tervals, 9.3–10.4–11.1–11.8 GeV. The $t-t_{\text{min}}$ distributions in each interval are fitted by a production cross section of form $A\exp(bt)$ after the calculated backgrounds are removed. The results in the respective energy regions are 0.94 ± 0.20, 1.10 ± 0.17, and 0.60 ± 0.12 nb/GeV^2 for A, and 0.97 ± 0.25, 1.31 ± 0.19, and 0.92 ± 0.21 GeV^{-2} for b. One may conclude that either the elastic cross section coefficient, A, is independent of energy and there is no statistically significant evidence for inelastic production, or the coefficient has an energy dependence which just compensates the inelastic production so that it appears to be constant.

The effect of the internal motion of the nucleons in the beryllium target nucleus has been studied. Using a phase-space distribution, $f(p)dp = 375 \times p^2 dp$ ($p < 0.2$ GeV/c), $f(p)dp = 0$ ($p > 0.2$ GeV/c),[3] for the target momentum spectrum, no significant change is found in the calculated $t-t_{\text{min}}$ or k distributions. The fitted values of A and b increase slightly to 1.01 nb/GeV^2 and 1.25 GeV^{-2}.

Taking into account the systematic experimental uncertainties as well as the possible inelastic contributions, our value for the elastic cross section parameters becomes $A = 1.01 \pm 0.20$ nb/GeV^2 and $b = 1.25 \pm 0.20$ GeV^{-2} at an energy of 11.0 GeV. There have been two other recent measurements of the photoproduction cross section at somewhat higher energies.[4,5] These results, together with ours, indicate that the forward cross section, starting above 11.8 GeV, rises rapidly with energy, reaching a value of 20 nb/GeV^2 at $k = 20$ GeV and 50 nb/GeV at 100 GeV.[6] This behavior was qualitatively suggested by Harari[7] based on arguments relating Zweig's rule and charm threshold.

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Magnetic Moment of the Proton in H_2O in Bohr Magnetons*

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We have measured the ratio of the magnetic moment of the electron in hydrogen to the magnetic moment of the proton in H_2O to be $g_j(\mathrm{H})/g_p' = 658.216\,0091(68)$ [10 parts per billion (ppb)], at a temperature of 34.7°C. This yields a value for the proton moment in Bohr magnetons of $\mu_p'/\mu_B = 0.001\,520\,992\,983(17)$ (11 ppb). Our result differs from the currently accepted value by 150 ppb.

Most fundamental experiments in atomic physics which involve determination of a magnetic field yield results in terms of an NMR frequency; in order to relate the results to useful atomic constants the magnetic moment of the proton as observed under NMR conditions must be known in terms of the Bohr magneton. Thus the ratio $R' = \mu_B/\mu_p'$, where μ_p' is the effective moment of the proton, is frequently needed as a natural conversion constant. The currently accepted value of R' has a reported accuracy[1] of 66 ppb (parts per billion), high enough to have allowed R' to be treated as a fixed constant in most previous experiments. However, a number of experiments are approaching a precision where uncertainty in R' becomes important. Among these are a new determination of the gyromagnetic ratio of the proton aimed at yielding a more precise value of the fine structure constant,[2] and forthcoming redeterminations of the electron g-factor anomaly[3] and the magnetic moment of the neutron.[4] In addition, an improved value of R' allows a more precise absolute determination of the diamagnetic shielding constant for molecules, a problem of current theoretical interest.[5] In this Letter we report a new value for the ratio of the magnetic moment of the proton in a spherical sample of water to the Bohr magneton. The accuracy is 10 ppb; the measurement differs from the currently accepted value by 150 ppb, about 2.5 times the latter's estimated uncertainty.

The measurement was made by comparing the NMR frequency of protons in a nearly spherical sample of water with the spin-flip frequency of electrons in an atomic hydrogen maser.[6] The experiment involved interchanging samples in a fixed magnetic field of 0.35 T. The proton frequency was 15 MHz; the electron transition $(m_s = -\frac{1}{2}, m_I = -\frac{1}{2}) \rightarrow (m_s = +\frac{1}{2}, m_I = -\frac{1}{2})$ occurred at 9.2 GHz. The two frequencies were compared by repeatedly interchanging the samples and observing their free precession decay signals. The electron frequency was corrected for effects of hyperfine structure and nuclear spin using the accurately known values of the hydrogen hyperfine separation and the ratio $g_j(\mathrm{H})/g_p(\mathrm{H})$.[6, 7] The ratio of the corrected electron frequency to the proton frequency is $g_j(\mathrm{H})/g_p'$ from which μ_p'/μ_B is calculated.

The atomic hydrogen and NMR samples were contained in spherical quartz bulbs 1.3 cm in diameter with 2-mm-diam entrance apertures. The bulbs were on 5-cm centers. The hydrogen bulb was coated with $(CF_3CH_2CH_2)_2SiCl_2$ to inhibit wall relaxation.[8] A thin collimator at the beam entrance served to prolong the storage time. The water sample had a similar collimator; the two sample holders were as similar as possible. Coating the water bulb and collimator prevented the water from wetting the surface of the bulb, and made it easier to contain the water in the bulb and to determine its shape accurately. [This was the motivation for using water rather than organic NMR standards such as $(CH_3)_4Si$. Such liquids were impossible to contain completely in the bulb; they would wet the surface, lose a well-defined meniscus, and be drawn into the collimator.]

The sample interchange mechanism was designed so that the only moving part was the quartz sample assembly. The active sample was centered in a double cavity, a microwave cavity whose cylindrical portion was made of a tightly wound copper strip that formed the NMR pickup coil. This arrangement resulted in a poor NMR filling factor but assured that the rf field was uniform over the sample volume.

The resonance signals were converted to an audio frequency which was digitally sampled and stored. Successive signals were averaged until the desired signal-to-noise ratio (S/N) was obtained. The data were later Fourier analyzed using a fast-Fourier-transform algorithm.

The maser signal was fitted by an absorptive and dispersive Lorentzian. The full width at half-maximum was typically 600 Hz. 5 sec of signal averaging typically yielded a S/N of 30 and a standard deviation of 4 ppb, consistent with the known magnitude of the magnetic-field fluctuations.

Because the NMR line shape is the convolution of a relaxation-broadened line with the inhomogeneous magnetic field distribution, the NMR line could not be fitted accurately by a simple function. Instead, the real and imaginary parts of the transform were combined to produce a purely absorptive line shape whose centroid was calculated. Computer modeling confirmed that this procedure was valid for a wide range of possible field distributions. NMR linewidths were typically 1.5 Hz. A S/N of approximately 100 was obtained with 30–60 sec of averaging. The standard deviation of points taken over a period of a few minutes was approximately 13 ppb.

A complete run comprised four or five hydrogen-NMR interchange cycles, each cycle taking approximately 6 min. After correction for the small magnetic field drift, usually 1 or 2 ppb per minute, the standard deviation for individual ratio measurements in a run was typically 8 ppb.

Most of the potential sources of error are due to the problem of assuring that both samples experience the same field distribution. The inside of each bulb was spherical to ~0.1% and diameters of different bulbs were identical to the same precision. In order to eliminate effects of uncontrollable features such as small variations in wall thickness the bulbs were interchanged, so that each bulb was used *both* as an NMR sample holder and as a maser storage bulb.

Several corrections remained in spite of the interchange procedure. The calculated values of these corrections and their estimated uncertainties are shown in Table I for each of the bulb configurations, along with uncertainties caused by statistical scatter and uncalculated systematic effects. The statistical error represents the scatter (standard deviation) of the results of all runs for each bulb configuration.

The meniscus correction is due to the roughly hemispherical intrusion of the meniscus into the otherwise spherical sample of water, which causes a shift from the average field experienced by a diamagnetic sphere. Gradient corrections arise because the meniscus and collimator regions of the bulb were accessible only to atomic hydrogen and not to the water. Field gradients in these regions, caused by the bulb stem and collimator diamagnetism, shift the average field seen by the hydrogen from that seen by the water. The diamagnetism of the water sample changes the field seen by the hydrogen, resulting in a correction that is the same for every configuration. Other effects whose magnitude was estimated include shifts due to bulb asymmetries, errors in positioning the bulbs in the field, regions other than the collimator and meniscus which were inaccessible to water, and possible effects of a nonuniform filling factor over the NMR sample.

It should be pointed out that most of these corrections are not needed to measure a field by NMR to the accuracy of our result. The major problems we encountered arose from the necessity of using sample holders that were suitable for both atomic hydrogen and water.

Bulbs 1 and 2 were used together as bulb pair *A*, and 3 and 4 were pair *B*. The unweighted average of the two results from each pair constitutes the final result for that pair. Care must be taken in computing the uncertainty of the result for one pairing since the errors for the two bulbs are not independent. This is because the bulbs in a given pair were very similar and the corrections shown in Table I were calculated using the same method for each bulb. Only the statistical error is combined as if it were independent for the two bulbs. A summary of the results is given in Table II.

TABLE I. Corrections and uncertainties in $g_j(\mathrm{H})/g_p'$, ppb.

Configuration	1	2	3	4
Statistical	(19)	(5)	(4)	(8)
Meniscus	− 47(16)	− 41(14)	− 15(5)	− 9(2)
Gradients	− 22(11)	− 19(10)	− 8(5)	− 1(3)
H_2O diamagnetism	− 8(2)	− 8(2)	− 8(2)	− 8(2)
Other	(13)	(13)	(6)	(6)

The difference between the results for the two pairs is only 1.5 ppb. Since the corrections for set *A* were much larger than for set *B* because of the larger meniscus, the good agreement suggests that the correction procedure was valid.

The final result is the weighted mean of the results of the two pairs of bulbs. We assume most of the errors to be independent since they arise mainly from statistics or meniscus-related corrections for different meniscuses. For a spherical sample of water at 34.7°C we obtain

$$g_j(\mathrm{H})/g_p' = 658.216\,0091(68)\ (10\ \mathrm{ppb}).$$

The uncertainty represents a 70% confidence interval and includes statistical error as well as our best estimate of the uncertainty introduced by all known systematic effects.

The most accurate previous measurement of $g_j(\mathrm{H})/g_p'$ was made by Lambe and Dicke.[1] They observed an electron spin-flip transition in hydrogen by microwave absorption of atomic hydrogen in a buffer gas. Their result is $g_j(\mathrm{H})/g_p' = 658.215\,9088(436)$ (66 ppb). The quoted error is twice the statistical error, and takes into account possible frequency shifts due to the buffer gas. Our method differs from theirs chiefly in the use of a hydrogen maser which yields a linewidth 30 times narrower and a S/N 10 times larger than the microwave-absorption method, as well as eliminating the buffer gas. In addition, the NMR linewidth and S/N have each been improved by a

TABLE II. Corrected values of $g_j(\mathrm{H})/g_p'$.

Single-configuration result
1: 658.216 0263(195) (30 ppb)
2: 658.215 9936(143) (22 ppb)
3: 658.215 9561(70) (11 ppb)
4: 658.216 0619(74) (11 ppb)

Pair result
A: 658.216 0100(171) (26 ppb)
B: 658.216 0090(71) (11 ppb)

Final result
658.216 0091(68) (10 ppb)

factor of 3. The two results differ by 152 ppb, more than twice the estimated error in the Lambe-Dicke experiment. Part of the discrepancy might be attributable to temperature effects. The proton moment in water decreases with rising temperature at a rate of approximately 10 ppb/°C.[9] The temperature of our samples was maintained accurately at 34.7(1)°C by the magnet's thermal control system. Lambe and Dicke did not measure the temperature.[10] Another possible source of error is the cylindrical neck of the Lambe-Dicke sample holder. We estimate that the bulk diamagnetic effect of the Teflon sample holder would reduce the field in the neck by roughly 1000 ppb. The neck volume was ~15% of the total volume which could give an error equal to the observed discrepancy.

From the value of $g_j(\mathrm{H})/g_p'$ which we obtain one can easily calculate the proton moment in Bohr magnetons. Using the value calculated by Grotch and Hegstrom,[11] $g_j(\mathrm{H})/g_e = 1 - 1.7705\times10^{-5}$, and the accepted value[12] $g_e/2 = 1.001\,159\,6567(35)$, we obtain

$$\mu_p'/\mu_\mathrm{B} = 0.001\,520\,992\,983(17) \quad (11 \text{ ppb}).$$

Using the theoretical value[11] $g_p(\text{free}) = 1 - 1.7733\times10^{-5}$ and the experimental result[2] $g_j(\mathrm{H})/g_p(\mathrm{H}) = 658.310\,706(6)$ we obtain for the diamagnetic shielding factor for water at 34.7°C

$$\sigma = 25.790(14)\times10^{-6}.$$

This result establishes a scale for the absolute values of shielding factors to an accuracy of 14 ppb. Determination of the relative shielding of H_2 and H_2O to a comparable precision by standard NMR techniques will now allow a critical test of recent theories[5] of diamagnetic shielding.

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