Both n and ϵ are functions of β, and $n(0)=1$. By explicit evaluation, $n(\beta)$ is exponentially decreasing in β.

A simple exercise in contour integration is to evaluate $\bar{\rho}_1(\beta)=n(\beta)/a$ as

$$\bar{\rho}_1(\beta)=\theta_1(2\pi\beta|2\tau)/2r\theta_1'(2\tau)\sinh(\pi\beta/r). \tag{31}$$

As an explicit calculation, we evaluate the long-range portion of the density-density correlation:

$$\bar{\rho}_2(x,y;x,y)\underset{|x-y|\to\infty}{\sim}N(x,x)N(y,y)=a^{-2}\frac{\theta_3(\pi(x/a+i\epsilon))\,\theta_3(\pi(x/a-i\epsilon))\,\theta_3(\pi(y/a+i\epsilon))\,\theta_3(\pi(y/a-i\epsilon))}{\theta_3^{\,2}(2\pi i\epsilon|2\tau)}. \tag{32}$$

The modulus of the θ functions is τ unless otherwise indicated. Then, by a fundamental relation for θ functions,

$$\bar{\rho}_2(x-y)\sim[2a^2\theta_3^{\,2}(2\pi i\epsilon|2\tau)]^{-1}\left\{\theta_3\left(\pi\left(\frac{x-y}{a}\right)\right)\theta_3(2\pi i\epsilon)\theta_3(0)+\theta_4\left(\frac{\pi}{a}(x-y)\right)\theta_4(2\pi i\epsilon)\theta_4(0)\right\}$$

$$=a^{-2}\left\{\theta_3\left(\frac{\pi}{a}(x-y)\right)+\frac{\theta_4(2\pi i\epsilon)\theta_4(0)}{2\theta_3^{\,2}(2\pi i\epsilon|2\tau)}\left[\theta_4\left(\frac{\pi}{a}(x-y)\right)-\theta_3\left(\frac{\pi}{a}(x-y)\right)\right]\right\}. \tag{33}$$

Since the theta functions are periodic, the long-range crystalline order of the ground state has been exhibited. It is noted that other quantities of interest may be readily evaluated.

This work suggests further generalization in a number of directions: (a) higher-order calculations and general statements on cluster properties; (b) calculations for other values of λ^3; (c) evaluation of the partition function for a two-dimensional classical plasma, which will also show long-range crystalline order; (d) availability of a reasonable and feasible trial wave function for the three-dimensional plasma problem. These points will all be pursued in a later publication.

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Origin of Cosmic Rays

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With the use of recent observations of the galactic gas and γ-ray distributions, the galactic cosmic-ray distribution is deduced. This distribution is identical to that of supernova remnants (within experimental error), strongly supporting the hypothesis that most observed cosmic rays are produced by supernovae in our own galaxy. The average age of the cosmic-ray sources is suggested from the character of their distribution to be about 30 million years.

The problem of the origin of cosmic rays has been the central problem of high-energy astrophysics for over a generation. Over four decades have passed since Baade and Zwicky[1] first proposed that supernova explosions could provide the energy for accelerating cosmic rays. The supernova-origin hypothesis gained observational support in the early 1950's when Shklovskii[2] proposed that cosmic-ray electrons radiating in the magnetic field of the Crab Nebula produced its optical continuum radiation. This suggestion led to the prediction that such synchrotron radiation should be linearly polarized[3,4] and shortly thereafter such polarization was indeed detected.[5-7] It became apparent that cosmic-ray electrons were present in the Crab Nebula and it was natural to assume that protons and nuclei of cosmic-ray energy were produced there also. The recent unambiguous detection of 100-MeV γ rays from the Crab Nebula and the Vela supernova remnant has provided evidence that these two young, nearby remnants produce cosmic rays.[8-11] The nature of the γ-ray emission from these objects as to whether nucleons are indeed involved, however, needs further observational clarification. Also, the question has been raised as to whether one can consider the Crab Nebula to be a typical galactic supernova remnant and to extrapolate to the conclusion that galactic supernovae are the main source of the observed cosmic rays.[12]

Shortly after the discovery of the 3-K microwave blackbody background radiation, it was first noted by Fazio, Stecker, and Wright[13] that such radiation precluded the existence of cosmic-ray electrons outside the galaxy with the same intensity as that observed locally. They noted that such electrons would produce more 100-MeV γ rays than observed if their distribution extended more than ~ 30 kiloparsecs (kpc) from the earth. Thus, it became apparent that cosmic-ray electrons were of galactic origin.

It was further noted by Greisen[14] and Zatsepin and Kuz'min[15] that the blackbody background radiation would interact with ultrahigh-energy cosmic rays of extragalactic origin to produce a cutoff in their energy spectrum. The lack of an observed cutoff allowed one to rule out the universal-origin hypothesis for ultrahigh-energy cosmic rays and to place limits on the extent of their source region as being within 300 Mpc.[16] Galactic origin of even ultrahigh-energy cosmic rays has been advocated[17,18] and support of this hypothesis has been recently provided by indications of the anisotropy of ultrahigh-energy cosmic rays.[19,20]

Attempts to place limits on the extragalactic cosmic-ray flux using the γ-ray background observations have not been conclusive.[21] They still allow a universal cosmic-ray nucleon flux provided that the mean intergalactic gas density satisfies $n_{IG} \leq 10^{-7}$ cm^{-3}, and allow origin within the local supercluster even if $n_{IG} \simeq 10^{-5}$ cm^{-3}. Thus, the discussion of the extragalactic- versus galactic-origin hypothesis has continued down to the present.[12,22]

It has long been realized that observations of galactic γ rays could provide important information for resolving this problem, but until now it has not been possible because of insufficient γ-ray data and an incomplete knowledge of the amount and distribution of an important component of the interstellar gas, viz., molecular hydrogen. Recent observations of the large-scale galactic distribution of γ radiation[23] and molecular clouds[24] have now made it possible to investigate the large-scale distribution of galactic cosmic rays. Using the new observations, Stecker *et al.*[25] have determined that the cosmic-ray distribution in the galaxy is not uniform as would be indicated by the extragalactic-orgin hypothesis. These results indicated that there is a weak correlation of the cosmic-ray flux with gas density (mostly H_2 clouds) in the inner part of the galaxy. Also apparent was a falloff of the cosmic-ray flux in the outer galaxy.[25,26] The striking similarity between the cosmic-ray distribution deduced in Ref. 25 and the supernova distribution in the galaxy[27,28] provides new evidence that supernovae produce the bulk of the cosmic-ray flux.

The galactic γ rays are primarily the result of the decay of π^0 mesons produced in cosmic-ray interactions with interstellar gas.[29] Their flux is therefore proportional to the product of gas density and cosmic-ray intensity integrated along the line of sight within the solid angle subtended by the γ-ray telescope. If a cosmic-ray flux distribution is assumed and Compton and bremsstrahlung γ rays are also included in the calculation (a 30% correction at most), one can calculate the flux expected to be observed by the SAS-2 γ-ray telescope of Fichtel *et al.*[23] integrated over $\pm 10°$ in galactic latitude and averaged over $5°$ longitude. This can only be done over the half of the galaxy for which the molecular-cloud distribution has been determined.[24] The details of this calculation are given in Ref. 25. A uniform cosmic-ray flux distribution leads to a γ-ray flux which is a factor of ~ 2 too high compared to the observations in the anticenter direction and which is too low in the direction of the galactic center.

By the same methods, a calculation of the γ-ray flux distribution can also be made under the assumption that the cosmic-ray distribution is proportional to the supernova remnant distribution in the galaxy, as would be expected if (1) su-

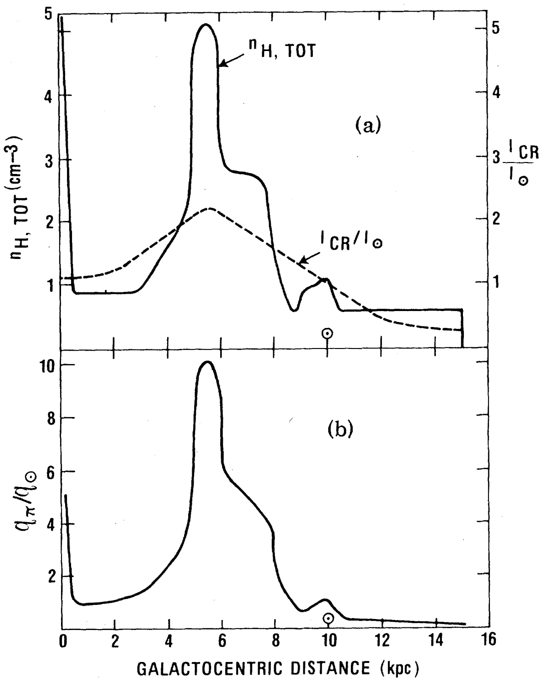

FIG. 1. (a) Galactic distributions of cosmic-ray intensity using the supernova remnant distribution (Ref. 28) and the total gas density deduced in Ref. 25. (b) Distribution of π^0 production rate in the galaxy based on (a).

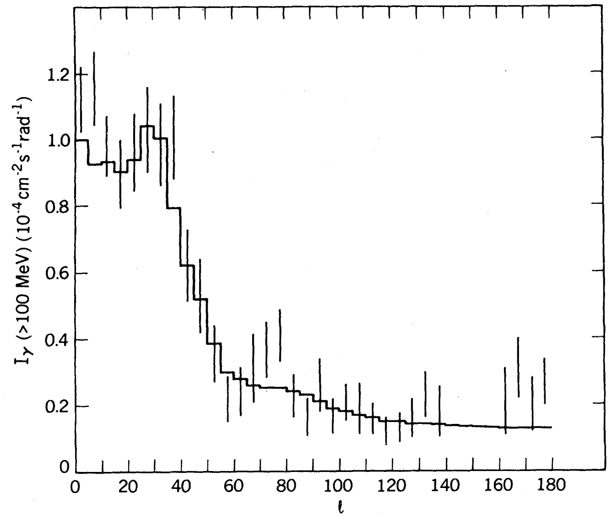

FIG. 2. Calculated longitude distribution of galactic γ rays under the supernova-origin hypothesis (histogram) compared with the observations (Ref. 23) (vertical lines).

pernovae are the principal source of cosmic rays, and (2) cosmic rays diffuse only a few hundred parsecs before leaking out of the galactic disk.[29,30]

With the supernova distribution obtained by Kodaira[28] taken as representative of the galactic cosmic-ray (CR) flux distribution (normalized so that $I_{CR}/I_{\odot} = 1$ at 10 kpc), the longitude distribution of γ rays as would be observed by the SAS-2 telescope has been calculated. Figure 1(a) shows the cosmic-ray and total-matter distributions used. Figure 1(b) shows the calculated γ-ray emissivity from π^0 decay. The results of the calculation are shown by the histogram in Fig. 2, along with the data actually obtained[23] indicated by the vertical lines. The calculated distribution is in remarkable agreement with the data, providing strong support for the hypothesis that supernovae produce most of the observed cosmic rays.

A further refinement of these calculations can be made by considering the fine-scale clumpiness of the nearby molecular clouds. This generally leads to small corrections which account for some of the finer features in the γ-ray data, the most important of which is an additional flux in the 35° to 40° longitude range due to a large cloud (Kh 3) which accounts for the apparent discrepancy between calculation and observations there.[31]

The supernova remnant distribution used here can be correlated with the distributions of various other galactic objects to estimate the average age of the remnants. For this purpose, I turn to the detailed discussion of the correlation of gas and associations of type-*O* and type-*B* stars (*OB*) made recently regarding M31.[32] These results show that the surface densities of H II regions and atomic hydrogen (H I) are related by $\sigma_{\rm H\,II} \propto (\sigma_{\rm H\,I})^{2.23}$. For our own galaxy, as a result in part of optical-depth effects, the H I distribution obtained from 21-cm observations may be to some extent misleading in the inner galaxy.[24,25,33] However, I find a correlation between the supernova (SN) remnant distribution[28] and the H II region distribution[31] of the form $\sigma_{\rm SN} \propto (\sigma_{\rm H\,II})^{0.4}$. Assuming, as for M31, that $\sigma_{\rm H\,II} \propto (\sigma_{\rm H\,I})^{2.23}$, it then follows that $\sigma_{\rm SN} \propto (\sigma_{\rm H\,I})^{0.89}$. If the correlation between *OB* associations and H I gas is also expressed in the form $\sigma_{OB} \propto (\sigma_{\rm H\,I})^m$, it is found that the older the association, the smaller the value of m (Ref. 31). The decrease of this correlation with age is generally attributed to a spreading out of the stars in the *OB* associations with time, all the stars in the association having been spawned from the same cloud complex. The correlation defined by $m = 0.89$ is close to the mean correlation with gas of all *OB* associations and consistent with an age of 3×10^7 yr.

Two important points should be kept in mind regarding the above discussion. (1) The γ rays are produced mainly in regions of high gas density which may bias this analysis to cosmic rays from younger (population I) sources, and (2) the cosmic-ray distribution shown in Fig. 2 shows only the large-scale variation of cosmic rays in the galaxy. Finer-scale correlations with arm features are beyond the scope of this work.

With these points in mind, nevertheless, it is important to note that the distribution of sources used here refers to that of radio remnants and not the supernovae themselves. Arguments given in Ref. 28 suggest that these remnants are indeed population-I objects which are restricted in distribution to the H I disk. A point of confusion has arisen as to whether supernovae of type I are population-II objects or population I.[28,34,35] This controversy blurs the interpretation as to the type of supernova producing most of the cosmic rays, but in view of the overall correlation with the gas it appears that most of the cosmic-ray sources responsible for the γ-ray production are population-I objects. It is, in any case, apparent that the cosmic-ray distribution in the galaxy is determined by the structure and evolution of the galaxy itself rather than by external sources.

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