NMR measurements of bulk nuclei in *Cu*Mn demonstrated that $\tau^{-1} \propto T$ for $T \gg T_K$ as expected from perturbation theories of the $s$-$d$ model.[7] Here a constant value for $\tau$ has been obtained for $T \ll T_K$ in *Cu*Fe. Further measurements at higher $T$ might determine how these two regimes are connected to each other.

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# Quantum Electrodynamical Effects in Kerr-Newmann Geometries

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Following the classical approach of Sauter, of Heisenberg and Euler and of Schwinger the process of vacuum polarization in the field of a "bare" Kerr-Newman geometry is studied. The value of the critical strength of the electromagnetic fields is given together with an analysis of the feedback of the discharge on the geometry. The relevance of this analysis for current astrophysical observations is mentioned.

Deruelle and Ruffini[1] have recently pointed out how the positive- and negative-root solutions[2] of test particles in the background field of a collapsed object have to be identified with the classical limit of the "positive" and "negative" energy solutions of a relativistic quantum field.[3] From this immediately follows that in the ergosphere[4] or in the effective ergosphere[5] of a black hole the conditions are encountered for the possibility of particle creation through the Klein process[6] as analyzed in the framework of a flat-space theory by Sauter,[7] Heisenberg and Euler,[8] and Schwinger.[9] In this Letter we present the results of the analysis of the vacuum polarization of a Kerr-Newman[10] geometry with particular emphasis on (a) the limitations imposed by vacuum polarization on the strength of the electromagnetic field of a black hole,[11] (b) the efficiency of extracting rotational[12] and Coulomb energy[13] from a black hole by pair creation, and (c) the possibility of having observational consequences of astrophysical interest. It is important to stress that the processes here analyzed differ in three major respects from the one recently analyzed by Hawking.[14] The process of particle creation in the present framework (1) does not require a time-varying background geometry, (2) can occur for masses of current astrophysical interest ($M \gtrsim 10^{33}$

g), and (3) always increases the irreducible mass[2] of the black hole, and therefore the area of the horizon ($S=16\pi M_{ir}^2$).

We study the creation of electrons (or positrons) of mass $\mu$ and charge $\epsilon$ in the field of a charged rotating black hole described by the Kerr-Newman geometry.[10] The main point here is to realize that at each point we can introduce the following orthogonal tetrad, as defined by Carter[15]:

$$\omega^{(0)}=(\Delta/\Sigma)^{1/2}(dt-a\sin^2\theta d\varphi), \tag{1a}$$

$$\omega^{(1)}=(\Sigma/\Delta)^{1/2}dr, \tag{1b}$$

$$\omega^{(2)}=\Sigma^{1/2}d\theta, \tag{1c}$$

$$\omega^{(3)}=\sin\theta\Sigma^{-1/2}[(r^2+a^2)d\varphi-a\,dt], \tag{1d}$$

where $\Delta=r^2-2Mr+a^2+e^2$ and $\Sigma=r^2+a^2\cos^2\theta$, $M$ being the mass, $e$ the charge, and $a$ the angular momentum per unit mass of the black hole.

In the local Lorentz frame defined by this tetrad the electric and magnetic fields are parallel. Their invariantly defined values, derived from the potential

$$A=-er(\Sigma\Delta)^{-1/2}\omega^{(0)}, \tag{2}$$

are then given by

$$E_{(1)}=e\Sigma^{-2}(r^2-a^2\cos^2\theta) \tag{3a}$$

and

$$B_{(1)}=e\Sigma^{-2}2ar\cos\theta. \tag{3b}$$

The probability of pair creation, in the limit of very small creation rates, has been shown to be proportional to the transmission coefficient $T=\exp(-\xi)$ of the relativistic electron-positron field between the positive- and negative-energy states. The function $\xi$ is defined to be the opacity of the barrier against pair creation.

If $\mu M\gg 1$ it is possible to consider the electric and magnetic fields defined by Eqs. (3) as constant in a neighborhood of a few wavelengths of the point $r$, $\theta$, $\varphi$, $t$ we are considering.

We can then *locally* apply[16] the classical results of Sauter,[7] and Heisenberg and Euler,[8] on uniform and parallel electric and magnetic fields, to evaluate the opacity $\xi$ against the production of electron-positron pairs. We then obtain

$$\xi=\pi\mu^2/\epsilon E_{(1)}+2\pi(n+\tfrac{1}{2}+\sigma_{(1)})B_{(1)}/E_{(1)}. \tag{4}$$

Here $n$ is the quantum number associated with harmonic oscillation in the plane orthogonal to $E$ and $B$ and $\sigma_{(1)}$ is the spin of the electron in the common direction of $E$ and $B$. The minimum opacity is then obtained for $n=0$ and $\sigma_{(1)}=-\frac{1}{2}$.

The condition $n=0$ is equivalent to requiring that the created pair have $u_{(2)}=u_{(3)}=0$ which in turn implies physically that *the particles move, in the frame defined by Eqs. (1), along the common direction of the electric and magnetic field.*

This same result can be reinterpreted and given a different physical meaning by going back to the coordinate frame. We then have that the condition $n=0$ is equivalent to requiring that the angular momentum of the particle, $m=\mu u_\varphi+\epsilon A_\varphi$, is linked to its energy, $\omega=-(\mu u_t+\epsilon A_t)$, by the relation

$$m=a\omega\sin^2\theta. \tag{5}$$

Moreover the further dependence of Eq. (4) on $\sigma$ shows that the pairs created are polarized and the ratio of production of particles with spin $\sigma_{(1)}=-\frac{1}{2}$ and $\sigma_{(1)}=\frac{1}{2}$ is $\exp(2\pi B_{(1)}/E_{(1)})$ which can be very significant if the Kerr-Newman geometry is endowed with a large value of $a/M$ (~0.1).

Finally, the energy of the particle of the pair created is given by

$$\omega=e\epsilon r/\Sigma \tag{6}$$

which implies $\omega\sim10^{20}M/M_\odot$ eV for $M\lesssim10^7M_\odot$ and $\omega\sim10^{27}$ eV for $M\gtrsim10^7M_\odot$.

Still capitalizing on the frame defined by Eqs. (1) we can proceed to compute the rate $R$ of pair creation using Schwinger's approach.[9] Generalizing the formula given in Ref. 9 to the case in which both an electric and a magnetic field are present[16] we find that the number of pairs created is

$$N=\int 2\,\mathrm{Im}\mathcal{L}(|g|)^{1/2}d^4x, \tag{7}$$

where $(|g|)^{1/2}=\Sigma\sin\theta$ and

$$2\,\mathrm{Im}\mathcal{L}=(4\pi)^{-1}(E_{(1)}\epsilon/\pi)^2\sum_{n=1}^{\infty}n^{-2}(n\pi B_{(1)}/E_{(1)})\coth(n\pi B_{(1)}/E_{(1)})\exp\{-n\pi\mu^2/\epsilon E_{(1)}\}. \tag{8}$$

The total rate of particles created is simply given by $R=N$. Since for each pair created the particle (or antiparticle) with the same sign of charge as the background geometry is expelled at infinity with an energy $\omega=\epsilon er/\Sigma$ and an angular momentum $m=a\omega\sin^2\theta$ and the antiparticle (or particle) with the opposite sign is absorbed by the collapsed object we can give an explicit estimate of the decrease of charge, mass, and angular momentum of the black hole associated with this process of particle creation. We have

$$-\dot{e} = R\epsilon, \tag{9a}$$

$$-\dot{M} = R\langle\omega\rangle, \tag{9b}$$

$$-\dot{J} = R\langle m\rangle, \tag{9c}$$

where $\langle\omega\rangle$ and $\langle m\rangle$ represent some suitable mean value for the energy and angular momentum carried by the pairs. We then obtain

$$-\dot{e}/e \simeq \pi^{-3}(\epsilon\mu)^2 M_{\rm ir}(E/E_c)^2 \exp-(\pi E_c/E), \tag{10}$$

where $E_c = \mu^2/\epsilon$, $E \simeq e/4M_{\rm ir}^2$, and $(\epsilon\mu)^2\pi^{-3}M_{\rm ir} \simeq 7.0\times10^{32}M_{\rm ir}/M_\odot\,\sec^{-1}$, where $M_{\rm ir}$ is the irreducible mass of the black hole.[2] We can then estimate, corresponding to a $\Delta e = -e$, the change in the total mass energy[2] as well as the changes in irreducible mass and angular momentum of the black hole. We have $-\Delta M/M \simeq e^2/4M_{\rm ir}^2 \simeq 3\times10^{-14}(M_{\rm ir}/M_\odot)^2\,\sec^{-1}$ for $M_{\rm ir} \lesssim 10^7 M_\odot$ and from Eq. (5) follows $\Delta J/J \simeq \frac{2}{3}\Delta M/M$.

Finally, we expect the change of irreducible mass to be relatively small; we do find[16] $\Delta M_{\rm ir} \lesssim 10^{-2}\Delta M$. It is also interesting to notice that for fields $E \ll E_c$ the pair creation, though much reduced, approaches more and more reversibility.[16]

We can then reach the following conclusions: (1) For any black hole smaller than $7.2\times10^6 M_\odot$ the vacuum polarization and particle-creation processes by the Klein[6] mechanism can drastically modify their electromagnetic structure; as a direct consequence we can never have a magnetic field larger than $2\times10^{12}$ G (assuming the mass of the black hole $M \gtrsim 1.0 M_\odot$).[17] (2) If the electric field of a black hole reaches the critical strength $E \sim \mu^2/(24\epsilon)$ then an abrupt discharge by pair creation could occur releasing an energy $\Delta M \gtrsim 10^{41}$ erg (see Table I). (3) The process of particle creation is an irreversible transformation in the sense of Ref. 2 and carries away both the charge and part of the angular momentum from the black hole (compare and contrast the enormous difference from the case $e = 0$, see Ref. 12). The entire treatment presented here applies to the case of a "bare" solution in vacuum. It is important to approach the more general problem of the electrodynamics of a collapsed object sur-

TABLE I: Critical values of the electromagnetic fields and of the energy extractable from a Kerr-Newman geometry as a function of the irreducible mass. $B$ (bare) indicates the limits imposed by the geometrical condition $a^2 + e^2 \leq M^2$, $P$ (polarized) indicates the limits imposed by vacuum polarization. Vacuum-polarization effects are important in the absence of plasma only if $M_{\rm ir} \lesssim 7.2\times10^6 M_\odot$.

| $M_{ir}/M_\odot$ | Maximum strength of electromagnetic field in Gauss | Maximum net Charge in Electron Charge | Maximum energy Extractable in erg |
|---|---|---|---|
| 1 | B $1.18\times10^{19}$ / $2.00\times10^{12}$ P | B $2.14\times10^{39}$ / $3.63\times10^{32}$ P | B $1.79\times10^{54}$ / $5.15\times10^{40}$ P |
| $10^2$ | $1.18\times10^{17}$ / $1.88\times10^{12}$ | $2.14\times10^{41}$ / $3.41\times10^{36}$ | $1.79\times10^{56}$ / $4.55\times10^{46}$ |
| $10^4$ | $1.18\times10^{15}$ / $1.77\times10^{12}$ | $2.14\times10^{43}$ / $3.21\times10^{40}$ | $1.79\times10^{58}$ / $4.03\times10^{52}$ |
| $10^6$ | $1.18\times10^{13}$ / $1.67\times10^{12}$ | $2.14\times10^{45}$ / $3.03\times10^{44}$ | $1.79\times10^{60}$ / $3.59\times10^{58}$ |
| $10^8$ | $1.18\times10^{11}$ / $1.18\times10^{11}$ | $2.14\times10^{47}$ / $2.14\times10^{47}$ | $1.79\times10^{62}$ / $1.79\times10^{62}$ |

rounded by a plasma[18] for two different and opposite reasons: (1) The presence of a plasma can modify the conditions under which vacuum polarization occurs and allow it to reach higher values for the magnetic fields, and (2) one must examine the possibility of a suitable mechanism to build up the electromagnetic field needed for this process of vacuum polarization to occur.[18]

It is by now clear that processes of this kind can be of fundamental importance for the understanding of the physics of binary x-ray sources and possibly of galactic nuclei. In particular this work naturally leads to a most simple model for the explanation of the recently discovered $\gamma$-rays bursts.[19] It is desirable that possible coincidences between $\gamma$-ray bursts and changes in the spectrum and intensity of x-ray sources of the kind noticed in Cygnus X1 or Cygnus X3 be analyzed with great care.[20]

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