it cannot be larger than q_0^2 ($=4$ GeV^2), nor should it be smaller than the typical hadron mass scale 1 GeV^2. Its value can either be simply chosen within this narrow range or again left to be determined by experiment.

Just for orientation, strict scaling would imply that all lines in Fig. 1 be horizontal. One cursory look at Fig. 1 immediately tells us that the patterns for violation of scaling implied by the two types of theories are quite similar. The only real difference lies in the horizontal scale. Setting this aspect aside for the moment, we first turn our attention to the common trend. By in-

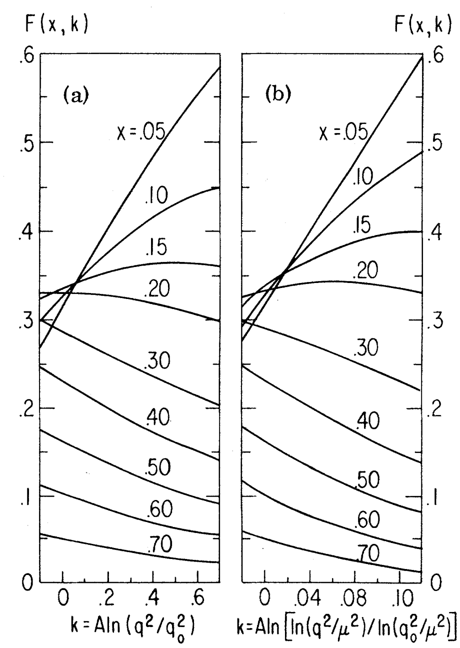

FIG. 1. Predicted $F(x,q^2)$ as function of q^2 for various values of x in (a) CT and (b) AF field theories.

specting these curves and similar ones obtained from other possible forms of $\lambda(n)$, the following general pattern for violation of scaling implied by both types of theory emerges[11]: (1a) For $0.25 \lesssim x < 1$, $F(x,q^2)$ is a decreasing function of q^2; (1b) the rate of decrease in q^2 is greatest for $x \simeq 0.4$ and tapers off at both ends. (2) In the vicinity of $x \simeq 0.2$, $F(x,q^2)$ has little or no dependence on q^2; hence this is the *apparent scaling region.* (3a) In the region $0 < x \lesssim 0.15$, $F(x,q^2)$ is an increasing function of q^2; (3b) the rate of increase grows monotonically as x approaches zero. The indicated ranges of x are approximate to within about 0.05.

These predictions are surprisingly specific[11] yet are quite independent of the theoretical uncertainties about $\lambda(n)$. They are largely consequences of two general requirements on $\lambda(n)$, (i) $\lambda(2)=0$ and (ii) $\lambda(n+1)>\lambda(n)$, plus, of course, the general shape of the input function $F(x,q_0^2)$. [Point (i) follows from the fact that the energy-momentum tensor is the dominant second-rank tensor. Point (ii) is a consequence of constraints imposed by positivity.[12]] This very specific pattern of scaling violation offers an excellent opportunity for forthcoming experiments to determine the viability of renormalizable field theories as the underlying theory for deep-inelastic-scattering phenomena.

Turning to more details, we need to establish the horizontal scales in the two parts of Fig. 1. This can be done by using one experimental point (preferably at very large q^2) as input. That will determine the unknown constant A and no more freedom will be left. The experimental reference point $x=0.57$, $q^2=10$, was used for this purpose. (For simplicity I picked $\mu=1$ GeV. The number would not be much different if we used a second experimental point to determine μ.) The value of A obtained is $\simeq 0.25$ for the CT case and $\simeq 0.085$ for the AF case.[13] The quantitative predictions of these theories are now plotted in Fig. 2 (both on linear q^2 scale). Also plotted on these graphs are published data points from Miller *et al.*[7] and Bodek *et al.*[7]

Let us first discuss the comparison of the theoretical results with data. The general impression is that within the very restricted available experimental range, there is broad agreement.[14] Going down the list of features previously stated, only points (1a) and (1b) are tested. Even here the agreement of the rate of decrease in q^2 for the higher values of x with data is not as impressive as it first appears since the one experimen-

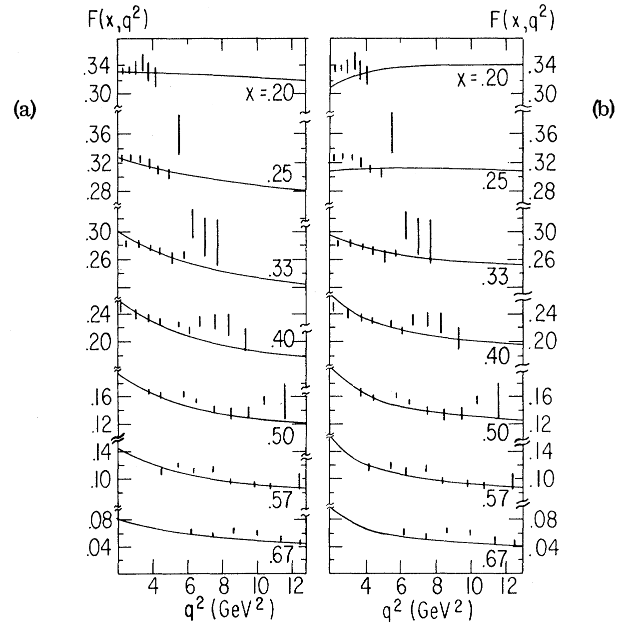

FIG. 2. Comparison of experimental data (Ref. 7) with predictions of (a) CT and (b) AF field theories. Theoretical predictions are normalized to the experimental reference point $x=0.57$, $q^2=10$. For simplicity, $\mu=1$ GeV was used.

tal input point lies in this region. Clearly, in order for the comparison to be really meaningful, data at large values of q^2 for $0<x<0.4$ are keenly needed. Fortunately, this is exactly the region where most data of the on-going μN experiments at Fermi National Accelerator Laboratory (FNAL) are expected to lie.[15] Hence the chance for a decisive confrontation between these predictions and experiment looks quite good.

In fact, very preliminary data from FNAL[15] already exist. No meaningful comparison can be made at this stage, however, as the structure functions are not extracted. To the extent that there seems to be excess events for small x (corresponding to smaller q^2 in this experiment[15]) and depreciation of events for large x (higher q^2 in this experiment) at high energies, there is indication of agreement with the predicted general trend. But this has to be confirmed by more refined data. In the analysis of these new data, it is very important to look at the q^2 dependence of measured quantities separately for the three regions of x mentioned before. Averaging over all values of x could wash out the real interesting effects completely.

Finally, I must comment on the feasibility of experimentally distinguishing CT from AF theories. Inspection of the corresponding curves in Fig. 2 shows that there are only very small differences between the two cases within the range covered. More study shows that these differences are well within the range of uncertainties of each type of theory by itself. Real differences between these two classes of theories do not develop until q^2 becomes very large—well beyond the range of any foreseeable experiment. Thus it appears that the often forcefully expressed statement that deep-inelastic-scattering phenomena should clearly separate AF theories from the other type of theories is somewhat too optimistic. *What is feasible, as this study shows, is a definitive confrontation between experiment and the rather specific pattern of scaling violation impled by both types of field theories.*

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Note added.—Since this paper was submitted for publication more definitive results from the first FNAL μN deep-inelastic-scattering experiments have become available [C. Chang *et al.*, Michigan State University–Cornell University–Lawrence Berkeley Laboratory–University of California at San Diego Collaboration Report No. MSU-CSL-23, CLNS-308, LBL-388b (to be published)]. Data spanning the kinematic range $1<q^2<20$ (GeV/c)^2 and $0.03<x<0.3$ were compared with predictions based on low-energy data[7] and scaling. A clear pattern of scaling violation was observed. The pattern agrees remarkably well with that specified in points (1a)–(3b) in the text of this Letter. Supplemented by data[7] for $x>0.3$ (discussed in the text), all the features of the theoretical predictions are seen.

It is also worth pointing out that the observed pattern of scaling noninvariance is incompatible with that implied by modified parton models [M. S. Chanowitz and S. D. Drell, Phys. Rev. Lett. 30, 807 (1973); V. Barger, Phys. Lett. 49B, 43 (1974); G. West and P. Zerwas, Phys. Rev. D 10, 2130 (1974)]. In these models the structure functions have the factorized form $G(q^2)f(x)$. Consequently, the slope of the q^2 dependence should have the same sign for all x.

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[1]For useful reviews see K. Wilson, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Ithaca, New York, 1971,* edited by N. Mistry (Cornell Univ. Press, Ithaca, N.Y., 1972); Y. Frishman, Phys. Rep. 13C, 1 (1974).

[2]G. Parisi, Phys. Lett. 42B, 114 (1972); C. Callan and D. Gross, Phys. Rev. D 8, 4383 (1973). Actually the second paper stated that asymptotically free theories are consistent with scaling, thereby ignoring the logarithmic breaking effects. This is an overstatement in view of the later emphasis [cf. G. Parisi, Phys. Lett. 43B, 207 (1973), and 50B, 367 (1974); D. Gross and F. Wilczek, Phys. Rev. D 9, 980 (1974); D. Gross, Phys. Rev. Lett. 32, 1071 (1974); H. Georgi and H. D. Politzer, Phys. Rev. D 9, 416 (1974); and A. De Rújula *et al.*, Phys. Rev. D 10, 2141 (1974)] that these same scaling-violation effects provide the best test for the validity of these theories.

[3]N. Christ, B. Hasslacher, and A. Mueller, Phys. Rev. D 6, 3543 (1972).

[4]G. Parisi, Phys. Lett. 43B, 207 (1973), and 50B, 367 (1974).

[5]Violation of scaling in asymptotically free theories has been discussed by Gross and Wilczek, Ref. 2; Gross, Ref. 2; Georgi and Politzer, Ref. 2; De Rújula *et al.*, Ref. 2.

[6]In a more precise formulation, it turns out that to determine F at (x,q^2) only $F(\xi,q_0^2)$ for $x\leq\xi\leq1$ is needed. This is important in practice since for q_0^2 in the scaling region $F(\xi,q_0^2)$ is not measured for small ξ.

[7]G. Miller *et al.*, Phys. Rev. D 5, 528 (1972); A. Bodek *et al.*, Phys. Lett. 52B, 249 (1974), and references cited therein.

[8]All previous investigations used as input the scaling function $F(x)$ which is some *average of $F(x,q^2)$ over strongly x-dependent q^2 ranges.* Results so obtained cannot provide the basis for quantitative comparison with experiment. I also note that although there is no good data at $q_0^2=4$ for small x, the bulk of the predicted results are not affected, for the reason mentioned in footnote 6.

[9]This is typical of results obtained from both perturbation-theory calculations (Refs. 3 and 4) and Wilson's $4-\epsilon$ dimension calculations [K. Wilson, Phys. Rev. D 8, 2911 (1973)]. Other possible forms [cf. Refs. 3 and 4 and C. Lovelace (to be published)] are less singular and yield results closer to the AF case (i.e., in between the two examples presented here).

[10]This is chosen to satisfy the requirements (i) $\lambda(2)=0$ and (ii) $\lambda(n)\simeq A[-3-2/n(n+1)+4\sum(1/m)]$ for $n>4$ (cf. Gross, Ref. 2). Other expressions satisfying the same requirements give essentially similar results.

[11]These statements only hold within the range of q^2 covered in Fig. 1. As I will show shortly, this already extends far beyond the forseeable experimental range. For really asymptotic q^2, all curves (except $x=0$) eventually go to zero.

[12]O. Nachtman, Nucl. Phys. B63, 237 (1973).

[13]The values for A obtained this way depend sensitively on the particular form of $\lambda(n)$ used, unlike the general shape of the curves in Fig. 1. Another way of saying this is that the effects due to a reasonable change in the form of $\lambda(n)$ can usually be compensated by a re-

scaling of the normalization constant A, leaving the general pattern of scaling violation unchanged.

[14]The apparent discrepencies for the AF case at $x \simeq 0.20$ are not to be taken seriously as there is no reason for the theory to hold at such low q^2.

[15]D. J. Fox *et al.*, Phys. Rev. Lett. 33, 1504 (1974).

Evidence for Intermediate Structure in the Inelastic Scattering of Polarized Protons from ^{26}Mg and ^{27}Al†

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Structure of variable widths from about 0.3 to 1.5 MeV has been observed in analyzing-power excitation functions for inelastic proton scattering from ^{26}Mg and ^{27}Al. Statistical tests and significant cross correlations show that it is very unlikely that this structure is the chance effect of random fluctuations. Thus these data are evidence for intermediate structure.

Since Block and Feshbach[1] and Kerman, Rodberg, and Young[2] suggested that simple modes of excitation of the nucleus might lead to structures with widths intermediate between those for the compound nuclear states and those for single-particle states, many attempts have been made to identify these states in nuclear reactions. Apart from isobaric analog resonances, however, only two isolated examples of intermediate structure in the elastic or inelastic scattering of protons are considered well-established.[3-5] The lack of selectivity in the (p,p') reaction mechanism presumably makes it difficult to observe definitively nonstatistical peaks in cross-section excitation functions, even when there is some evidence for intermediate structure.[6-8] In this Letter we report convincing evidence for intermediate structure in the elastic and inelastic scattering of low-energy protons from ^{26}Mg and ^{27}Al from measurements of analyzing-power excitation functions. The ease with which the structure is identified suggests that the method should have wide applicability.

The analyzing power A_y is a sensitive indicator of coherent structure; its magnitude depends on the interference between different partial waves. For direct reactions, A_y may be nonzero, but its value should vary smoothly over an energy region of several MeV. For compound reactions, the value of A_y averaged over a sufficiently large energy region should be zero because of the random phases of the contributing partial widths. In a region of high level density where the average width Γ of the compound states is much greater than the average spacing D, the value of A_y measured in small intervals should oscillate rapidly about zero in a manner characteristic of Ericson fluctuations. Intermediate structure is indicated by peaks in the excitation function of A_y which remain significant when the data are averaged over a region much larger than Γ.

The experiments were performed at the Rutgers FN tandem Van de Graaff accelerator, using the atomic-beam polarized-ion source. Solid-state detectors were mounted at symmetric angles on each side of the incident beam. Beam polarization was monitored continuously with a helium polarimeter. Targets were 0.5- to 1.0-mg/cm^2 self-supporting foils of ^{26}Mg and ^{27}Al. Data were recorded in 50-keV steps from 5.5- to 9.4-MeV incident proton energy for ^{26}Mg and in 50- and 100-keV steps from 6.1- to 12.0-MeV bombarding energy for ^{27}Al at several angles between 60° and 160°. Excitation functions of A_y for several final states in ^{26}Mg at 140° and for ^{27}Al at 145° are shown in Figs. 1 and 2.

The qualitative features that indicate intermediate structure are apparent from these figures. The raw data in 50-keV steps [e.g., Fig. 1(b)] show the expected Ericson fluctuations; a coherence width Γ of about 50 keV has been determined from previous cross-section measurements.[7,9] However, considerable structure remains when these fluctuations have been smoothed by averaging over larger intervals. The p_1 and p_2 data (p_i refers to the proton group leading to the ith state) for ^{26}Mg at 140° are probably the most striking. The p_1 data show two negative peaks about 750 keV wide separated by 1.5 MeV. The p_2 data reveal one positive bump of about the same width; the value of A_y rises to 0.7 at the peak compared to values close to zero everywhere else. The