will, to a first approximation, result in a phase change of $\pi$ for the $4(C+N)$ amplitude. Thus for large angles (small impact parameters) one will observe strongly constructive interference, but near $\theta_{c.m.} = 75°$ (outside $d_g$) completely destructive interference will occur but it will not be a deep minimum since the $4(C+N)$ amplitude is much smaller than the $2(C+N)$ amplitude. Changing the sign of $\beta_2$ (not yet observed experimentally) should, to a first approximation, not alter the interference structure since the $L=2$ amplitude, being a two-step process, depends on $\beta_2^2$. Calculations supporting these predictions are shown in Fig. 1.

Finally, for $\beta_4 = 0$ the interference is best described as Coulomb-nuclear interference since the $L=4$ amplitude is negligible. Here, since the $0^+ \to 2^+$ cross section is reduced because of destructive Coulomb-nuclear interference, it is evident that the $0^+ \to 2^+ \to 4^+$ cross section will have a similar behavior.

I am grateful to J. Raynal for furnishing his code ECIS and to W. G. Love for discussions of this problem.

[1]W. Brückner, J. C. Merdinger, D. Pelte, U. Smilansky, and K. Traxel, Phys. Rev. Lett. 30, 57 (1973).

[2]C. E. Bemis, Jr., P. H. Stelson, F. K. McGowan, W. T. Milner, J. L. C. Ford, Jr., R. L. Robinson, and W. Tuttle, Phys. Rev. C 8, 1934 (1973).

[3]I. Y. Lee, J. X. Saladin, C. Baktash, J. E. Holden, and J. O'Brien, Phys. Rev. Lett. 33, 383 (1974).

[4]W. Brückner, D. Husar, D. Pelte, K. Traxel, M. Samuel, and U. Smilansky, Nucl. Phys. A231, 159 (1974).

[5]F. T. Baker, T. H. Kruse, and W. Hartwig, Bull. Am. Phys. Soc. 19, 577 (1974).

[6]F. T. Baker, T. H. Kruse, W. Hartwig, I. Y. Lee, and J. X. Saladin, Bull. Am. Phys. Soc. 19, 1016 (1974), and to be published.

[7]J. Raynal, unpublished.

[8]R. A. Broglia, in *Proceedings of the International Conference on Reactions between Complex Nuclei, Nashville, Tennessee, 1974*, edited by R. L. Robinson *et al.* (North-Holland, Amsterdam, 1974), Vol. 2, p. 303.

# NMR Relaxation Time in Superfluid $^3$He†

R. Combescot
*Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853*
(Received 12 May 1975)

The temperature dependence of the NMR relaxation time introduced by Combescot and Ebisawa, $\tau_{\rm CE}$, is studied from the microscopic theory. It diverges like $(1-T/T_c)^{-1/2}$ near $T_c$, in agreement with Leggett and Takagi's prediction. At low temperature, it grows like $1/T^4$ in the $A$ phase (except at extremely low temperature where it grows like $1/T^5$) and like $T^{-1/2}e^{\Delta/T}$ in the $B$ phase.

It is now fairly clear that the main contribution to the NMR linewidths in superfluid $^3$He is coming from the finite rate of Combescot and Ebisawa,[1] $\tau_{\rm CE}$, at which the normal fluid is relaxing toward the local equilibrium defined by the superfluid. A theory[1] based on this idea and using the kinetic equation formalism has given results in good agreement with experiment for the transverse linewidth in the $A$ phase. More recently, Leggett and Takagi[2] have given a phenomenological theory which agrees[3] with the results of Ref. 1, but has the advantage of suggesting that the relaxation time $\tau_{\rm CE}$ introduced in Ref. 1 should diverge at $T_c$. Ambegaokar[4] has pointed out that this divergence results from spin conservation during collisions between (normal) quasiparticles. In this note, Ref. 1 is supplemented by showing that the behavior of $\tau_{\rm CE}$ near $T_c$ as well as its low-temperature behavior can be obtained from the microscopic theory by simple arguments. Near $T_c$, the result agrees with Leggett and Takagi's prediction. At low temperature, I obtain $\tau_{\rm CE} \sim 1/T^4$ in the $A$ phase except at extremely low temperature where we have $\tau_{\rm CE} \sim 1/T^5$ and $\tau_{\rm CE} \sim T^{-1/2}e^{\Delta/T}$ in the $B$ phase.

Let us consider the longitudinal NMR in the $A$ phase, for simplicity. To find $\tau_{\rm CE}$, we have to solve the kinetic equation

$$-i\omega\varphi_k'(\xi_k/E_k)X = I(\delta\tilde{\nu}_k), \qquad (1)$$

where $I(\delta\tilde{\nu}_k)$ is the collision term for Bogoliubov quasiparticles[5]; $X$ is the total effective field (external field plus molecular field plus Josephson field), $\varphi_k'$ is the derivative of the Fermi distribution, and $\delta\tilde{\nu}_k$ is the departure of the quasiparticle distribution from the local equilibrium corresponding to $X$ (for the other notations, see Ref. 1). Once $\delta\tilde{\nu}_k$ is known from Eq. (1), the relaxation time $\tau_{\rm CE}$ used in Ref. 1 is given by definition by

$$i\omega X\tau_{\rm CE}(T)\sum_k (-\varphi_k')\left(\frac{\xi_k}{E_k}\right)^2=\sum_k \frac{\xi_k}{E_k}\delta\tilde{\nu}_k . \qquad (2)$$

*For the sake of the argument,* let us assume that $\delta\tilde{\nu}_k$ has the same dependence on $\xi_k$ as the left-hand side of Eq. (1):

$$\delta\tilde{\nu}_k=\varphi_k'(\xi_k/E_k)C , \qquad (3)$$

where $C$ is a constant independent of $k$. Actually, this is only true at $T_c$, but taking the exact solution of Eq. (1) would modify the coefficient in $\tau_{\rm CE}(T)$, and not the temperature dependence near $T_c$ and at low temperature. With this hypothesis, $\tau_{\rm CE}$ is proportional to $C$, that is to $1/I(\delta\tilde{\nu}_k)$, for a fixed scattered quasiparticle.

Equation (3) corresponds to an equilibrium distribution except that there is a difference in chemical potential between spin up and spin down. At $T_c$, because of spin conservation during collisions between normal quasiparticles, this difference in chemical potential cannot relax to its equilibrium value. Therefore, $I(\delta\tilde{\nu}_k)$ is zero and $\tau_{\rm CE}$ is infinite. Below $T_c$, $I(\delta\tilde{\nu}_k)$ describes the collisions between *Bogoliubov quasiparticles,* whose spin is not conserved during collisions.[6] Since, at $T_c$, Bogoliubov quasiparticles are going into normal quasiparticles, $I(\delta\tilde{\nu}_k)$ is small just below $T_c$. The scattering amplitude for Bogoliubov quasiparticles is obtained from the scattering amplitude for normal quasiparticles by expressing normal quasiparticle operators in terms of Bogoliubov quasiparticle operators. This brings in coherence factors. Just below $T_c$, the more important corrective terms coming from these coherence factors are of order $|\Delta_i|^2/E_i^2$, where $i=1,2,3$ ($i=1$ corresponds to the scattering quasiparticle and $i=2,3$ to the final states). After integration of the collision term on $\xi_i$, the resultant correction on $I(\delta\tilde{\nu}_k)$ is of order $\Delta\sim(1-T/T_c)^{1/2}$ which makes $\tau_{\rm CE}\sim(1-T/T_c)^{-1/2}$ near $T_c$. This agrees[7] with taking the relaxation time introduced by Leggett and Takagi, $\tau_{\rm LT}$, to be a nonzero constant at $T_c$.[8] This result was rather likely since the corrections to the relaxation times for viscosity and thermal conductivity are known[5] to be of order $(1-T/T_c)^{1/2}$ near $T_c$. The same result holds naturally for the $B$ phase.

At low temperature, we can obtain the behavior of $\tau_{\rm CE}$ by a scaling argument. The collision integral is an integral over $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$ with two $\delta$ functions ensuring momentum and energy conservation in the collision. Changing the variables, we obtain

$$I(\delta\tilde{\nu}_k)\sim\int d\xi_i \sin\theta_i\, d\theta_i\, d\varphi_i\, \delta(\vec{k}_i)\,\delta(E_i)\,F(\xi_i,\theta_i,\varphi_i) , \qquad (4)$$

where $\delta(\vec{k}_i)$ represents the momentum-conserving $\delta$ function and $\delta(E_i)$ the energy-conserving one; $\theta_i$ and $\varphi_i$ are the angular variables corresponding to the unit vector $\hat{k}_i$. $F(\xi_i,\theta_i,\varphi_i)$ contains the scattering amplitude, the coherence factors, and the Fermi distributions. We consider first the $A$ phase and take the $z$ axis along the gap anisotropy axis. At low temperature, we will have all the $\theta_i$ small and the gap will be proportional to $\theta$. From this, it follows that by taking $x_i=\beta\xi_i$ and $\alpha_i=\beta\theta_i$ as new variables, we get rid of the temperature dependence in the Fermi distributions without introducing it in the coherence factors, so that $F(x_i/\beta,\alpha_i/\beta,\varphi_i)$ is temperature independent. In this way, the temperature dependence of $I(\delta\tilde{\nu}_k)$ comes simply from Eq. (4): $d\xi_i\sim T$, $\sin\theta_i\, d\theta_i\sim T^2$, $\delta(E_i)\sim 1/T$; and because of the momentum-conserving $\delta$ function, we have only to integrate on $\vec{k}_1$ and $\vec{k}_2$, for example. Therefore, we obtain $\tau_{\rm CE}\sim 1/T^5$ in the $A$ phase at low temperature. However, this result holds only at extremely low temperature.[9] To see this, we have to consider explicitly the momentum-conserving $\delta$ function. The conservation of the $z$ component of the momentum gives $\delta(k_i\cos\theta_i)\sim\delta(\xi_i/E_{\rm F}+\theta_i^2)$. Since $\xi_i\sim T$, and $\theta_i\sim T/T_c$, the term $\xi_i/E_{\rm F}$ dominates at very low temperature where $T/T_c\lesssim T_c/T_{\rm F}$. This gives the $1/T^5$ law. But in the range $T_c/T_{\rm F}\ll T/T_c\ll 1$, the term $\theta_i^2$ dominates so that $\delta(k_i\cos\theta_i)\sim 1/T^2$ and not $1/T$, which gives $\tau_{\rm CE}\sim 1/T^4$. Physically one can say that the relaxation time goes from $1/T^2$ in the normal state to $1/T^4$ in the $A$ phase because in $\vec{k}$ space one can find the scattering quasiparticle $\vec{k}_1$ only in a small region around the nodes of the gap, whose volume is of order $T^3$, whereas in the normal state, one can find it in a shell of thickness $T$ around the Fermi surface.

In the $B$ phase, there is no restriction on the angular variables. At low temperature, we have essentially $\xi \ll \Delta$ which makes $\beta E \simeq \beta\Delta + \beta\xi^2/2\Delta$ so that $\xi$ scales like $T^{1/2}$. From Eq. (4) we obtain a factor $(T^{1/2})^3/T = T^{1/2}$, but we must take into account that we have only a probability $\exp[-\Delta/T]$ of finding a scattering quasiparticle. Finally, we obtain $\tau_{CE} \sim T^{-1/2}e^{\Delta/T}$. This behavior actually agrees with the low-temperature behavior of the relaxation time[5] for viscosity and thermal conductivity. This is not surprising since this behavior could be deduced from the same scaling arguments. In the same way, we expect the viscosity and thermal-conductivity relaxation times to behave like $1/T^4$ in the $A$ phase. (Naturally, to obtain the viscosity, for example, one has to take into account other factors than the relaxation time, but their temperature dependence can be easily deduced from a relaxation-time approximation.[10])

Finally, it is remarked that when the gap is completely established, say $T/T_c \sim 0.7$–$0.8$, the spin-conserving character of the normal quasiparticle collisions is no longer felt so that in this range $\tau_{CE}$ should be of order of a typical relaxation time at $T_c$.

I am very grateful to V. Ambegaokar, T. Ho, A. J. Leggett, M. Levy, H. Smith, and C. J. Pethick for numerous discussions on this problem.

†Work supported by the National Science Foundation under Grant No. DMR 74-23494.

[1]R. Combescot and H. Ebisawa, Phys. Rev. Lett. 33, 810 (1974).

[2]A. J. Leggett and S. Takagi, to be published.

[3]R. Combescot, to be published.

[4]V. Ambegaokar, in Proceedings of the International Symposium on Quantum Statistics and the Many-Body Problem, Sanibel Island, Florida, 1975 (to be published).

[5]C. J. Pethick, H. Smith, and P. Bhattacharyya, Phys. Rev. Lett. 34, 643 (1975).

[6]For a more complete discussion, see Ref. 3.

[7]P. Bhattacharrya, C. J. Pethick, and H. Smith, following Letter [Phys. Rev. Lett. 35, 473 (1975)] have also derived this result together with the exact coefficient by solving the kinetic equation (1).

[8]Note that, although $\tau_{CE}$ is diverging, $\omega_0\tau_{CE} \ll 1$ when $T$ goes to $T_c$ since the longitudinal frequency $\omega_0$ goes to zero like $(1-T/T_c)^{1/2}$. For the transverse linewidth, one could have situations where the hydrodynamic condition $\omega\tau_{CE} \ll 1$ is no longer satisfied. But this would require such high magnetic fields and temperature so near $T_c$ that the linewidth would be very small anyway and any effect would be difficult to observe.

[9]I am very grateful to O. Valls for pointing out this fact to me.

[10]R. Combescot, to be published.

# Spin Relaxation in Superfluid $^3$He†

P. Bhattacharyya
*Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801*

and

C. J. Pethick
*Department of Physics and Materials Research Laboratory, University of Illinois, Urbana, Illinois 61801, and Nordita, Copenhagen, Denmark*

and

H. Smith
*Physics Laboratory I, H. C. Ørsted Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark*
(Received 27 May 1975)

We show that near the transition temperature $T_c$ the relaxation time introduced in Leggett and Takagi's phenomenological theory of spin relaxation in superfluid $^3$He is equal to the relaxation time of a normal-state quasiparticle at the Fermi energy at $T_c$, and is independent of the superfluid state. Combescot and Ebisawa's relaxation time is found to diverge as $(T_c - T)^{-1/2}$. These results are obtained by deriving and solving exactly the Boltzmann equation for quasiparticles in the superfluid.

The authors of two recent Letters[1,2] in which spin relaxation in superfluid Fermi liquids is treated phenomenologically arrive at different conclusions about NMR linewidths close to the transition temperature $T_c$. The differing results reflect differences in the assumptions made about