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[1]For a recent experimental and theoretical review, see A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975); J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).

[2]As it is well known (see Ref. 1), the dipole interaction actually breaks this invariance which results in a gap opening in the spin-wave spectrum at low frequency.

[3]P. W. Anderson, Phys. Rev. Lett. 30, 368 (1973); P. G. de Gennes, Phys. Lett. 44A, 271 (1973); P. Wolfle, Phys. Lett. 47A, 224 (1974).

[4]R. Graham, Phys. Rev. Lett. 33, 1431 (1974).

[5]The angular momentum conservation law cannot be invoked since it is not really independent from the momentum conservation law. It implies simply that the stress tensor is symmetric (see Ref. 4).

[6]A more detailed account of this work will be given elsewhere.

[7]O. Betbeder-Matibet and P. Nozières, Ann. Phys. (N.Y.) 51, 392 (1969).

[8]Actually, this limit is already assumed in deriving the right-hand side of Eq. (7), but it can be proved, before taking the limit $\omega/qv_F \to 0$, that the solution corresponds to this limit: For a nonzero ω/qv_F, the solution would be obtained by equating to zero the left-hand side of Eq. (7), which has no solution for $\omega/qv_F \neq 0$.

[9]As cited by P. W. Anderson and W. F. Brinkman, in Lecture Notes of the 1974 Scottish Universities Summer School (to be published).

[10]R. Graham and H. Pleiner, Phys. Rev. Lett. 34, 792 (1975).

[11]This result has also been obtained independently by M. C. Cross and P. W. Anderson, in *Proceedings of the Fourteenth International Conference on Low Temperature Physics, Otaniemi, Finland, 1975,* edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975).

[12]P. Bhattacharyya, C. J. Pethick, and H. Smith, to be published.

[13]G. Baym and C. J. Pethick, to be published.

[14]D. N. Paulson, R. L. Kleinberg, and J. C. Wheatley, to be published.

Formation of Mesonic Atoms in Condensed Matter

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The formation of mesonic atoms in condensed matter is calculated with a parameter-free classical model. The probability for Coulomb capture is found to be proportional to $Z^{1/3}\ln(0.57Z)$. It is shown by a χ^2 test performed for metal halides that the new formula reproduces well the experimental data.

The capture of negative meson particles such as muons, pions, kaons, or antiprotons in condensed matter has been treated many times since the pioneering paper by Fermi and Teller.[1] Older work was summarized by Burhop[2] and by Ponomarev.[3] Recently Leon and Seki[4] and Haff, Vogel, and Winther[5] investigated modifications of the Fermi-Teller theory. The experimental data on the distribution of muons in the different components of a chemical compound does not follow the "Z law" as originally proposed,[1] nor any other proposed relationship between the values of the atomic number Z of the individual constituents. It is the aim of this Letter to present a calculation of the Coulomb capture probability of negative mesonic particles in condensed matter, performed with a purely classical model based to a large extent on the treatment by Fermi and Teller.[1] Special emphasis will be given to a simple formula describing the gross features of the process.

Consider a negative meson particle of mass M and (kinetic plus potential) energy $W>0$, which is traveling at low speed v through condensed matter. This particle will be trapped in a potential well if it loses enough energy while traveling inside the well so that W becomes negative. We approximate the potential in matter by a spherically symmetric screened Coulomb potential around the nucleus up to a radius r_0 where r_0 is determined, e.g., by the dimension of the elementary cell in the case of a simple lattice.

Let us now consider the process quantitatively. For the screened Coulomb potential we take[1]

$$U=-e^2 b\,dZ^{2/3}/r^2, \tag{1}$$

where $b=(9\pi^2/128)^{1/3}a_0$ (a_0 is Bohr radius), $d=0.4$, and r is the distance from the nucleus. The energy loss of the meson particle with $W\approx 0$ when traveling through this field, if the electrons are treated as Fermi gas, is

$$\Delta W = -\int_{r_1}^{r_2} \frac{dW}{ds}\,ds = -\int_{r_1}^{r_2} \frac{dW}{dt}\frac{dt}{ds}\,ds = -\frac{4}{3\pi}\frac{m^2e^4}{M\hbar^3}\ln\left(\frac{v_0}{\alpha c}\right)\int_{r_1}^{r_2}\frac{T}{v}\,ds, \qquad (2)$$

where ds is an element of the trajectory of the meson particle between r_1 and r_2, m the electron mass, e the elementary charge, v_0 the mean velocity of the atomic electrons, α the fine structure constant, c the speed of light, and T the kinetic energy of the mesonic particle. For dW/dt the expression given by Fermi and Teller was taken.[6] In the further evaluation of Eq. (2) we set $T=-U$ and obtain

$$\Delta W = -(2/3\pi)m\alpha^2c^2(2d)^{1/2}(m/M)^{1/2}(9\pi^2/128)^{1/6} \times Z^{1/3}\ln[v_0/(\alpha c)]\int_{r_2}^{r_2} ds/r. \qquad (3)$$

It is easily seen that there is no stable bound state in the potential Eq. (1). The slowly incoming particle comes close to the nucleus if, and only if,

$$q^2 < q_{\max}^2 = e^2 b\, dZ^{2/3}/W_0, \qquad (4)$$

where q is the impact parameter and W_0 the value of W with no potential. According to Eq. (3) only particles whose trajectories satisfy the condition Eq. (4) will lose an appreciable amount of energy.

If Eq. (4) is fulfilled we may replace ds by dr in Eq. (3); with the additional approximation

$$v_0 = \alpha c f Z^{2/3},$$

where f has the numerical value[7] 0.688, we obtain

$$\Delta W = -C(Z)[\ln(r_1/r_{\min}) + \ln(r_2/r_{\min})],$$

where

$$C(Z) = (2/3\pi)m\alpha^2c^2(2d)^{1/2}(m/M)^{1/2} \times (9\pi^2/128)^{1/6}Z^{1/3}\ln(fZ^{2/3}) = 0.23(\mu/M)^{1/2}Z^{1/3}\ln(0.57Z)\ \text{eV},$$

μ meaning the mass of the muon, and $r_{\min}$ is the minimum value of r when the particle passes the nucleus on its path between r_1 and r_2, $r_{\min}$ exceeding zero in reality, for $q>0$, because Eq. (1) does not hold for very small distances r. For a particle not captured we have with $r_1 = r_0$ also $r_2 = r_0$,

$$\Delta W = -2C(Z)\ln(r_0/r_{\min}), \qquad (5)$$

and, if Eq. (4) is fulfilled, $r_0 \gg r_{\min}$. In the gross structure of the Coulomb capture we are treating in this paper, we shall be allowed to neglect the dependence of $r_{\min}$ on Z, q, and W_0.

In the above approximation ΔW depends only on Z, and a particle entering the atom will be captured if, and only if, Eq. (4) holds and simultaneously the condition $W_0 \leqslant -\Delta W$ is fulfilled. A numerical estimate with $r_{\min}$ of the order of the Bohr orbit of the K electron or smaller shows that $r_0 \lesssim q_{\max}$, i.e., Eq. (4) is, at least approximately, fulfilled for all particles with $W_0 \leqslant -\Delta W$, where ΔW is calculated with the help of Eq. (5); it will be assumed from now on that Eq. (4) holds. Hence a particle with $q < r_0$ will be captured ex-

TABLE I. Ratio $A(Z_1/Z_2)$ of Coulomb capture probabilities per atom; Z_1 and Z_2 refer to metal and halogen, respectively. Column 3 lists the experimental values of the references given in column 2. Column 4 lists the values calculated with Eq. (6). Column 5 lists the values from the "Z law." For the meaning of χ_ν^2 see text.

Compound 1	Ref. 2	Experiment 3	This work 4	"Z-Law" 5
LiF	13	0.28 ± 0.03	0.23	0.33
LiI	11	0.06 ± 0.01	0.06	0.06
NaF	13	1.56 ± 0.81	1.20	1.22
NaCl	9	0.68 ± 0.06	0.70	0.65
NaI	13	0.29 ± 0.03	0.32	0.21
AlF_3	10	0.90 ± 0.30	1.38	1.44
$AlCl_3$	10	0.63 ± 0.21	0.81	0.75
AlI_3	10	0.48 ± 0.18	0.37	0.24
KCl	9	1.16 ± 0.03	1.09	1.12
KBr	10	0.47 ± 0.06	0.65	0.54
KI	13	0.50 ± 0.05	0.50	0.36
$CaCl_2$	13	1.56 ± 0.18	1.13	1.18
AgI	13	1.42 ± 0.25	0.93	0.89
CdF_2	10	3.98 ± 0.54	3.53	5.34
$CdCl_2$	10	2.26 ± 0.26	2.06	2.82
$CdBr_2$	10	1.06 ± 0.10	1.23	1.38
CdI_2	10	1.00 ± 0.12	0.94	0.90
$SnCl_2$	10	1.98 ± 0.22	2.11	2.94
$SnCl_4$	10	2.36 ± 0.40	2.11	2.94
SbF_3	10	3.69 ± 0.42	3.67	5.67
$SbCl_3$	10	2.55 ± 0.39	2.14	3.00
$SbCl_5$	10	2.30 ± 0.45	2.14	3.00
PbF_2	13	4.70 ± 0.40	4.91	9.12
$PbCl_2$	13	3.16 ± 0.24	2.86	4.82
PbI_2	13	1.22 ± 0.12	1.30	1.54
BiF_3	12	4.74 ± 0.45	4.94	9.21
UF_4	12	6.08 ± 0.60	5.25	10.24
		χ_ν^2	1.61	15.98

actly if

$$W_0 \leq 2C(Z)\ln(r_0/r_{\min}).$$

Since, in sufficient approximation,[8] the mesonic particles enter the atom with a white energy spectrum up to an energy $W_w > -\Delta W$, $-\Delta W/W_w$ is just the capture probability for particles with $W_0 < W_w$ entering the atom, i.e., having a q smaller than r_0. As a further approximation we assume that r_0 has the same value for all atoms in a chemical compound. The capture in a constituent of the compound is then proportional to $-\Delta W$, i.e., proportional to $Z^{1/3}\ln(0.57Z) \approx Z^{1/3}\ln(Z/2)$, and we obtain as ratio of capture probabilities per atom in the case of a binary compound with elements Z_1 and Z_2

$$A\left(\frac{Z_1}{Z_2}\right) = \frac{Z_1^{1/3}\ln(0.57Z_1)}{Z_2^{1/3}\ln(0.57Z_2)}. \qquad (6)$$

As a large amount of new experimental data on metal halides[9,10] is available we compared the predictions of Eq. (6) with *all* experimental data on metal halides[9-13] except those old values which could be replaced by new values of higher accuracy. This comparison is presented in Table I. The quality of the representation of the experimental data by the theory is characterized by the normalized value χ_ν^2 of the quantity χ_ν^2, which must not appreciably exceed unity for a good representation.[14] For comparison the predictions of the "Z law" are included. Considering the crude approximations in the derivation of Eq. (6), the agreement between experiment and our theory is excellent whereas the "Z law" does not well reproduce the data.

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[1]E. Fermi and E. Teller, Phys. Rev. 72, 399 (1947).

[2]E. H. S. Burhop, in *High Energy Physics*, edited by E. H. S. Burhop (Academic, New York, 1969), Vol. 3, p. 109 ff.

[3]L. I. Ponomarev, Annu. Rev. Nucl. Sci. 23, 395 (1973).

[4]M. Leon and R. Seki, Phys. Rev. Lett. 32, 132 (1974).

[5]P. K. Haff, P. Vogel, and A. Winther, Phys. Rev. A 10, 1430 (1974).

[6]In Ref. 1, v_0 means the maximum electron velocity at the instantaneous position in space of the mesonic particle. As this maximum velocity depends on r, and as the mesonic particle does not move slowly compared to the electrons at small r values, we use for v_0 the electron velocity averaged over the whole atom.

[7]C. A. Coulson and N. H. March, Proc. Phys. Soc., London, Sect. A 63, 367 (1950).

[8]The question of the *exact* form of the energy spectrum is difficult to answer. The approximately white energy spectrum follows from the approximately energy-independent *mean* energy loss of a beam of particles.

[9]L. F. Mausner, R. A. Naumann, J. A. Monard, and S. N. Kaplan, Phys. Lett. 56B, 145 (1975).

[10]A. Brandão d'Oliveira, Thesis, Technical University of Munich, Munich, 1975 (unpublished); A. Brandão d'Oliveira, H. Daniel, and T. von Egidy, to be published.

[11]J. F. Lathrop, R. A. Lundy, R. A. Swanson, V. L. Telegdi, and D. D. Yovanovitch, Nuovo Cimento 15, 813 (1960).

[12]M. Eckhause, T. A. Filippas, R. B. Sutton, R. E. Welsh, and T. A. Romanowski, Nuovo Cimento 24, 666 (1962).

[13]V. G. Zinov, A. D. Konin, and A. I. Mukhin, Yad. Fiz. 2, 859 (1965) [Sov. J. Nucl. Phys. 2, 613 (1966)].

[14]Ph. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York, 1969), p. 89.