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COMMENTS

Possible Effects of Decays of Charmed-Particle Resonances

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Under the assumption that production of the charmed vector meson D^* is important in the charm threshold region, the soft cascade pion (photon) from $D^*\to D\pi$ ($D^*\to D\gamma$) for $M_{D^*}-M_D>M_\pi$ ($M_{D^*}-M_D<M_\pi$) could serve as a very useful clue for charmed particles. Also the pions produced in e^+e^- collisions together with (strong-interaction–stable) charmed particles must obey energy equipartition. This strongly suggests other new heavy-quark thresholds and/or heavy-lepton production.

The discovery of even-charge-conjugation states in $\psi\to\gamma\chi$ decays[1,2] and of the $\eta_c(2.85\text{ GeV})$[3] strongly suggest that ψ spectroscopy is that of a fermion-antifermion system. If the structure in $\sigma_{tot}(e^+e^-\to\text{hadrons})$ in the region $4\leq W\leq 4.6$ GeV reflects $c\bar{c}$ continuum states decaying into charmed-particle pairs, many such $\bar{c}q$ ($q=u, d, s$) composites have already been produced at SPEAR and DESY. The failure to find charmed particles there (and elsewhere) is attributed to multibody decays and to the possible almost simultaneous onset of a heavy-lepton-pair threshold[4] which tends to reverse the charmed-particle signal in the K/π ratio.[5] The small mass difference between D^* (1^{--}) and D,[6] the corresponding 0^{-+} state, leads to soft π and γ emissions for $\Delta\equiv M_{D^*}-M_D>m_\pi$ and $\Delta<m_\pi$, respectively. The possible implications, particularly of the $\Delta>0$ case, for charmed-particle searches are discussed below.

Simple potentiallike models for $\bar{c}c$ bound states tend to predict ψ-η_c mass differences much smaller than the observed value of 250 MeV. The presumably short-range spin-spin forces which cause ^3S-^1S splittings are likely to be less effective in the more extended ($\bar{c}q$) systems D and D^* than in the ψ and η_c systems which are composed of two heavy quarks. Thus it is expected that $\Delta=m_{D^*}-m_D\leq m_\psi-m_{\eta_c}$ and in the following I will assume $\Delta\leq 200$ MeV. On the other hand comparison with the larger light-quark system (i.e., the ordinary mesons) suggests that $(m_{D^*})^2-(m_D)^2\geq m_{K^*}{}^2-m_K{}^2$. Thus for $m_{D^*}\approx 2$ GeV as indicated by the large rise in R at $W\approx 4$ GeV we find $\Delta\geq 0.14\approx m_\pi$.

Specific model calculations[7,8] predict almost complete dominance of the $D^*\bar{D}^*$ mode in the charm threshold region. Even if the detailed prediction of a huge jump ($\delta R\approx 3$) in R over a very small (10–20 MeV) range due to $D^*\bar{D}^*$ is not verified experimentally I believe the qualitative result that $D^*\bar{D}^*$ states are important just at the point where R rises to ≈ 5.5. This is corroborated by naive statistical-weight considerations which are 1:6:9 for $D\bar{D}$, $D^*\bar{D}+\bar{D}^*D$, and $D^*\bar{D}^*$. Thus a rough estimate for $f_{D^*\bar{D}^*}$, the fraction of δR rise due to the $D^*\bar{D}^*$ final state at $W\approx 4.2$ GeV, is $f_{D^*\bar{D}^*}\sim\frac{9}{16}\approx 0.55$ and $f_{D^*\bar{D}^*}\approx 1$ in the model of Ref. 7.

An important point for later discussion is that the rise of the $e^+e^-\to D^*\bar{D}^*$ cross section above threshold can be much faster than the corresponding rise for a pointlike $e^+e^-\to U^+U^-$ heavy-lepton competing process because of the existence of resonances and virtual states in the $c\bar{c}\to D^*\bar{D}^*$ system.[7,8] We may therefore expect that for $\epsilon \equiv W-2M_{D^*}\approx 0.2$ GeV, a major part of the δR rise reflects charmed and in particular $D^*\bar{D}^*$ final states.

If $\Delta \leq 0.2$ GeV then the P-wave decay $D^*\to D\pi$ occurs with momenta $P_\pi \leq 0.14$ GeV/c in the D^* rest frame. Comparing with $K^*\to K\pi$ with $p' = 0.288$ GeV/c I estimate $\Gamma(D^*)\leq\Gamma(K^*)/8 \leq 6$ MeV, an effectively zero width. The Lorentz transformation $E\pi=\gamma_{D^*}(\Delta+\beta_{D^*}P_\pi^*)$ allows, for $\epsilon=0.2$ GeV, lab pion momenta to vary within ($P_\pi^{\max}$, $P_\pi^{\min}$) limits (0, −0.12), (0.03, −0.16), and (0.04, 0.21) GeV/c for $\Delta=0.16$, 0.18, and 0.20 GeV, respectively. These limits as well as the median value ($P_\pi^{\text{med}}=\gamma_{D^*}P_\pi^*$) are insensitive to detailed angular distributions in the decay chain "γ" $\to D^*\bar{D}^*\to D\pi\bar{D}\pi$. Thus the onset of $D^*\bar{D}^*$ threshold will (if $0.2\geq\Delta\geq m_\pi$) enhance, via the $D^*\to D\pi$ cascade pions, the inclusive $d\sigma/dP_\pi^{\pm}\approx d\sigma/dP_{\text{charged}}$ at low momenta.

The fraction of the integrated inclusive cross section reflecting the cascade pions is

$$y=\frac{\delta[\int(d\sigma/dp)dp]}{\int(d\sigma/dp)dp}=\frac{(\delta R)f_{D^*\bar{D}^*}}{R\langle n_c\rangle}\frac{4}{3}, \qquad (1)$$

where $\frac{4}{3}$ is the average number of charged pions in the $D^*\bar{D}^*\to D\pi\bar{D}\pi$ cascade. Using $\delta R\approx 2.5$, $f_{D^*\bar{D}^*}=0.55$, $R=5.5$, and $\langle n_c\rangle\approx 4$, the values appropriate at $W\approx 4.2$ GeV, I find $y\approx 0.09$. This means that around $W\approx 4.2$ GeV a sharp increase is expected in the integrated $d\sigma/dp$ at small p_π amounting to 9% of the total integrated inclusive cross section.

This effect should occur over and above the general nonscaling features, i.e., the observed monotonic increase at $x\leq 0.5$ of $S\,d\sigma/dx$ for W varying between 3 and 7.4 GeV. In order to see that such a "cascade pion" effect could indeed be distinguished from the background representing pions from noncharmed events and also the weak $D\bar{D}$ decay I present in Fig. 1 $W^3\,d\sigma/dp$ at $W=3.8$ GeV and the averaged result for $W=4$–4.4 GeV.[9] Both are drawn on an ordinary scale versus P_π.[10] The measurements do not extend below $P_\pi=0.10$ GeV/c because of the magnetic rigidity[11] and because presumably also the lowest available points are imprecise. This together with the averaging over $4\leq W\leq 4.4$ GeV prevents observation of the possible sharp onset of the low-P_π cascade-pion signal. However by smoothly extrapolating the available points to $P_\pi=0$ one may have an estimate of the background. I also indicate by dashed lines the low-P_π region where the cascade pions should contribute 9% of the integrated $W=4.2$-GeV curve, for $\Delta=0.200$ and $\Delta=0.160$ GeV, respectively ($\epsilon=0.200$ GeV).

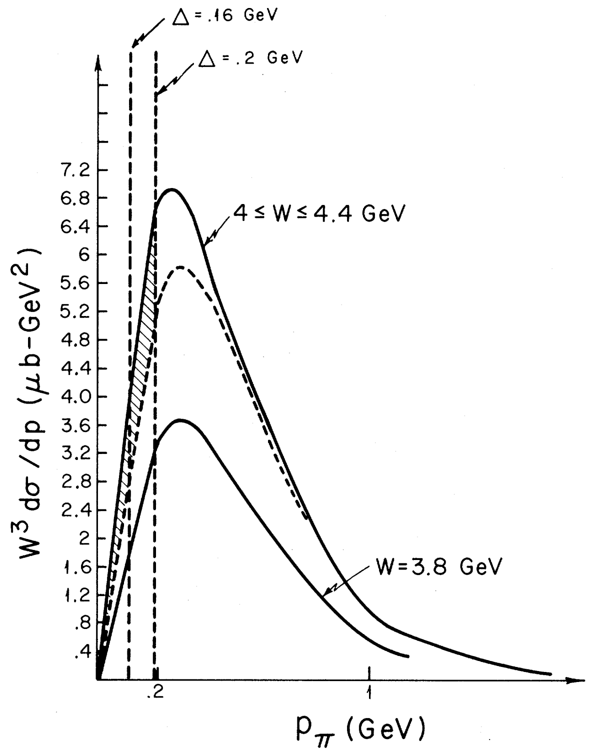

FIG. 1. The experimental $W^3 d\sigma/dp_{\text{charged}}$ spectra for e^+e^- collisions at $W=3.8$ and $4\leq W\leq 4.4$ GeV. The dashed curve indicates scaled-up 3.8-GeV data representing some minimal background at $W\approx 4.2$ GeV and the lines are the kinematic limits for the soft cascade pions for $\Delta=0.16$ and 0.2 GeV.

Note that the *total* fraction of the area under the 4–4.4-GeV curve to the left of these lines is 16% and 4%, respectively. Furthermore, most of it ($\approx 75\%$) is readily accounted for as background as indicated by the dashed curve which is the 3.8-GeV data scaled by the ratio of the curves at $P_\pi=0.5$ GeV/c, a region where the soft cascade pions certainly do not contribute. Thus even if the value of $y \sim 0.09$ is a considerable overestimate and $y \approx 0.03-0.06$ only, a careful study of the low-P_π region for beam energies $W \approx 4-4.2$ GeV, the region of the large R rise, should very clearly reveal the soft cascade pions by a "sudden" 50–100% increase of $d\sigma/dp$ at $P_\pi \leq 0.100-0.200$ GeV/c.[12]

Instead of using the data directly to estimate the background I could have resorted to some statistical fireball model. In such models the observed nonscaling inclusive π distribution can be reproduced by sequential decay of fireballs emitting pions with an average energy of ≈ 400 MeV. The decay $D^* \to D\pi$ has much lower energy and therefore a unique signature in the low-P_π region.

To date no feature of e^+e^- annihilation was found to display a marked sharp variation in the short region where R increases quickly.[13] The sole exception is a jump in $S\,d\sigma/dx$ between $W \cong 3.8$ and $W \simeq 4$ GeV for $0.8 \leq x \leq 12$, i.e., 160 MeV $\leq P_\pi \leq 240$ MeV/c.[9] This may very well be an indication that the cascade-pion effect suggested here is indeed observable.

However, in order to really verify the interpretation suggested here, the association of this phenomenon with the increase in R should be checked much more carefully. What I am suggesting in effect is that the jump in $S\,d\sigma/dx$ for low P_π should occur very sharply in the same energy bin of 20 MeV or so where R rises by roughly $\delta R = 2$. The averaging over a large W interval in the $S\,d\sigma/dx$ data may account for the intermediate leveling suggested by the data. If the predicted soft cascade pions are found they could serve as strong indications of the existence of charmed particles.

The discussion above regarding estimates of Δ does not rule out $\Delta < m_\pi$ which indeed we may have to assume if the slow-cascade-pion search fails (short of giving up the charm schemes altogether). In this case D^* decays into $D\gamma$ with $E_\gamma{}^* \approx \Delta$ (≈ 100 MeV in the subsequent estimates) which transforms to $\Delta + \Delta\beta \geq E_\gamma \geq \Delta - \Delta\beta$ in the lab frame ($\beta \approx 0.3$ for $\epsilon = 0.2$ GeV). This still constitutes a narrow region of very low-energy ($E_\gamma \approx \Delta \approx 100$ MeV) photons, which as indicated in Fig. 2 stands clearly above the background estimated from π^0 decays. Under the assumption $d\sigma/dp_{\pi^c} \approx d\sigma/dp_{\pi^0}$ the integrated signal-to-background ratio is, in analogy with (1),

$$y^0 = 2\delta R f_{D^*D^*}/\langle n_\gamma\rangle R = 0.125.$$

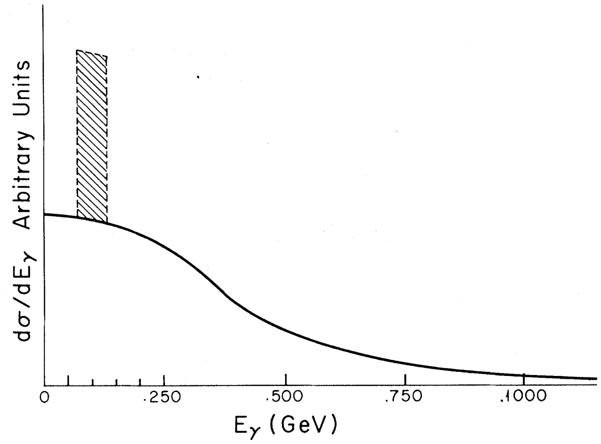

FIG. 2. The $D^* \to D\gamma$ photons (hatched area) versus background photons for $\Delta = 0.1$ GeV.

A good-resolution experiment with $\Delta E \leq 20$ MeV, if feasible, could very likely indicate the $D^* \to D\gamma$ photons.

Next we note a general result on the equality of the inclusive $\pi^+\pi^-$ and π^0 spectra for those pions which result from any charm-production events via a strong-interaction cascade down to $D\bar{D}$. It follows directly from the fact that the charmed-particle production occurs via an isoscalar part ($c\bar{c}$) of the electromagnetic current and, because it involves an eventual weak D decay, does not interfere with noncharmed production.

A formal proof of the identity of the inclusive cross section for the cascade $\pi^+\pi^-$ and π^0 is obtained by using the Mueller amplitude for the forward "scattering" of a $\pi^+\pi^-$ and π^0, respectively, from a fictitious isosinglet "$\gamma_{\bar{c}c}$" target. A similar argument leads to equality of K^+ and K^0 inclusive cross sections and hence eventually to

$$d\sigma/dp_{K^+} = d\sigma/dp_{K^-} = d\sigma/dp_{K_S} = d\sigma/dp_{K_L};$$

since $\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0) = 2$ and K_L also yields some ($\approx 20\%$) charged pions we find that also the kaonic cascade component of the inclusive cross section will contribute $\approx 70\%$ of its energy to the charged particles and only 30% to neutrals. The only mechanism for violating equipartition for the cascade particles involves the rather unlikely copious production of η's. This is particularly unlikely in a cascade because of the large η mass and even in the extreme case of exact SU(3) would occur only $\frac{1}{8}$ of the time.

This implies a slow increase of $\langle E_{\text{charged}}/W\rangle$ in the region $4 \leq W \leq 8$ GeV if additional isosinglet quark pairs ($c\bar{c}$) are introduced. This is so because far above the threshold region an increasing amount of energy appears as the strong-interaction cascade π's which do obey equipartition. Since no such trend is evident in the data[9] this tends to strengthen arguments based on the R value that other mechanisms involving heavy leptons which always decay to neutrinos and other particles are operative in the range $4 \leq W \leq 8$ GeV.

In the above we have indicated one particular short cut for indication of charmed-particle production. One other interesting possibility[14]—if charged multiplicity distributions should become available—is to compute the second correlation coefficient. It should reflect the existence of a two-cluster ($D\bar{D}$, $D^*\bar{D}^*$, etc.) component which at $W \approx 4.2$ GeV adds to the "old physics," presumably one-cluster mechanism.

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[10]To translate back from x to p I used the average value of $W \approx 4.2$ GeV.

[11]B. Wiik and M. Breidenbach, private communication. The spectrometer field can be reduced to avoid trapping of low-momentum particles allowing a more careful study of the threshold region.

[12]A unique feature of the soft-pion ($p \leq 0.2$ GeV) signal from the $D^* \to D\pi$ cascade is that it disappears once we are sufficiently above the $D^*\bar{D}^*$ threshold. This is so because $D^*\bar{D}^*$ production will be damped by form factors and competing D^{**}, D^{***}, etc., and continuum production dominates. Also the higher charmed resonances are likely to decay strongly, emitting relatively large-momentum pions (e.g., $D^{**} \to D\pi$ may dominate over the $D^{**} \to D^*\pi$ both being D-wave decays with a higher Q value for the first) and will not contribute very strongly at $p_\pi \leq 0.2$ GeV. Finally $D^* \to D\pi$ decays, if they still occur at higher W's via direct $D^*\bar{D}^*$ production or some complicated cascade process, will tend to suffer stronger Doppler spreading. This is in marked contradistinction to the general $x \leq 0.5$ background which appears to be monotonically increasing.

[13]The data (Ref. 9) may indicate a statistically not too significant structure in charged-particle multiplicity at this point. The tendency to increase the charged multiplicity at this point may reflect in part the cascade pions.

[14]L. Van Hove, private communication.