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Asymmetry of Beta-Ray Angular Distribution in Polarized Nuclei and G-Parity Nonconservation

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We have derived an equation for the β -ray angular distribution including Coulomb corrections, radiative corrections, induced effect, and higher-order nuclear matrices. With this equation and the experimental data on β -ray asymmetries in polarized ¹²B and ¹²N, we conclude that the strength of the second-class induced tensor is $f_T/f_A = -(0.96 \pm 0.35) \times 10^{-3}$ in the limit of the impulse approximation. A possible modification of this value due to mesonic corrections is discussed.

Since Weinberg proposed a measurement of the ft -value ratio in mirror β decays to test a possible existence of the second-class currents in weak interactions,¹ there have been a number of articles published on this subject. Among those, Wilkinson and his co-workers have made an extensive search for the asymmetries of the ft values experimentally.² The results were originally thought to be a direct indication for the induced tensor interaction. Later on, these were, however, recognized as the sum of the effects due to nuclear structure, the induced tensor term f_T [see Eq. (4)], and possible meson-exchange currents. In a model calculation,³ Kubodera, DeLorme, and Rho adopted the meson-exchange effect due to $\omega \rightarrow \pi e \nu$, and they gave a ratio

$$(ft)_+ / (ft)_- - 1 = \delta_{\text{exp}} = \delta_{\text{sc}} + \delta_{\text{nucl}}$$

Here

$$\delta_{\text{sc}} = -4(\lambda/f_A)J + (2/3f_A)(\lambda L - 2\zeta)(E_0^- + E_0^+)$$

and δ_{nucl} represents the nuclear-structure effect. The effect of the second-class current, δ_{sc} , is also dependent on the nuclear model through J

and L , while λ is a combination of the strong-coupling constants including the ω - ρ mixing parameter, and ζ is nearly equal to f_T . Wilkinson made an analysis of δ_{exp} in the region of mass number $A = 8$ -30 systematically, using available nuclear models. He obtained as limits for the parameters²

$$|\zeta| \leq 2.5 \times 10^{-3} \text{ MeV}^{-1} \text{ and } |\lambda| \leq 10 \times 10^{-3}.$$

Here the combination $\zeta = \lambda = 0$ is not necessarily excluded, while the theory does not give us a single value of λ , since it contains a logarithmic divergence.

Less ambiguous information for the second-class current can be obtained from the measurement of the β -ray asymmetries in polarized nuclei. A long-awaited experiment on ¹²B and ¹²N was recently performed successfully by Sugimoto, Tanihata, and G6ring.⁴ In order to derive a conclusion about G -parity nonconservation from this experiment, we have to be careful to include all induced effects, higher-order corrections, etc., in the equation for the angular distribution of β rays.⁵⁻⁸ In particular, the radiative corrections

are important, at least for the β -ray spectrum,⁹ and the Coulomb corrections are nonnegligible for the electron waves, especially with $j \geq \frac{3}{2}$.⁵ Since we found several features insufficient for our purpose, first we state the method for determining the equation correctly, and next, we find the strength of G -parity nonconservation. Finally, we make a comment on possible modification due to meson-exchange effects, since our equation is based on the impulse approximation.

The higher-order corrections including Coulomb corrections are given in extended form by one of the present authors whose equations⁵ include no induced tensor coupling. The effect of this G -parity-nonconserving second-class current is, however, easily taken into account by replacing in the published equations⁵ the factors [in terms of Eq. (4) below]

$$\begin{aligned} C_A \int \vec{\sigma} &\text{ by } (f_A - E_0 f_T) \int \vec{\sigma} \text{ and} \\ C_A (i \int \gamma_5 \vec{r}) &\text{ by } (f_A + 2M f_T) (i \int \gamma_5 \vec{r}), \end{aligned} \quad (1)$$

where $C_V = f_V(0)$ and $C_A = f_A(0)$. We also adopt the nonrelativistic approximation for nuclear matrix elements:

$$\begin{aligned} \int \vec{\alpha} \times \vec{r} &= x[(1 + \mu_p - \mu_n)/M] \int \vec{\sigma}, \\ i \int \gamma_5 \vec{r} &= -y(1/2M) \int \vec{\sigma}. \end{aligned} \quad (2)$$

Here $1 + \mu_p - \mu_n = 4.7$, and M is the nucleon mass. The parameters x and y are nuclear-model dependent, and they are given by

$$\begin{aligned} x &= 1 + (1/4.7) (\int \vec{r} \times \vec{p}) / (\int \vec{\sigma}), \\ y &= 1 + i2c [\int \vec{r} (\vec{\sigma} \cdot \vec{p})] / (\int \vec{\sigma}). \end{aligned} \quad (3)$$

Here c is 1 for f_A and 0 for f_T . The reason for no nucleon-recoil term for f_T is seen in Eq. (7) below. In the case of ^{12}B and ^{12}N , we have $x \approx 1$,¹⁰ and $y \approx 1.5$ for $c = 1$.¹¹

We adopt the interaction Hamiltonian density

$$H = \{ \bar{\psi}_p [\gamma_\lambda (f_V - f_A \gamma_5) + \sigma_{\lambda\nu} k_\nu (f_W + f_T \gamma_5) + i k_\lambda (f_S + f_P \gamma_5)] \psi_n \} [\bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_\nu] / \sqrt{2} + \text{H.c.}, \quad (4)$$

with $k = k_p - k_n$ and $\sigma_{\lambda\nu} = [\gamma_\lambda, \gamma_\nu] / 2i$. The coupling constants for β decay are

$$\begin{aligned} f_V(0) &= (3.001 \pm 0.002) \times 10^{-12} \hbar^3 / m^2 c, \\ f_A(0) &= - (1.239 \pm 0.011) f_V(0), \\ f_W &= - (\mu_p - \mu_n) f_V / 2M = -3.7 f_V / 2M, \quad f_S = 0, \end{aligned} \quad (5a)$$

from conservation of vector current,

$$f_P \approx f_A / 25, \quad (5b)$$

from partial conservation of axial-vector current. The induced tensor term, f_T , introduced by Weinberg¹ is now being revealed. Equation (1) can be derived by comparing the space and time components of f_A and f_T in Eq. (4),

$$(f_A - E_0 f_T) (\psi_p^\dagger \vec{\sigma} \psi_n) \cdot [\psi_e^\dagger \vec{\sigma} (1 + \gamma_5) \psi_\nu] \text{ and } - [f_A (\psi_p^\dagger \gamma_5 \psi_n) + i f_T (\psi_p^\dagger \vec{\gamma} \gamma_5 \psi_n) \cdot \vec{p}] [\psi_e^\dagger (1 + \gamma_5) \psi_\nu], \quad (6)$$

where \vec{p} is the differential operator, and adopting the nonrelativistic approximation with two-component spinor u ,

$$(\psi_p^\dagger \gamma_5 \psi_n) = - (\frac{1}{2} M) [(u_p^\dagger \vec{\sigma} u_n) \cdot \vec{p} + 2 (u_p^\dagger \vec{\sigma} \cdot \vec{p} u_n)], \quad (7)$$

$$i (\psi_p^\dagger \vec{\gamma} \gamma_5 \psi_n) \cdot \vec{p} = - (u_p^\dagger \vec{\sigma} u_n) \cdot \vec{p} \approx 2M (\psi_p^\dagger \gamma_5 \psi_n). \quad (8)$$

Equation (8) is useful for deriving rule (1), by neglecting the nucleon recoil. In practice, a correction in Eq. (3) should be considered.

The explicit form of angular distribution of β rays in polarized nuclei is very lengthy; one must take into account Coulomb corrections, higher-order nuclear matrices of momentum type as well as coordinate type, radiative corrections,¹² and f_T terms simultaneously. These can be obtained from Ref. 5 by the replacements (1) and (2). To save space, we only give an expression for the $1^+ \rightarrow 0^+$ transition by omitting the contributions of $\int \vec{\sigma} r^2$ and $\int (\vec{\sigma} \cdot \vec{r}) \vec{r}$ (the finite de Broglie wavelength effect is, however,

included in the main L_0 and Λ_1 terms⁵):

$$\begin{aligned}
 W(\theta) = & |f_A|^2 \int \vec{\sigma}^2 \left\{ [1 \mp 2E_0(f_T/f_A)] [1 + F(E, E_0)] \left[\frac{1}{2}(1 + \gamma) \mp \frac{10}{3} VR^2 E \mp (2/3E) VR^2 - (p^2/3) R^2 \right] \right. \\
 & \mp \frac{4}{3} a [E_0 - 2E + (1/E) \mp (\frac{2}{3}) b [E_0 - (1/E) \pm 3V]] \\
 & \mp P(p/E) \left\{ [1 \mp 2E_0(f_T/f_A)] [1 + G(E, E_0)] \left[\frac{1}{2}(1 + \gamma) \mp \frac{10}{3} VR^2 E - (p^2/3) R^2 \right] \right. \\
 & \mp \frac{2}{3} a [2E_0 - 5E \mp 6V] \pm \frac{2}{3} b [E_0 - E \pm 3V] \mp a' VR [p + (4/p)] \left. \right\} \cos \theta \\
 & \pm A \left\{ \frac{2}{3} (a - b) [E - (1/E)] \mp a' VR p \right\} \frac{1}{2} (3 \cos^2 \theta - 1) \left. \right\}. \tag{9}
 \end{aligned}$$

with

$$\begin{aligned}
 \gamma &= (1 - \alpha^2 Z^2)^{1/2}, \quad V = \alpha Z / 2R, \\
 P &= a_1 - a_{-1}, \\
 A &= 1 - 3a_0, \\
 a &= x \operatorname{Re}(f_A g_W^*) / |f_A|^2, \quad g_W = f_W - (f_V / 2M), \\
 b &= \operatorname{Re}(f_A g_T^*) / |f_A|^2, \quad g_T = f_T \pm y(f_A / 2M), \\
 a' &= \operatorname{Im}(f_A g_W^*) / |f_A|^2. \tag{10}
 \end{aligned}$$

Here F and G are radiative corrections,¹² and V is half of the Coulomb energy at the nuclear surface. p , E , and E_0 are the momentum, energy, and maximum energy of the electron, respectively. The upper sign refers to the electron, and the lower sign refers to the positron. The angle θ is the direction of the electron emission with respect to the nuclear polarization axis; p is the nuclear polarization and A is the nuclear alignment. The a_i 's are the magnetic substate densities ($\sum_i a_i = 1$). The units $\hbar = m_e c = 1$ are adopted. Assuming small real values for a and b , we can rewrite Eq. (9) as

$$\begin{aligned}
 W(\theta) = & \operatorname{const} [1 + F(E, E_0)] \left[1 \pm \frac{2}{3} aE \right] \\
 & \times \left\{ 1 \mp P(p/E) \left[1 \pm \frac{2}{3} (a - b)E \right] \cos \theta \pm A \frac{2}{3} (a - b)E \frac{1}{2} (3 \cos^2 \theta - 1) \right\}. \tag{11}
 \end{aligned}$$

Here the energy-dependent radiative corrections¹² $F(E, E_0)$ and the V and E_0 terms are factorized in the first line, while the minor terms, $1/E$, $\alpha Z/E$, and R^2 , are simply omitted. It is noticed here that the factorization of the electromagnetic interactions in Eq. (11) is by no means self-evident without calculations. In fact, this is not the case, if $\int (\vec{\sigma} \cdot \vec{r}) \vec{r}$ and $\int \vec{\sigma} r^2$ are explicitly taken into consideration.⁵

Experimentally, the asymmetry of the β -ray angular distribution is given in a form

$$\mathcal{A} = [W(0) - W(\pi)] / [W(0) + W(\pi)] = \mp P(p/E) [1 + \alpha_{\mp} (1 - A)E]$$

and

$$\alpha_{-} (^{12}\text{B}) - \alpha_{+} (^{12}\text{N}) = (0.52 \pm 0.09)\% / \text{MeV}. \tag{12}$$

From this we have

$$f_T / f_A = - (0.96 \pm 0.35) \times 10^{-3} \tag{13}$$

with $x = 1$. It is interesting to note that this is also given by $f_T = - (3.5 \pm 1.3) f_A / 2M$, which is closely related to the strength of weak magnetism. The $f_A / 2M$ term in g_A of Eq. (10) has a reasonable magnitude but with an opposite sign from the experimental data, $\alpha_{-} + \alpha_{+}$, at the present stage⁴ [$\alpha_{-} (^{12}\text{B}) = (0.31 \pm 0.06)\% / \text{MeV}$ and $\alpha_{+} (^{12}\text{N}) = - (0.21 \pm 0.07)\% / \text{MeV}$]. A possible small effect of $\int (\vec{\sigma} \cdot \vec{r}) \vec{r}$ and $\int \vec{\sigma} r^2$ is under investigation.

Up to this point, we have made the impulse approximation. We can take into account possible meson-exchange effects by regarding x , y , and c

in Eqs. (3) as parameters. (y for f_T is no longer unity.) Equivalently, we regard a and b in Eqs. (13) as parameters, and the coupling constants in Eqs. (10) as those modified by the exchange currents. In fact, Eq. (13) coincides with the equation in the elementary-particle approach.⁷ In this case, f_T in Eqs. (10) may be written as f_T^{eff} which is not equal to f_T in Eq. (4) anymore. Although the relation of f_T to f_T^{eff} is model dependent, the experimental data in Eq. (12) do not lose their importance by giving us a nonzero value for f_T^{eff} , which is of the second class anyhow. This is because the magnitude of a including the mesonic corrections is known in the experiment.¹⁰ Adopting the experimental value [$\frac{16}{3} a = (1.07 \pm 0.24)\% / \text{MeV}$], we have $f_T^{\text{eff}} / f_A^{\text{eff}} = - (3.5 \pm 1.5) / 2M$.

We have assumed the point nuclear charge in Eq. (9). The effect of the finite nuclear size can be evaluated in a way described in the references.⁵ A contribution of the pseudoscalar term in Eq. (4) is very small as is given elsewhere.¹³

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Hyperfine Structure of $2s\ ^3\text{He}^+$ by an Ion-Storage Technique*

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An electrostatic confinement device has provided resonance linewidths ≈ 1 kHz for the hyperfine transition $F=1, m_F=0$ to $F=0$, in metastable $2s\ ^3\text{He}^+$. The state-selection and resonance-detection scheme is the same used in an earlier ion-beam experiment; however, ion storage has yielded a resonance linewidth narrower by a factor of 100. Our result for the $2s$ hyperfine structure is $\Delta\nu_2 = 1083.354\,969(30)$ MHz. Comparison with the $1s$ hyperfine structure yields a test of state-dependent terms in the theory.

It is well known that in the theory of the hyperfine structure of atomic hydrogen, uncertainty in the size of the nuclear-structure correction limits comparison with experiment to the level of about 3 ppm. This far exceeds the experimental precision of $\approx 1 \times 10^{-6}$ ppm, and, for example, precludes a good test of the quantum-electrodynamic (QED) correction term proportional to $\alpha(Z\alpha)^2$ which is calculated¹ to be 2.27(62) ppm. It is possible, however, to reduce the importance of nuclear structure if one compares the hfs in the $2s$ and $1s$ states. In particular the quantity $D_{21} \equiv (8\Delta\nu_2 - \Delta\nu_1)$, where $\Delta\nu_2$ and $\Delta\nu_1$ are the $2s$ and $1s$ hfs, has a contribution in hydrogen due to the $\alpha(Z\alpha)^2$ term of about 2%, whereas the nucle-

ar structure is not expected to contribute more than about 0.01%. The obvious drawback to this strategy is the requirement for two precision measurements.

In the case of $^3\text{He}^+$, in a unique and pioneering experiment, Novick and Commins² measured $\Delta\nu_2 = 1083.354\,99(20)$ MHz and, by a novel ion-storage technique, Schuessler, Fortson, and Dehmelt³ measured $\Delta\nu_1 = 8665.649\,867(10)$ MHz. This yields $D_{21} = 1.1901(16)$ MHz. One sees that the uncertainty in $\Delta\nu_2$ is responsible for virtually all the uncertainty in D_{21} . It was the goal of the present work to determine $\Delta\nu_2$ more accurately. Our experiment uses the same method of state selection and resonance detection as the work of No-