cross section calculation for the LMM potential. One of us (R.A.E.) wishes to thank the staff of LAMPF for a year's gracious hospitality.

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*On leave at Clinton P. Anderson Meson Physics Facility, Los Alamos, N. M. 87545.

‡Present address: Lawrence Berkeley Laboratory, Berkeley, Calif. 94720.

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Polarization in Proton-Proton Scattering at 10 MeV*

J. D. Hutton and W. Haeberli
University of Wisconsin, Madison, Wisconsin 53706

and

L. D. Knutson
University of Wisconsin, Madison, Wisconsin 53706, and University of Washington, Seattle, Washington 98195

and

P. Signell
Michigan State University, East Lansing, Michigan 48824

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The polarization in proton-proton scattering at 10.0 MeV has been measured at seven angles with an accuracy of $\pm 2\times 10^{-4}$. Model-independent values of the phase shifts at 10 MeV are deduced from an analysis of the cross-section and polarization data. The implications for existing energy-dependent phase-shift sets are examined.

Measurements of the polarization in proton-proton scattering at low energies can provide information about the p-wave phase shift caused by the nucleon-nucleon tensor and spin-orbit interactions. However, such polarization measurements are useful only if they are of very high accuracy because the low-energy p-wave phase shifts are small. In fact, Noyes and Lipinsky[1] have argued that polarization measurements near 10 MeV are not likely to be useful since the required accuracy of $\pm 4\times 10^{-4}$ seemed impractical to achieve. In the absence of such experimental data, p-wave phase shifts in this energy region have traditionally been obtained by extrapolation from higher energies by use of phenomenological representations of unknown accuracy. In this Letter we present measurements of the polarization in p-p scattering at 10.0 MeV accurate to $\pm 2\times 10^{-4}$, which is more than an order of magnitude more accurate than any previous measurement.[2] The results are used to deduce values of the 10-MeV s- and p-wave phase shifts which are independent of any model or assumption about the energy dependence of the phase shifts. The data thus bear on the recent controversy surrounding the low-energy cross-section normalizations and p-wave phase shifts.

The measurement was carried out by bombarding a gaseous hydrogen target with polarized protons and observing the left-right asymmetry of scattered protons in two detectors located symmetrically to the left and right of the incident beam. The experiment thus in fact determines the analyzing power rather than the polarization. However for elastic scattering of strongly interacting particles the two quantities are equivalent.[3] The energy at the center of the target was 10.00 ± 0.05 MeV. The polarized beam was obtained from a tandem electrostatic accelerator equipped with a polarized-ion source. The beam polarization (75–80%) was monitored continuously by observing p-^{4}He scattering in a polarimeter mounted at the beam-exit port of the scattering chamber.[4] For each measurement, scattered protons were counted for spin up and spin down. The direction of the polarization was reversed by reversing the current in a spin-precession solenoid located between the ion source and the accelerator.

The scattered protons were detected by silicon surface-barrier detectors located 32 cm from the center of the target. The angular acceptance of the detection system was $\pm 0.5°$. In order to measure the analyzing power accurately, it is important to obtain a clean pulse-height spectrum which has no significant background or contaminant peaks under the peak of interest. In the observed spectra, the background was flat and unstructured on the low-energy side of the peak. The measured asymmetry of this background was less than 5×10^{-3} and the peak-to-background ratio was 10^3. The background on the high-energy side was negligible. The dead time of the detection system was measured; the resulting correction to the analyzing power was less than 5×10^{-5}.

The target consisted of 1 or 2 atm of hydrogen gas in a cylindrical cell 20 cm in diameter. At most angles, small peaks corresponding to elastic scattering from contaminants (C, N, O) and deuterium could be seen in the spectrum. The level of impurities (< 0.01%) was monitored by observing protons scattered at 60°. For the measurement at $\theta_{\mathrm{lab}} = 10°$ elastic scattering from the contaminants was not resolved from the p-p scattering, and the measured analyzing power was corrected by 5×10^{-5}. The effects of inelastic scattering by the contaminants and elastic scattering by deuterium are insignificant.

The experiment is insensitive to intrinsic small left-right asymmetries in the detection geometry,[5] but is extremely sensitive to variations in the position and direction of the incident beam, particularly if these variations are correlated with the reversal of the beam polarization. The incident beam was defined by one slit 2.6 m from the target, and a second slit at the target entrance window 0.15 m from the center of the target. The first slit was 1.0 mm wide for measurements at $\theta_{\mathrm{lab}} = 10°$ where the asymmetry is most sensitive to the beam shifts, and 3 mm wide for the remaining measurements. The second slit was 0.6 mm wide. For both slits, a feedback system sensed the beam current on the left and right slit jaws and kept the beam centered by controlling trim magnets some 3 m away. Additional slits inside the gas cell prevented protons scattered by the entrance foil and by the beam-defining-slit edges from illuminating the detector-slit systems.

To investigate the effects of beam shifts, the beam polarization was set to zero, and the asymmetry resulting from the reversal of the current in the spin-precession solenoid was measured. Eleven measurements of the asymmetry were made, mostly at forward angles where the effect of beam displacements are greatest. The mean asymmetry was $(2\pm 9)\times10^{-5}$.

For each scattering angle the analyzing-power measurement was divided into a number of separate runs, during which about 10^6 counts were collected in each detector for spin up and spin down. By comparing the scatter in these individual measurements with the statistical errors, one can detect the presence of random fluctuations beyond the statistical fluctuations. It was found that, averaged over the entire data set, the scatter was slightly larger than expected. To account for the possibility that the measurements may be subject to nonstatistical random fluctuations, an additional error of 2.5×10^{-4} was added in quadrature with the statistical error for each individual measurement. The magnitude of this

TABLE I. Measured values of the analyzing power for proton-proton scattering at 10.0 MeV. The column labeled N gives the number of individual measurements at each angle.

$\theta_{\mathrm{c.m.}}$ (deg)	N	$10^4 A$
20	9	-19.4 ± 1.7
30	5	-16.1 ± 1.5
40	4	-6.5 ± 1.8
50	5	-1.8 ± 1.5
60	3	-2.5 ± 2.2
70	6	-1.7 ± 1.4
80	4	2.0 ± 2.2

TABLE II. Phase-shift parameters for p-p scattering at 10.0 MeV. The superscript E indicates that the phase parameters are of the "electric" type (Refs. 8 and 9).

${}^1S_0{}^E$	$55.38° \pm 0.15°$
${}^3P_0{}^E$	$2.62° \pm 0.40°$
${}^3P_1{}^E$	$-1.94° \pm 0.10°$
${}^3P_2{}^E$	$0.64° \pm 0.09°$

additional error contribution was chosen to make the errors in the individual measurements consistent with the observed scatter. The individual measurements were then combined to obtain a single analyzing-power value and uncertainty at each angle. The results are given in Table I.

The present measurements permit for the first time a model-independent determination of the phase parameters at an energy below 25 MeV. The simultaneous analysis[6] of the 9.918-MeV cross sections of Jarmie *et al.*[7] and of the data in Table I yields the phase parameters presented in Table II. These parameters fit the ten cross-section measurements with $\chi^2 = 6.5$ and the seven analyzing-power measurements with $\chi^2 = 5.6$ (total χ^2 per degree of freedom is 0.93). The calculated analyzing power is shown as the solid line in Fig. 1. Also shown is the analyzing power predicted from the "energy-dependent" analysis of MacGregor, Arndt, and Wright[10] (dashed curve). The prediction from the phase shifts measured by Seamon *et al.*[11] is essentially identical.

The analyzing power in low-energy p-p scattering depends primarily on the tensor p-wave phase-shift combination[8] Δ_T and the spin-orbit p-wave phase-shift combination Δ_{LS}. The cross section is relatively insensitive to these parameters, as it depends primarily on the ${}^1S_0$ phase shift and the central p-wave combination Δ_C. From our analysis we find

$$\Delta_C = -0.003° \pm 0.034°, \quad \Delta_T = -0.812° \pm 0.055°,$$

$$\Delta_{LS} = 0.31° \pm 0.11°.$$

The value of Δ_T is consistent with $\Delta_T = -0.91° \pm 0.28°$ at $E_p = 9.69$ MeV which Noyes and Lipinski[1] deduced on the basis of a single A_{yy}/A_{xx} spin-correlation measurement. Recent energy-dependent analyses[12] predict values of Δ_T and Δ_{LS} in the range $-1.50° \leq \Delta_T \leq -0.96°$ and $0.02° \leq \Delta_{LS} \leq 0.40°$.

Recently, Arndt, Hackman, and Roper[13] (AHR) proposed that the absolute normalization of the high-accuracy p-p cross sections between 1 and 10 MeV should be "floated" (i.e., treated as adjustable parameters), because, by doing so, they obtained a significantly improved fit with their energy-dependent phase-shift representation. However, the value of Δ_T obtained from the AHR floated 1–27.6-MeV analysis ($\Delta_T = -1.04° \pm 0.07°$ at 10 MeV) turned out to be strikingly different from that of their corresponding unfloated one ($\Delta_T = -1.50° \pm 0.05°$). Our analysis shows that the larger value of $|\Delta_T|$ is entirely inconsistent with the new 10-MeV data set independent of whether the cross-section normalization is floated or not.[14] Thus our results strongly reinforce the preference for AHR's floated phase-shift solution over their unfloated one. However, it should be emphasized that the cross-section norm from the AHR 1–500-MeV floated analysis is inconsistent with the norm of Jarmie *et al.*'s cross-section measurements,[7] and that subsequent to AHR's analysis Jarmie *et al.* have found no reason to doubt their absolute cross-section normalization or its uncertainty.[15] In addition, the value of Δ_C from AHR's floated 1–500-MeV analysis is in substantial disagreement with the Δ_C from their 1–27.6-MeV analysis. These matters will certainly bear further investigation.

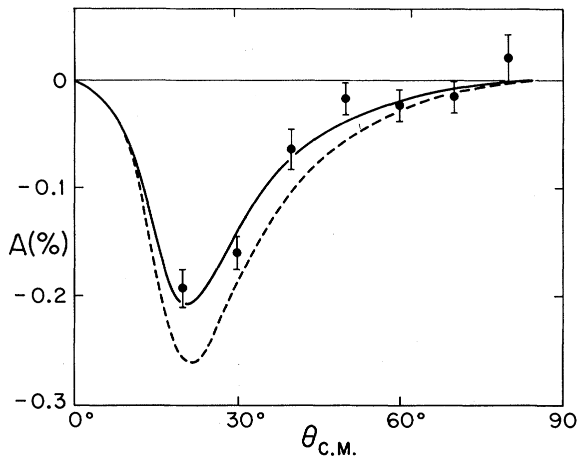

FIG. 1. Analyzing power for proton-proton scattering at 10.0 MeV. The dashed curve shows the analyzing power predicted from the energy-dependent analysis of MacGregor, Arndt, and Wright (Ref. 10). The solid curve is the result of a single-energy phase-shift analysis of the present analyzing-power data and the cross-section data of Jarmie *et al.* (Ref. 7).

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[14] If the cross-section normalization is "floated" we obtain $\Delta_C = 0.032° \pm 0.060°$, $\Delta_T = -0.847° \pm 0.069°$, and $\Delta_{LS} = 0.30° \pm 0.11°$ with a χ^2 per degree of freedom of 0.94.

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Phase Transitions, Two-Level Atoms, and the A^2 Term

K. Rzążewski* and K. Wódkiewicz*
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627, and Institute of Theoretical Physics, Warsaw University, 00-681, Warsaw, Poland

and

W. Żakowicz
Institute of Nuclear Research, 00-681, Warsaw, Poland

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We show that the presence of the recently discovered phase transition in the Dicke Hamiltonian is due entirely to the absence of the A^2 terms from the interaction Hamiltonian.

Consider the well-studied Hamiltonian

$$H_1 = \frac{\hbar\omega_{ba}}{2}\sum_{j=1}^{N}\sigma_j^{z} + \hbar\omega a^\dagger a + \frac{\lambda}{\sqrt{N}}\sum_{j=1}^{N}(\sigma_j^{+} a + \sigma_j^{-} a^\dagger). \tag{1}$$

This Hamiltonian describes the collective interaction of a single mode of radiation (frequency ω) with a single transition between levels a and b (frequency $\omega_{ba} > 0$) in N identical two-level atoms. Operators a and $a^\dagger$ denote here the annihilation and creation operators of the photons; σ_j^z, σ_j^+, σ_j^- are Pauli matrices used to describe the jth atom. The Hamiltonian (1), sometimes called the Dicke Hamiltonian,[1] may be derived[2] from the more familiar one

$$H = \sum_{j=1}^{N}\left[\frac{1}{2m}\left(\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right)^2 + V(\vec{r}_j)\right] + \hbar\omega a^\dagger a \tag{2}$$