an edge penetration depth. If this is indeed the case, then a study of perpendicular diffraction patterns in Josephson tunnel junctions provides a means of investigating the temperature dependence of the edge penetration depth in superconductors, in a similar manner as has been done with the surface penetration depth.[8]

We wish to thank Professor M. Tinkham and Professor D. N. Langenberg for stimulating discussions.

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[5]The bulk lower critical field for our sample is calculated to be ~300 G. See *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), p. 860. Because of demagnetizing effects (*op. cit.*, p. 986), $H_{c1} \simeq 0.5$ G which is still considerably higher than the fields used in our work.

[6]I_J is a function of the energy gap, Δ (see V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. 10, 486 (1963), and 11, 104(E) (1963). The applied field is too small to affect Δ. This has been verified through measurement of Δ from the *I-V* characteristic.

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Limiting Flux-Passage Time in Narrow Superconductors

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A limiting value for the supercurrent response of narrow superconductors upon flux passage has been deduced from measurements of the 9.2-GHz impedance of thin cylindrical films. The observed times are in good agreement with the predicted relation $\tau_1 \simeq \hbar/2\Delta(T)$, where $2\Delta(T)$ is the BCS energy gap for the material.

In the last several years much work has been devoted to the understanding of the dynamic behavior of superconductors under nonequilibrium conditions. Times characteristic of the coupled responses of the Cooper pairs, the quasiparticles, and the phonons have been extensively studied. In a recent paper[1] Langenberg has surveyed many of these characteristic times and concluded that there still remains considerable uncertainty in both theory and experiment. Several years ago Mercereau introduced a "finite relaxation time for the flux change" in a model for the operation of rf-biased thin-film superconducting quantum interference devices (SQUID's).[2] The flux movement in these devices is localized at a weak-link section of the film. In the limit of small weak links, where the weak-link dimensions are less than a coherence length, Mercereau deduced that the flux-passage time would have a minimum limiting value, $\tau_1 \sim \hbar/2\Delta(T)$, where $2\Delta(T)$ is the temperature-dependent BCS energy gap. The minimum response time of a SQUID, therefore, is found to be limited by the lifetime of the Cooper pairs in the weak link. This limiting response time may simply reflect the charateristic delayed response of a supercurrent to an electric field.[3] For SQUID's operating in the frequency range from 1 to 30 MHz, τ_1 can be ignored since the time between flux passages never approaches the flux-passage time. In studying SQUID's at microwave frequencies, however, we find that τ_1 can no longer be ignored, but can in fact be directly inferred from the data. We find that the magnitude and temperature variation of our experimental values for the flux-passage time, τ_1, agree quite well with Mercereau's predicted gap dependence of the limiting response time.

Our cylindrical films, which were about 1000 Å thick, were made by evaporation onto a rotating quartz rod. To localize the region where the flux passages occur, weak links were fabricated into the films. Two of the samples reported here contained tin microbridges which were bisected by narrow (2 μm) gold strips deposited normal to the direction of current flow, producing proximity-effect or Notarys-type weak-link SQUID's. Another sample was made from a lead–40%-thallium alloy and contained a tapered microbridge section which was approximately 2 μm in width at the narrowest portion.

These samples were mounted in a TE_{101}-mode microwave cavity at a position of maximum microwave magnetic field intensity and the sample response was detected by a standard superheterodyne spectrometer operating at 9.24 GHz.[4] The samples were aligned so that the applied microwave magnetic field was directed along the cylindrical axis of the films.[5]

According to the Mercereau model[2] and the usual theories for SQUID operation[6,7] the number of flux quanta being cycled through a sample can be inferred from the number of discrete increments or steps[8] in the microwave power absorbed by the sample as the microwave magnetic field is increased. Each step corresponds to the cycling of an additional flux quantum into and out of the SQUID during each microwave cycle. When the detected microwave signal reflected from the cavity is observed with a lock-in amplifier tuned to an audio-frequency (af) flux modulation, then, following Mercereau,[2] we can approximate the resulting signal by the equation

$$|V_{af}| \propto \sum_{n=1}^{\infty} \frac{\sin(n\pi\tau_1/\bar{t}\,')J_1(2n\pi\Phi_{rf}/\Phi_0)J_1(2n\pi\Phi_{af}/\Phi_0)}{n}, \tag{1}$$

where $J_1(x)$ is the first-order Bessel function, τ_1 is a characteristic time identified with the limiting flux-passage time, $\bar{t}\,'$ is an average time between flux passages, Φ_{rf} is the amplitude of the microwave flux applied axially to the sample, and Φ_{af} is the audio-frequency flux-modulation amplitude.[2] (Experimentally $\Phi_{af} \sim \Phi_0/20$ at ~100 Hz.) The observed signals can be closely fitted by Eq. (1) (see Fig. 1). From the modulation of the envelope of the lock-in signal [given by the $n=1$ sine term of Eq. (1)] we can infer the characteristic time, τ_1. Intuitively we would expect a degradation in the signal as $\bar{t}\,'$ approaches τ_1. From our data we note that a significant signal-to-noise degradation does occur when the microwave field amplitude is increased beyond the first $n=1$ sine lobe. The appearance of this signal degradation can be used to infer τ_1 independently of Eq. (1).

Figure 2 is a plot of τ_1 versus the inverse BCS gap ratio derived both from Eq. (1) and from the appearance of signal degradation for a proximity-effect sample. For this sample the signal degradation is further evidenced by the appearance of double steps[4] in the direct absorption signal of the SQUID observed in the absence of lock-in detection. We find that the data for this sample imply $\tau_1(T)$ which can be represented as

$$\tau_1(T) = \tau_1(0)\Delta(0)/\Delta(T), \tag{2}$$

where $\tau_1(0) = (0.7 \pm 0.1)\times 10^{-12}$ sec.

Equation (2) compares favorably with the theoretical expression for the limiting flux-passage time as given by Mercereau:

$$\tau_1(T) \simeq \hbar/2\Delta(T) = \tau_1(0)\Delta(0)/\Delta(T), \tag{3}$$

where $\tau_1(0) \simeq 0.6\times 10^{-12}$ sec and 2Δ has been as-

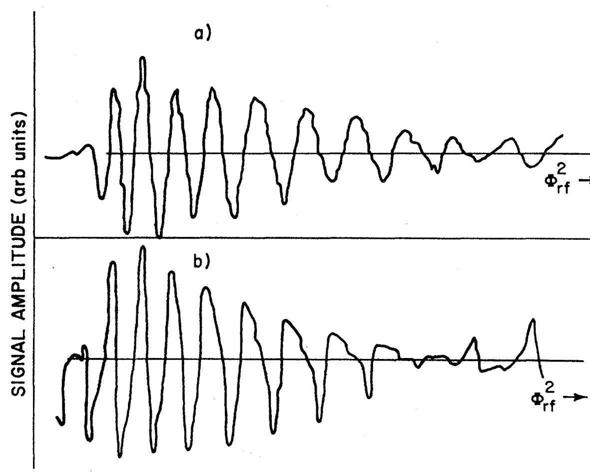

FIG. 1. (a) The initial portion of the signal detected by a lock-in amplifier versus the microwave pump power ($\propto \Phi_{rf}^2$) for a Pb-Tl alloy sample. (b) A computer-generated curve of signal amplitude versus Φ_{rf}^2 calculated with use of Eq. (1). $\bar{t}\,'$ was assumed to be the shortest time between successive flux entries. τ_1 and the amplitude have been adjusted for comparison with the experimental curve in (a).

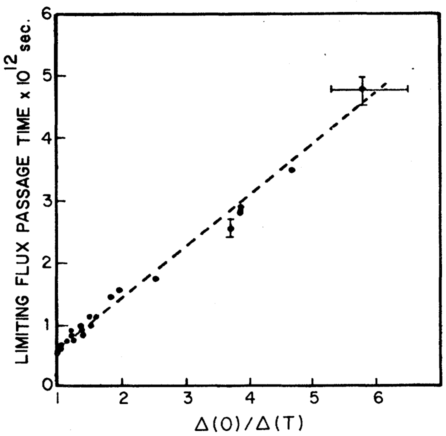

FIG. 2. The limiting flux-passage time for a Au-Sn proximity-effect sample plotted versus the reciprocal of the reduced energy gap where the reduced energy gap is $\Delta(T)/\Delta(0)$. Uncertainties are indicated on two of the points.

sumed to be the BCS gap parameter for pure tin. (T_c is reduced in the proximity-effect region by only 0.3% of the pure-film T_c.)

The second Notarys-type SQUID exhibited the same type of behavior and yielded a low-temperture value for the limiting flux-passage time of $\tau_1(0) \simeq (0.7 \pm 0.05) \times 10^{-12}$ sec in good agreement with the result for the previous sample.

Similar results were obtained in the case of a lead-40%-thallium sample. In the low-temperature limit for this sample the experimental flux-passage time is found to be $\tau_1(0) \simeq (0.6 \pm 0.1) \times 10^{-12}$ sec, whereas the theoretical value for $\tau_1(0)$ is 0.4×10^{-12} sec under the assumption that the BCS gap can be calculated as $2\Delta(0) \simeq 4.0 k_B T_c$.

In summary, we have found that the microwave response of certain types of rf-biased SQUID's is in good agreement with the Mercereau model which explicitly takes into account a finite flux-passage time. However, it must be stressed that the existence of this finite flux-passage time can be inferred directly from the data and is not just an artifact of the model. It is also clear that the unique temperature dependence of the observed characteristic time precludes the possibility that this time is associated with the normal resistance of the weak link and the geometric inductance of the device.

In addition, the magnitude and the temperature dependence of the deduced time is in very good agreement with the value proposed for the limiting flux-passage time $\tau_1 \sim \hbar/2\Delta(T)$. The results reported here are the first direct observation of this characteristic superconducting response time.

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