mance of our system given in terms of the mean effective noise energy $\overline{D} \simeq 0.034kT_r$ compares favorably with the best results reported in higher-frequency bands, we have inevitably arrived at a rather large limit for the GR energy spectrum density $F(\nu_0)$ at 145 Hz given by Eq. (14).

We are grateful to Mr. Kenji Tsunemoto and Mr. Masa-Katsu Fujimoto for their contributions during various phases of the experiment. We thank Mr. Y. Kamiya and Mr. K. Tsubono for their cooperation.

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[3]Details of our GR detectors will be published elsewhere.

[4]On the assumption that the resonance width ν/Q of the antenna cross section $\sigma(\theta, \varphi, \nu)$ is much smaller than the bandwidth of the incoming GR pulse spectrum $F(\nu)$, Eq. (3) is derived from $D_G = \int \sigma(\theta, \varphi, \nu) F(\nu)\, d\nu = F(\nu) \int \sigma(\theta, \varphi, \nu)\, d\nu$.

[5]These time constants were chosen so that the lowest possible temperatures are achieved for D_i.

[6]J. L. Levine and R. L. Garwin, Phys. Rev. Lett. 31, 173 (1973).

[7]In this case $\phi(-1)$ and $\phi(+1)$ have a finite magnitude, $0.11\phi(0)$.

Simple Quark-Counting Predictions for Band Structure in $p + \mathrm{Be}^9 \rightarrow (h^+ + h^- + X, h^+ + h^+ + X, h^- + h^- + X)$*

Alan Chodos and Jorge F. Willemsen
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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A model based on simple quark counting is presented, which describes the inclusive production of charged hadron pairs at large transverse momenta. Predictions are given for the ratios of cross sections for $p + \mathrm{Be}^9 \rightarrow h_1 + h_2 + X$, where h_1 and h_2 are any of p, $\bar{p}$, π^+, π^-, K^+, and K^-. The essential feature of the model is that the valence quarks of the initial particles must be included in the counting.

Recently, preliminary data have been reported[1] on the reaction

$$p + \mathrm{Be}^9 \rightarrow h^+ + h^- + \text{anything} \qquad (1)$$

at 28 GeV incident energy. Here $h^+ + h^-$ represents a pair of oppositely charged hadrons which are detected coming off at approximately 90°, back to back, in the center-of-mass system. The observed positively charged hadron is p, π^+, or K^+; the negatively charged hadron can be any of the corresponding antiparticles. Thus, in all, nine reactions have been studied.

Because the data have not been fully analyzed, only the gross features are known. These are as follows:

(i) For each reaction, the cross section $d\sigma/dM$ is observed to fall exponentially as a function of M in the region $2 \lesssim M \lesssim 5$ GeV/c^2. Here M is the invariant mass of the emitted pair. All the cross sections fall with apparently identical slopes.

(ii) Instead of nine different curves, one observes three bands with an order-of-magnitude separation between bands. The $p + \pi^-$ points form the top band. The cross sections for $\pi^+ + \pi^-$, $K^+ + \pi^-$, $p + \bar{p}$, and $p + K^-$ cluster to form the second band. The spread of this band is about a factor of 2. In the bottom band are the reactions producing $\pi^+ + K^-$, $K^+ + K^-$, $\pi^+ + \bar{p}$, and $K^+ + \bar{p}$. These are spread by a factor of perhaps $2\frac{1}{2}$ or 3.

In this Letter we present a simple model which accounts quite well for feature (ii) of the data, and which is consistent with feature (i). In addition the model is easily generalized to the cases of ++ or −− charges in the final state, and we make predictions for these. We also discuss the changes to be expected if experiments using a π beam instead of a p beam are performed.

The model is most simply motivated by first introducing a rule, allegedly due to Feynman,[2] which summarizes the band structure of feature (ii): Assign the number zero to the proton, one to the π^+, K^+, and π^-, and two to the $\bar{p}$ and K^-. Then the top band contains those h^+-h^- pairs whose numbers sum to one, the middle band to two, and the bottom band to three.

We propose that the physical meaning of the rule may be understood by asking the following question: How many new kinds of quarks (i.e., quarks not brought into the reaction by the incident particles) must be created in order to produce the given particle? Since the incident quarks are four u quarks and two d quarks or three u quarks and three d quarks depending on whether a proton or neutron in the Be^9 target is struck, the answer is zero for p; one for π^+, K^+, and π^-; two for K^-; and three for $\bar{p}$.

Though this assignment for $\bar{p}$ disagrees with Feynman's rule, we are led to make two assumptions about the reactions (1):

(I) The incident quarks participate fully in forming the final state. In other words, these are reactions in which the incident particles are *stopped.* There should be no leading-particle effect, and no fragments of the target or beam go sailing through the reaction. This should be an experimentally testable assumption.

(II) Quark-antiquark pair production is suppressed in this regime. For if pair production were copious, it would swamp the observed correlation between incident and emitted quarks.

In addition to these principal assumptions, which are crucial to the success of our model, we must make two additional technical assumptions; the physical implications of these added assumptions are discussed below:

(III) In order to be able to quantify our predictions, we must assume a definite distribution function for the probability of producing N q-$\bar{q}$ pairs. As a zeroth-order guess, we assume that the emission of each pair is independent of any other, and therefore follows a Poisson distribution:

$$P(N) = (\alpha^N/N!)(u\bar{u}+d\bar{d}+\gamma s\bar{s})^N. \qquad (2)$$

α is the only important parameter in the model. It expresses the difficulty of producing pairs. The parameter γ which appears in (2) can be used to suppress further the production of strange pairs. At the rather crude level at which we are working, the introduction of γ is really only window dressing. We shall, however, include predictions for the band structure both with and without γ, just to illustrate what effect it has.

(IV) We assume that q-$\bar{q}$ pairs tend to be produced collinearly rather than back to back. Therefore, in calculating our cross sections, we systematically exclude configurations in which the members of the same q-$\bar{q}$ pair are used in making the two oppositely moving hadrons. The reasons for, and implications of, this assumption will be discussed further.

Having made our four assumptions, we proceed to calculate the relative weights of the nine processes, ignoring any further questions of kinematics. All we do is count. We assume that in order α^N, the collision produces a statistical goo composed of the incident quarks u^4d^2 for proton on proton (or u^3d^3 for neutron on proton), plus the conglomeration $(u\bar{u}+d\bar{d}+\gamma s\bar{s})^N$ of pairs. We then count the number of ways consistent with assumption (IV) of picking the final configuration of quarks [e.g., $(\bar{u}^2\bar{d})+(u\bar{s})$ in the case of $\bar{p}+K^+$] out of this goo, multiply by $\alpha^N/N!$ according to assumption (III), sum on N, and, finally, average the proton and neutron cases in the ratio 4:5 to get our result. Of course we do not predict the absolute cross section; only the ratios of these numbers are supposed to be significant in this consideration. We shall illustrate our method of calculation with the example $p+p\to\bar{p}+K^++X$. First we sum our distribution over N to get the full goo function

$$G_p(\alpha) = u^4d^2e^{\alpha S}, \qquad (3)$$

where $S \equiv u\bar{u}+d\bar{d}+\gamma s\bar{s}$. Then we project out the final configuration by taking partial derivatives with respect to the constituents of that configuration. Since $\bar{p}+K^+$ consists of $(\bar{u}^2\bar{d})+(u\bar{s})$,

$$F_p(\bar{p}K^+|\alpha\gamma) = \frac{1}{2}\frac{\partial^2}{\partial\bar{u}^2}\frac{\partial}{\partial\bar{d}}\frac{\partial}{\partial\bar{s}}\frac{\partial}{\partial u}G_p(\alpha)\Big|_{\text{all } q=\text{all } \bar{q}=1}. \qquad (4)$$

We set $q=\bar{q}=1$ at the end to take into account the inclusive nature of the process. Assumption (IV) is implemented by the rule that derivatives with respect to antiquarks are to be taken first, such

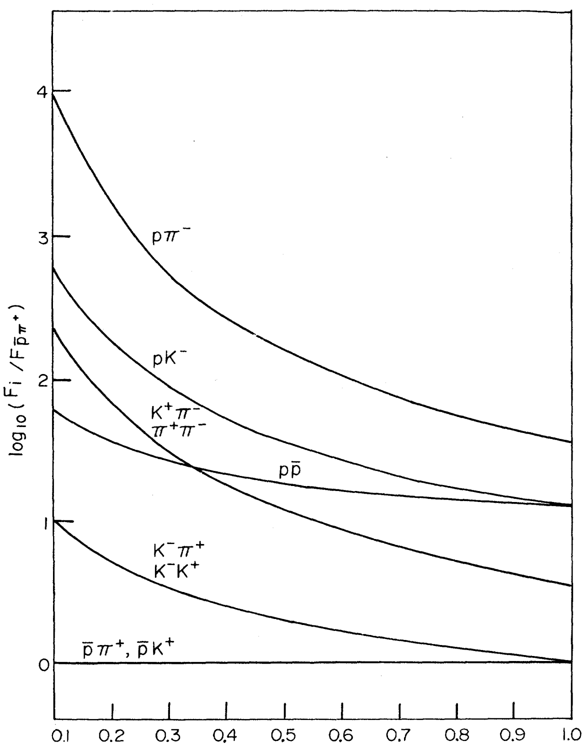

FIG. 1. The cross sections $F(h^+h^-)$ as a function of α, $0.1 \leq \alpha \leq 1.0$, for $\gamma = 1$. $F(\bar{p}\pi^+)$ has been normalized to 1.

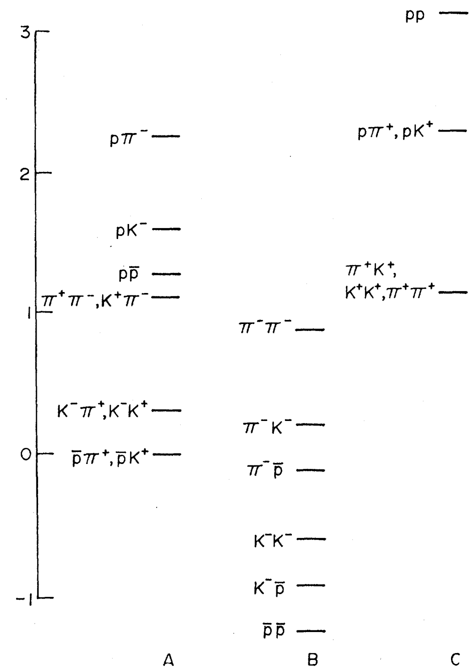

FIG. 2. A plot of $\log_{10}[F(h_1h_2)/F(\bar{p}\pi^+)]$ for $\alpha = 0.475$ and $\gamma = 1.0$. Column A contains the $+-$ predictions, column B the $--$, and column C the $++$.

that when S is differentiated the corresponding quark variable brought down is immediately set equal to 1. With these rules in Eq. (4) one finds

$$F_p(\bar{p}K^+|\,\alpha\gamma) = \tfrac{1}{2}\gamma\alpha^4(\alpha + 4)e^{\alpha S}. \qquad (5)$$

The reader can check that (5) is the same as the order-by-order procedure described earlier.

Finally we multiply (5) by a factor of 2 to count the spin degrees of freedom of the $\bar{p}$ in the final state:

$$\sum_{\text{spins}} F_p(\bar{p}K^+|\,\alpha\gamma) = \gamma\alpha^4(4 + \alpha)e^{\alpha S}. \qquad (6)$$

We then average this with the result for a neutron.

Once the nine functions have been computed in this manner, the eight available ratios are formed.

In Fig. 1 we plot the common logarithms of these eight ratios against α, with $\gamma = 1$. Note that certain degeneracies are present: $(\pi^+\pi^-) = (K^+\pi^-)$; $(K^-\pi^+) = (K^-K^+)$; and $(\bar{p}\pi^+) = (\bar{p}K^+)$. The main effect of assumption (IV) has been to suppress the predicted cross section for $K^+ + K^-$. Without assumption (IV), we could have produced $K^+ + K^-$ by creating one u-$\bar{u}$ pair and one s-$\bar{s}$ pair. This would have resulted in a $K^+ + K^-$ cross section about halfway between the two bottom bands. With assumption (IV) we need to create at least one more s-$\bar{s}$ pair. Thus the position of the $K^+ + K^-$ cross section gives some information about the q-$\bar{q}$ production mechanism.

From Fig. 1 we see that for $\alpha < 0.4$ the $p + \pi^-$ cross section becomes too large, as does the spread in band III.

For $\alpha > 0.6$, the bands lie too close together and the spread in band II is too large. Thus $0.4 < \alpha < 0.6$ defines an acceptable range. We have settled on $\alpha = 0.475$ as the value which optimizes the clustering within bands and the separation between bands.

Figure 2 shows our predictions for the nine $h^+ + h^-$ cross sections, as well as the six $h^+ + h^+$ and six $h^- + h^-$ with $\alpha = 0.475$ and $\gamma = 1$. Figure 3 is the same except that γ has been taken equal to 0.69 for purposes of illustration. We see that the fine structure of some of the clusters changes with γ, but the overall picture remains much the same.

Perhaps the worst feature of our predictions is the spread of band II. In the acceptable range of α, it cannot be made less than a factor of 3, and for any α-γ combination the spread between

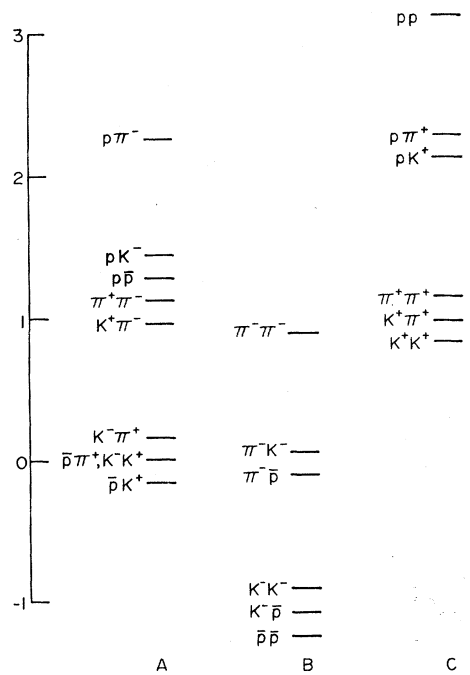

FIG. 3. A plot of $\log_{10}[F(h_1h_2)/F(\bar{p}\pi^+)]$ for $\alpha=0.475$ and $\gamma=0.69$. The columns are arranged as in Fig. 2.

$p+K^-$ and $K^++\pi^-$ is always at least 2.44. This is very likely the price paid for the simplicity of the Poisson distribution.

We remark that if a π beam were used to produce the same final states, our predictions would change drastically, since the contribution of the incident quarks changes from u^4d^2 to $u^2d^2\bar{u}$ in going from p on p to π^- on p. Thus *an important test of our ideas is to repeat the experiment using a pion beam.* Predictions for this case follow unambiguously from the model, and calculations are currently in progress.

We need not burden the reader with a catalog of the many dynamical and kinematical effects that we have simply neglected. We should just like to point out one effect which can be handled by our model and which we are also currently investigating. For simplicity, we have assumed that a $(u\bar{s})$ pair, for example, always emerges as a K^+, and have ignored the possibility that it is a K^{*+}, which then decays into either $K^++\pi^0$ or π^++K^0. Thus $(u\bar{s})$ should really count as K^+ some of the time and π^+ the rest of the time. This effect, extended to include all members of the pseudoscalar and vector octets and the baryon octet and decuplet, will rearrange the multiplicities of the final states. The extent to which our simple model matches the gross trends of the available h^++h^- data leads us to hope that these effects will alter the interband "fine structure" more significantly than the band-band structure.[3] However, it is difficult to extrapolate this hope to the doubly charged cases without more detailed calculations.

We close with a few speculations about the dynamics which may underlie our model. Since we are imagining that a fireball, composed of both incident particles and produced q-$\bar{q}$ pairs, gives rise to the final state, we are relatively comfortable with the observed exponential falloff as a function of M. We envision the q-$\bar{q}$ pairs to be produced by a mechanism akin to bremsstrahlung, which would naturally lead to a Poisson distribution[4] (although a Poisson distribution is probably not crucial; any distribution which favors a small number of pairs will do). Furthermore, assumption (IV) can be understood as the statement that the intermediate particle in the bremsstrahlung process itself carries a fair amount of momentum in the center-of-mass system, so that the likelihood of the members of any single pair coming off back to back at 90° in the center-of-mass system is small. Finally, the overall suppression of q-$\bar{q}$ pair production can be achieved if the coupling g of this intermediate particle to the quarks is not too large. Clearly, the value of α is related to g^2, but the extraction of the exact relation is presently outside the scope of our model. If our model continues to explain the trends of the data, this will be strong evidence that these reactions belong to a dynamical regime in which mechanisms different from those usually considered are likely to play an important role.[5]

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[1]These data have been taken by the Massachusetts Institute of Technology–Brookhaven National Laboratory group, and have been reported in various seminars by U. Becker and S. C. C. Ting.

[2]Communication from R. P. Feynman to S. C. C. Ting.

[3]In particular, a naive expectation is that π's will be enhanced relative to K's and baryons by resonance production.

[4]See, for example, J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York,

1965), pp. 202–207.

[5]We are aware that statistical quark-counting models have been around for a long time. Recent examples are S. Matsuda, Kyoto University Report No. KUNS 315, 1975 (to be published); J. D. Bjorken and G. R. Farrar, Phys. Rev. D 9, 1449 (1974). However, we believe that the dynamical assumptions underlying our model are quite different from those of previous models.

On-Shell Asymptotics of Non-Abelian Gauge Theories*

John M. Cornwall and George Tiktopoulos
Department of Physics, University of California, Los Angeles, California 90024
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In the leading-logarithm approximation, the high-energy fixed-angle elastic fermion-fermion amplitude and the asymptotic form factor have been calculated to sixth order for a Yang-Mills theory; some other processes have been calculated to fourth order. The results suggest properties of exponentiation and factorization which can be described in terms of momentum-dependent anomalous dimensions in the renormalization-group equations. We discuss the possible relation of the zero-mass limit of our results to particle confinement.

We report here calculations based on perturbation theory which suggest certain remarkable asymptotic features of S-matrix elements and form factors for Yang-Mills theories of vectons (vector particles) and fermions. In order that the S matrix exist, of course, it is necessary that the vectons have a mass; this we do by putting in a mass "by hand" in the Feynman gauge.[1] We define the *high-energy fixed-angle regime:* All particles are on shell, and all Lorentz squares of nontrivial momentum sums are large compared to masses, while the ratios of invariants are of order 1. In this regime there is only one large dimensionless quantity, so that our results can be interpreted in terms of the Callan-Symanzik equation (with exceptional momenta).

In the fixed-angle regime the following processes have been calculated in the approximation of retaining only the leading logarithms in each order: (a) fermion-fermion elastic scattering T_{FF} (to sixth order); (b) fermion-vecton elastic scattering T_{FV} (to fourth order); (c) vecton-vecton elastic scattering T_{VV} (to fourth order); (d) the form factor F_F for fermion-antifermion → group-singlet current[2] (to sixth order)[3]; (e) the form factor F_V for vecton-vecton → group-singlet current[2] (to fourth order). The results can be written in the following factored, exponentiated form, valid to the order given above:

$$T_{ij} \sim T_{ij}{}^{B} F_i{}^{n_i/2} F_j{}^{n_j/2}, \tag{1a}$$

$$F_i \sim \exp[(-g^2/16\pi^2)C_i \ln^2(-t/m^2)], \tag{1b}$$

where $i, j = F$ or V, and n_i is the number of legs of type i in T_{ij}. The superscript B denotes the Born approximation, t is the square of the momentum transfer, and we do not distinguish between s, $-t$, and $-u$. The mass scale m is arbitrary, except that if the fermion mass vanishes, m is the vecton mass; C_F and C_V are the eigenvalues of the quadratic Casimir operator for the fermion and vecton (adjoint) representations, respectively.

Details of the calculation will be published elsewhere. We note that the leading logarithms [occurring as powers of $g^2 \ln^2(-t/m^2)$] come only from infrared singular regions of integration in momentum space; ultraviolet logarithms [typically powers of $g^2 \ln(-t/m^2)$] are nonleading. The contribution from Higgs scalars, ghosts, ordinary scalars, closed fermion loops, and four-vecton vertices (seagull terms) are nonleading, but the three-vecton Yang-Mills coupling is quite important. There are miraculous conspiracies between graphs of very different topology which lead to the factored, exponentiated forms in (1a) and (1b).

For the scattering of *group-singlet* particles or currents (i.e., those which do not couple directly to vectons), we find the striking result that, in a general renormalizable field theory, there are no infrared-singular logarithms; in fact, such amplitudes obey the same renormalization-group equations in the fixed-angle regime that they do in the Euclidean region.[4] Equivalently, the form factors for singlets are identically 1, to *leading* logarithmic order [$g^2 \ln^2(-t/m^2)$]. An example which can be studied to all orders is massive