ic Pb, Eqs. (3) and (4) yield the $\alpha^2(Z\alpha)^2$ correction

$$\delta E = 0.76(3)\ \text{eV}. \qquad (5)$$

The value given by (5) for the $\alpha^2(Z\alpha)^2$ vacuum-polarization correction to the $5g$-$4f$ transition energies of muonic Pb is in complete agreement with the estimate obtained by Wilets and Rinker, and disagrees in both sign and order of magnitude with the result of Chen. The method of the present investigation is vastly different from those of the two previous calculations. Even with a generous allowance for systematic errors in the numerical evaluations, for refinements in the static-muon, massless-electron approximation scheme, and for corrections of higher order in $(Z\alpha)^2$, it seems reasonable to conclude that the order-α^2 two-photon vacuum-polarization correction for muonic transition energies is negligible; at most only about 1 or 2 eV in the $5g$-$4f$ transitions of muonic Pb.

The author is indebted to L. S. Brown for suggesting the present investigation and the crucial approximation scheme. Throughout this work, analytical advice has been provided by D. G. Boulware who has also patiently checked the calculations preceeding the computer work. Finally, very useful numerical techniques evolved from conversations with S. J. Brodsky, R. N. Cahn, and L. Wilets. It is a pleasure to thank all of these people for their help.

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[9] The type of variable change used here is described by M. J. Levine and J. Wright, Phys. Rev. D 8, 3171 (1973).

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Has ψ_4 Already Been Observed at Stanford Linear Accelerator Center?

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Okubo mass splitting is generalized to Han-Nambu color theory and used to predict a fourth ψ particle with mass about 4.8 GeV/c^2 and partial width into e^+e^- of 0.8 keV.

We consider the possibility that the newly discovered ψ particles[1] belong to (1, 8) and (8, 8) multiplets in the SU(3′)⊗SU(3″) strong-interaction scheme of Han and Nambu.[2] In this theory the photon is a U-spin scalar with respect to both SU(3′) and SU(3″), transforming like (1, 8) + (8, 1). If we suppose that the group breaks down to SU(2′)⊗U(1′)⊗SU(2″)⊗U(1″) and that there is mixing between the (1, 8) and (8, 8) multiplets [but not with (8, 1) or (1, 1)] the photon will couple to particles which are mixtures of the neutral states which we denote by[3]

$$(0,1)_{18} \pm (0,1)_{88}, \qquad (1)$$

and

$$(0,0)_{18} \pm (0,0)_{88}. \qquad (2)$$

The two former particles are stable against decay into hadrons on the assumption that they are the least massive bosons with nonvanishing I'' spin. They may be identified with $\psi_1(3.1)$ and $\psi_2(3.7)$. The other two particles decay strongly into hadrons and the lightest is $\psi_3(4.1)$. There are two other neutral particles which do not couple to the photon (colored K^* mesons),

$$(0, \tfrac{1}{2})_{18} \pm (0, \tfrac{1}{2})_{88}. \tag{3}$$

To make this scheme quantitative, we introduce a mass splitting and mixing mechanism which is the direct generalization of that proposed by Okubo[4] for the familiar "white" hadrons. On this basis the mass operator transforms as

$$M = M_{11} + M_{81} + M_{18}. \tag{4}$$

If we denote I'' spin by j $(=0, 1, \tfrac{1}{2})$ and the Okubo factor by

$$k_j = j(j+1) - y^2/4, \tag{5}$$

where y is the Y'' value corresponding to j in the octet representation, the relevant diagonal matrix elements of the mass operator may be written (assuming linear mass breaking)

$$\langle 1, 8, j|M|1, 8, j\rangle = m_{18} + k_j\delta \equiv m_1(j), \tag{6}$$

$$\langle 8, 8, j|M|8, 8, j\rangle = m_{88} + k_j\Delta \equiv m_8(j), \tag{7}$$

$$\langle 8, 1|M|8, 8\rangle = 0 \tag{8}$$

(i.e., no mixing with white hadrons), and

$$\langle 1, 8, j|M|8, 8, j\rangle = \langle 1, 8|M_{81}|8, 8\rangle = d, \tag{9}$$

independent of j. Thus the six particle masses and three mixing angles are determined by five parameters (m_{18}, m_{88}, δ, Δ, and d) implying four relations between them. For each j the mass matrix is

$$\begin{pmatrix} m_8(j) & d \\ d & m_1(j) \end{pmatrix}. \tag{10}$$

If these matrices are expressed in terms of their eigenvalues M_j^+, M_j^- and mixing angles[5] θ_j, the four relations can be shown to be the generalized Okubo formulas:

$$\cot 2\theta_1 + 3\cot 2\theta_0 = 4\cot 2\theta_{1/2}, \tag{11}$$

$$(M_1^+ + M_1^-) + 3(M_0^+ + M_0^-) = 4(M_{1/2}^+ + M_{1/2}^-), \tag{12}$$

$$(M_1^+ - M_1^-)\sin 2\theta_1 = (M_0^+ - M_0^-)\sin 2\theta_0 = (M_{1/2}^+ - M_{1/2}^-)\sin 2\theta_{1/2}. \tag{13}$$

As the five pieces of data to determine the system we take[1,6] the three observed masses (GeV/c^2)

$$M_1^- = 3.09, \quad M_1^+ = 3.68, \quad M_0^- = 4.15 \tag{14}$$

and two mixing angles. These can be determined from ratios of the three partial decay widths into e^+e^-, which depend on the admixture of U″ singlet state in the physical particle,[7]

$$\Gamma_j^- = C_j \sin^2\theta_j\, M_j^-, \qquad \Gamma_j^+ = C_j \cos^2\theta_j\, M_j^+, \tag{15}$$

where C_j is the same for all particles j. Ignoring the large experimental errors, we take[1,6]

$$\Gamma_1^- = 4.8 \text{ keV}, \quad \Gamma_1^+ = 2.2 \text{ keV}, \tag{16}$$

to find that

$$\Gamma_0^- = 2.2 \text{ keV}, \quad \sin\theta_0 \simeq \sin\theta_1 \simeq \sqrt{\tfrac{2}{3}}. \tag{17}$$

This is "magic" mixing familiar in the SU(3) interpretation of ρ, ω, and φ and predicts a fourth resonance[8]

$$M_0^+ = 4.75 \text{ GeV}/c^2, \tag{18}$$

with partial width into e^+e^- of

$$\Gamma_0^+ = 0.8 \text{ keV}. \tag{19}$$

This will have total width similar to ψ_3 and should give rise to a broad hump in the cross section of e^+e^- into hadrons with an integrated area above background somewhat less than half that of ψ_3. The last three points of the recently published data[6] (see Fig. 1) can easily be interpreted as evidence for such an effect, particularly if the

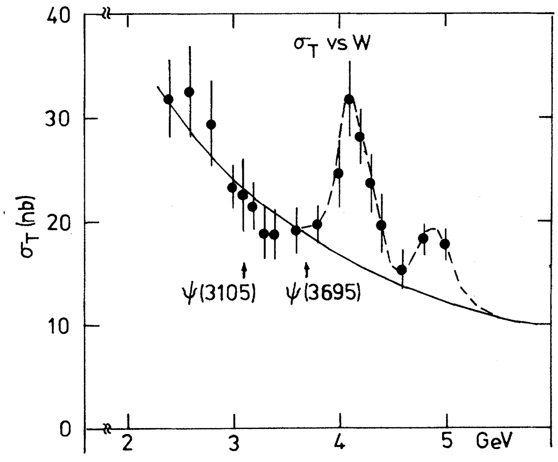

FIG. 1. Recently published data (Ref. 6) with the addition of two curves drawn by eye to aid the imagination.

nonresonant background is assumed to fall with increasing energy, as is to be expected on simple theoretical grounds.[9]

This theory makes a number of other simple predictions: (i) ψ particles are produced in pairs in white-hadron collisions. (ii) The main hadronic decay modes of ψ_1 contain a real photon.[10] (iii) There are charged counterparts (positive and negative) to ψ_1 and ψ_2 (colored ρ mesons) with the same masses (apart from electromagnetic effects). The heavier decays into the lighter like $\psi_2 \rightarrow \psi_1$. The lighter has only weak decay modes. (iv) There are two colored 1^- K^* mesons of masses 3.9 and 4.5 GeV/c^2 [from (11), (12), and (13)]. (v) There are 56 other colored vector mesons of the (8, 8) multiplet with masses in the 4–5 GeV/c^2 range all fixed by one further parameter. The lightest of these has a main decay mode into $\psi_1 + K$.

A more detailed discussion of the above points, together with an exposition of their weak interactions consistent with Salam-Weinberg theory and the observed selection rules of neutral currents for white hadrons will be published elsewhere.

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[2] M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).

[3] We signify the neutral states by their I' spin, I'' spin, and multiplets; e.g., $(I', I'')_{n'n''}$. The combinations ± indicate mixing but are not intended to imply that it is 50:50.

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[5] We have adopted the convention that the state corresponding to M_j^- is $|-, j\rangle = \cos\theta_j |8, 8\rangle - \sin\theta_j |1, 8\rangle$.

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[7] The dependence on the mass is that which gives the best fit of "magic" mixing to the observed e^+e^- partial widths of ρ, ω, and φ. D. R. Yennie, Phys. Rev. Lett. 34, 239 (1975).

[8] The errors on the observed partial widths are at present so large that M_0^+ cannot be accurately predicted. However, universal magic mixing is so appealing that one is tempted to take it seriously. If we had worked with squared masses, magic mixing would give a somewhat lower value for M_0^+ in which case interference effects with M_0^- would be important.

[9] See particularly Fig. 1 which may be interpreted as two wide bumps sitting on a falling background which passes smoothly through the data points for $E_c < 3$ GeV.

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Measurement of $\psi(3.1)$ Meson Production by Pions and Protons*†

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The production of $\psi(3.1)$ mesons is reported for the reactions $\pi^- + \mathrm{Fe} \rightarrow \mu^+ + \mu^- +$ anything, at 200 GeV, and $p + \mathrm{Fe} \rightarrow \mu^+ + \mu^- +$anything, at 240 GeV. For ψ production, distributions in $x \equiv P_L/P_{\mathrm{beam}}$ and $P_\perp$ are given. For $x \geq 0.5$, the ratio of the ψ production cross sections in iron for pions to that for protons is found to be 7.4 ± 2.0.

We report here results of an experiment carried out at the Fermi National Accelerator Laboratory (FNAL) in which enhancements are observed in the dimuon invariant-mass spectra at about 3.1 GeV. The reactions studied were

$$\pi^- + \mathrm{Fe} \rightarrow \mu^+ + \mu^- + \text{anything}, \quad P_B = 200 \text{ GeV}, \tag{1}$$

and

$$p + \mathrm{Fe} \rightarrow \mu^+ + \mu^- \text{ anything}, \quad P_B = 240 \text{ GeV}, \tag{2}$$

where P_B is the monoenergetic beam momentum. We interpret the enhancements, whose widths are consistent with the resolution of our apparatus, as the $\psi(3.1)$ meson.[1]

The μ-pair detector is shown in Fig. 1(a). The μ pairs were created at the front end of the first iron (Fe) absorber. Muons were identified by their traversal of 5.6 m of Fe. Muon momenta and angles were measured using a 56-kG-m gapless magnet and associated wire-chamber system.