nucleons, and (iii) that momentum is conserved. The validity and implications of these theories can be determined by comparison with the values of σ_0 measured by this experiment.

The work of Lepore and Riddell[5] is a quantum mechanical calculation that employs the sudden approximation with shell-model wave functions to predict $\sigma_0^2 = \frac{1}{8} m_p B^{1/3}[45B^{1/3} - 25]$ (MeV/c)^2. This expression, where m_p is the proton mass, gives qualitative agreement with the measured values as shown in Table II. Feshbach and Huang[6] assume sudden emission of virtual clusters and relate σ_0 to the Fermi momentum of the projectile, P_F. With use of the formulation due to Goldhaber,[10] the relation between P_F and σ_0 is $\sigma_0^2 = \frac{1}{20} P_F^2 B^2/(B-1)$. The values of P_F determined by quasielastic electron scattering[7] give predicted values of σ_0 that are generally 25% higher than the measured values as shown in Table II. An interesting point to note here is that through the predicted relationship between σ_0 and P_F, this experiment measures the projectile Fermi momentum via nuclear fragmentation (see Table II). By assuming that the projectile has come to thermal equilibrium at an excitation temperature T, Goldhaber[10] has shown that the parabolic shape is again predicted and relates σ_0 to T by the equation $kT = 4\sigma_0^2/m_N B$, where k is Boltzmann's constant and m_N is the nucleon mass. The measured values of σ_0 then reflect excitation energies which we have listed in Table II along with the average binding energies per nucleon as determined by the projectile masses. Because our measured excitation energies are essentially the binding energy per nucleon of the projectiles, we infer that the fragmentation process which results in bound fragments involves very little energy transfer between the target and fragment.

*Work performed under auspices of the U. S. Energy Research and Development Administration and the National Aeronautics and Space Administration, Grant No. NGR 05-003-513.

[1]P. J. Lindstrom, D. E. Greiner, H. H. Heckman, B. Cork, and F. S. Bieser, Lawrence Berkeley Laboratory Report No. LBL 3650, 1975 (to be published).

[2]D. E. Greiner, P. J. Lindstrom, F. S. Bieser, and H. H. Heckman, Nucl. Instrum. Methods 116, 21 (1974).

[3]H. H. Heckman, D. E. Greiner, P. J. Lindstrom, and F. S. Bieser, Phys. Rev. Lett. 28, 926 (1972).

[4]W. R. Frazer *et al.*, Rev. Mod. Phys. 44, 284 (1972).

[5]J. V. Lepore and R. J. Riddell, Jr., Lawrence Berkeley Laboratory Report No. LBL 3086, 1974 (unpublished).

[6]H. Feshbach and K. Huang, Phys. Lett. 47B, 300 (1973).

[7]E. J. Moniz, I. Sick, R. R. Whitney, J. R. Ficenec, R. D. Kephart, and W. P. Trower, Phys. Rev. Lett. 26, 445 (1971).

[8]Preliminary results based on eight isotopes not including ^{1}H indicated $\sigma_{P_\parallel} \approx 140$ MeV/c, independent of mass [H. H. Heckman, in *Proceedings of the Fifth International Conference on High Energy Physics and Nuclear Structure, Uppsala, Sweden, 1973,* edited by G. Tibbell (Elsevier, New York, 1974)].

[9]W. A. Wenzel, in Proceedings of the Lawrence Berkeley Laboratory Heavy-Ion Seminar, 1973 (unpublished).

[10]A. S. Goldhaber, Phys. Lett. 53B, 306 (1974).

Production Cross Sections of Be Isotopes in C and O Targets Bombarded by 2.8-GeV α Particles: Implications for Factorization

G. M. Raisbeck and F. Yiou

Laboratoire René Bernas du Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse, 91406 Orsay, France

(Received 31 March 1975)

We have measured the production cross sections of ^{7}Be in C and O targets, and ^{9}Be and ^{10}Be in C targets irradiated by 2.8-GeV α particles. The results are discussed in terms of the applicability of a factorization relationship proposed for high-energy nuclear cross sections.

The availability of relativistic nuclei at synchrotrons previously devoted exclusively to proton acceleration has opened up many new possibilities for studying nuclear interactions. One of the first observations to be made from such studies was the apparent applicability of factorization to high-energy nuclear cross sections.[1] This concept, as applied by Heckman and co-workers,[1,2] permits one to factor the fragmentation cross sections of relativistic nuclei into two parts, one depending on the projectile and product species, the other on the target. The latter dependence has been studied by the same group in targets ranging from H to Pb, and found to be approximately proportional to $A^{0.25}$, where A is the target mass. Such a conclusion has very important implications, not only for understanding the basic mechanism involved, but also in permitting the systematization and prediction of a large number of unmeasured cross sections for other applications.

One aspect of these results has bothered us for some time, and that is the applicability of such a relationship in the region of very light targets. The reason for this question is the ratios of some high-energy α- and proton-induced cross sections in carbon and oxygen that have been determined previously.[3-5] If we consider such reactions in the rest frame of the bombarding particle, they are equivalent to a carbon or oxygen projectile on H or He targets. Thus, from the results of Heckman *et al.*, we would expect the cross-section ratio σ_α/σ_p to be approximately $(4/1)^{0.25} = 1.41$. However, previous results for such ratios[3-5] were about 2. It appeared to us that there might be at least two explanations for this: (a) Factorization, at least in the form proposed by Heckman *et al.*, does not hold in the region of very light targets. (b) The α-induced cross sections were still not at their energy-independent "asymptotic" values, as required if the more general concept of limiting fragmentation is to apply.[1,2] This second possibility exists because the previous α-induced measurements, when considered in terms of the relevant velocity parameter, were only at ~220 MeV/nucleon.[3-5] As pointed out in Ref. 3, this is below the energy at which the nucleon-nucleon cross sections rise as a result of pion production, and thus may not represent a limiting value for nuclear cross sections. With the recent availability of α beams of energies up to 4.6 GeV (1.15 GeV/nucleon) at the Saturn synchrotron, we have undertaken to examine this aspect of the problem, and the data reported here are the initial results of that investigation.

The ^{7}Be cross sections in C and O have been measured by standard radiochemical techniques. A 1-mm-thick carbon target (6×20 mm) was irradiated in the internal α beam for 15 min, giving an integrated flux of $\sim 1\times10^{14}$ particles. The α energy was 2.8 GeV, and the relative intensity was monitored throughout the run. Beginning ~90 min after the irradiation the target was placed between two Al foils (1.5 mm thick), and counted with a calibrated Ge(Li) detector for the 20.4-min β^+ activity of ^{11}C by means of the 511-keV annihilation radiation. Several days later, in the same configuration, the target was counted for longer periods to measure the 478-keV γ activity from ^{7}Be. From these data, we measured $\sigma_C(^7\text{Be})/\sigma_C(^{11}\text{C}) = 0.39\pm0.03$. In a separate experiment, to be reported elsewhere,[6] the absolute value of $\sigma_C(^{11}\text{C})$ at 2.8 GeV has been measured to be 41.7 ± 2.5 mb. We are thus able to calculate directly $\sigma_C(^7\text{Be}) = 16.3\pm1.5$ mb.

In another experiment, a stack of C, Mg, and MgO targets was irradiated internally, also at 2.8 GeV, for a much longer time ($\sim 7\times10^{15}$ α's). In this case, the previously determined $\sigma_C(^7\text{Be})$ was used as the monitor cross section. By subtracting the ^{7}Be contribution of the Mg in the MgO (as measured in the pure Mg target), the ratio $\sigma_O(^7\text{Be})/\sigma_C(^7\text{Be})$ was determined.

The ^{9}Be and ^{10}Be cross sections in C were measured, relative to ^{7}Be, by a mass-spectrometric technique described in detail elsewhere.[3,7] Some improvements in sensitivity were necessary since, even with long irradiations, only about 10^{-11} g of these isotopes were formed in our targets. The targets, consisting of six separate pieces of 1-mm-thick, high-purity graphite, were irradiated internally for 50–100 h (in parasite with a biomedical experiment). Irradiations were made at both 4.6 and 2.8 GeV α energy. The most accurate results are, in fact, at 4.6 GeV. However, since we do not yet have an absolute monitor at this energy, and because the measured ratios are, within our experimental errors, the same at the two energies, we have averaged the results and present them as at 2.8 GeV. In fact on the basis of the energy dependence of the ^{11}C cross section between 1 and 2.8 GeV,[6] we believe that all the results presented here will also be approximately valid at higher energies. After irradiation, the targets underwent combustion, to remove the carbon, and then were heated under vacuum, to remove Li which masks the ^{7}Be. The relative quantities of ^{7}Be, ^{9}Be, and ^{10}Be were then measured in the mass spectrometer. These in turn were normalized to the $\sigma_C(^7\text{Be})$ given above, to get the absolute values of $\sigma_C(^9\text{Be})$ and $\sigma_C(^{10}\text{Be})$. Our final results and their estimated uncertainties are given in column 4 of Table I. Also given in Table I are earlier results for both protons and α's on C and O,[3,5,7-12] and the corresponding results of Lindstrom *et al.*[2] for ^{12}C and ^{16}O in a "H" target (the latter have actually been determined from a CH_2 target). We see that the

TABLE I. Cross sections for Be production in high-energy interactions of ^{12}C and ^{16}O with protons and α particles.

		σ_α (mb)		σ_p (mb)				
Target	Product	900 MeV	2.8 GeV	600 MeV	1.05 GeV[a]	2.1 GeV[a]	25 GeV	σ_α/σ_p
^{12}C	^{7}Be	20.0 ± 1 [b]	$16.3\ \pm1.5$	11.0 ± 1.1[d]	8.45 ± 0.81[g]	9.49 ± 0.99[g]	9.2 ± 0.4[d]	1.5–1.9
	^{9}Be	10.6 ± 1.7[c]	$10.0\ \pm1.4$	5.3 ± 0.8[e]	5.13 ± 0.54[g]	5.92 ± 0.54[g]	6.1 ± 0.9[i]	1.6–2.0
	^{10}Be	6.5 ± 1.4[c]	5.38 ± 0.93	2.8 ± 0.5[e]	3.41 ± 0.35[g]	3.42 ± 0.35[g]	3.6 ± 0.6[i]	1.5–1.9
^{16}O	^{7}Be	18.5 ± 1 [b]	$16.3\ \pm2.2$	$7\ \ \pm1.7$[f]	$9.4\ \pm1.5$ [h]	$10.1\ \pm1.2$ [g]	10.8 ± 1.4[f]	1.5–2.3

[a]These energies are in MeV/nucleon of the projectile nucleus.
[b]Ref. 5.
[c]Ref. 3.
[d]Ref. 8.
[e]Ref. 7.
[f]Ref. 9.
[g]Ref. 2.
[h]Ref. 10.
[i]Ref. 11.

anomaly mentioned at the beginning of this Letter is not explained by any substantial variation in the α-induced cross sections with energy. While the α-induced cross sections reported here are slightly smaller than at 220 MeV/nucleon, the ratio σ_α/σ_p is still significantly larger than predicted by an $A^{0.25}$ dependence. In the last column we give the range of σ_α/σ_p observed depending on the value of σ_p chosen. Since there does not appear to be a large systematic variation in the cross sections over the energies considered here, these fluctuations mostly represent the experimental uncertainties in the various measurements. However, in no case do the σ_α/σ_p values get as low as the expected 1.4, and an unweighted average, using all values, gives 1.74 ± 0.23.

In their latest work, Lindstrom *et al.*[2] have shown that their own data for a hydrogen target are not consistent with strict factorization. Therefore, in order to compare our results with the rest of their data, it is necessary to chose one of their heavier targets. If we use carbon, for example, we can calculate $\gamma_{He}=\gamma_C\sigma_{He}/\sigma_C$, where γ_T is given by the expression[2] $\sigma_B{}^F=\gamma_T\gamma_B{}^F$ and essentially gives the relative probability of producing a given fragment (F) from a certain projectile (B) in a target (T). Lindstrom *et al.*[2] have defined $\gamma_B{}^F$ in such a way that it represents the extrapolated or *hypothetical* cross section of producing the same fragment in a target of mass = 1. Using the data for γ_C and σ_C (2.1 GeV/nucleon) from Ref. 2, we calculate in Table II $\gamma_{He}=1.67\pm0.19$, where the error given is the standard deviation. The error of the mean would be about half this amount although there are not really sufficient data for these quantities to be meaningful. In Fig. 1 this value can be compared to the data of

TABLE II. Target factor for production of Be isotopes from ^{12}C and ^{16}O interactions with He.

Target	Product	σ_{He}[a] (mb)	σ_C[b] (mb)	γ_C[b]	$\gamma_{He}=\gamma_C\sigma_{He}/\sigma_C$
^{12}C	^{7}Be	16.3	18.6	1.92	1.68
	^{9}Be	10.0	10.6	1.92	1.81
	^{9}Be	5.38	5.81	1.92	1.78
^{16}O	^{7}Be	16.3	22.3	1.92	1.40
					1.67 ± 0.19

[a]This work.
[b]Ref. 2 (2.1 GeV/nucleon).

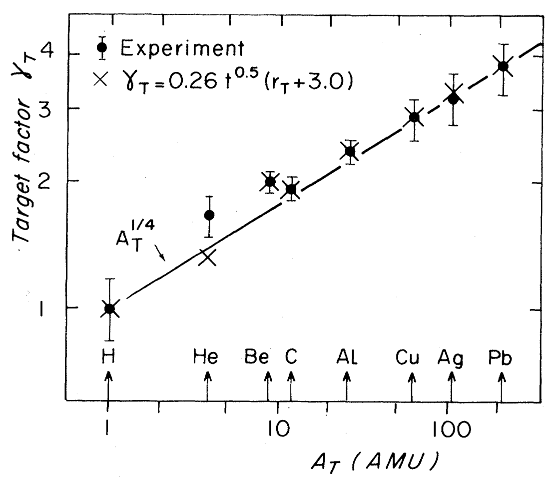

FIG. 1. Experimental and calculated values for target factor (γ_T), as defined in Ref. 2. This factor gives the relative probability of forming a given fragment from a given projectile in a target of mass A_T compared to the *extrapolated* probability for the same reaction in a target of mass = 1 (see text). The crosses are theoretical values calculated by use of the formula proposed in Ref. 2. Data for He are from this work, while all other results are from Ref. 2.

Ref. 2, where it is seen to lie above the general $A^{0.25}$ trend of the other points. Heckman *et al.*[1,2] have interpreted the $A^{0.25}$ target dependence as indicating that the observed fragmentation of relativistic nuclei is a peripheral process. In their most recent work[2] they have pursued this idea further by suggesting that a functional form for the target factor (γ_T),

$$\gamma_T = kt^n(r_T + b) \qquad (1)$$

(where r_T and t are measures of the target radius and skin thickness), can better explain a slight structure observed between Be and C targets. In order to see whether this could also explain our results, we have calculated γ_T for He according to (1), using their values for the constants k, t, and b, and data from Ref. 12 for r_T and t. This result is also plotted in Fig. 1, and we see that the disagreement is even worse than with the simple $A^{0.25}$ dependence. This is because He is a tightly bound nucleus, and therefore presents a lower than normal peripheral surface.

We thus conclude that our data are not consistent with the factorization picture of Heckman *et al.*[1,2] One possibility is that we are not measuring the same quantity as those authors. For example, if there were large-momentum-transfer events, the fragments might remain in our target, but not be seen in the Berkeley experiments. On the other hand, this seems inconsistent with the observation of Lindstrom *et al.*[2] that they see no evidence for such events, and in fact for light targets are observing most of the total inelastic cross section.

A second possibility is that the factorization relationship deduced from heavier targets simply does not hold in the region of very light targets. We believe that both the present data and the lack of strict factorization for a hydrogen target[2] support such a conclusion. This is perhaps not surprising since, in contrast to heavier nuclei, it is difficult to see how all reactions involving protons or α's can be considered "peripheral." Barshay, Dover, and Vary[13] have argued that a certain type of approximate factorization might be expected to hold on essentially geometric grounds if two interacting nuclei have roughly similar size.

In order to clarify some of these questions, it will obviously be necessary to have additional data. It would be interesting, for example, to carry out the Berkeley-type experiment with a He target. Such an experiment has already been proposed.[14] It would also be interesting to have additional measurements of the type reported here, for other targets and products. We have already begun such a program, and the results will be reported elsewhere. A preliminary analysis shows that with heavier targets many σ_α/σ_p ratios are indeed closer to the expected factorization value of 1.4, although again ^{7}Be is anomalous. Zebelman *et al.*[15] have reported σ_α/σ_p ratios between 3 and 4 for light fragments from a U target. These data suggest that it may be difficult to completely factor out a beam- and product-independent γ_T between H and He.

We wish to thank J. Lestringuez for his very important contribution to this work in carrying out the mass-spectrometric measurements. We are indebted to R. Bimbot and M. F. Rivet for the use of the Ge(Li) counting facilities. We are grateful to the staff of the Saturn accelerator, and particularly R. Schoen for assistance in arranging and carrying out the irradiations. We thank our colleqgues J. Radin and H. Quechon for permission to quote the ^{11}C monitor result before publication.

[1]H. H. Heckman, D. E. Greiner, P. Lindstrom, and F. S. Bieser, Phys. Rev. Lett. 28, 926 (1972).

[2]P. J. Lindstrom, D. E. Greiner, H. H. Heckman, B. Cork, and F. S. Bieser, Lawrence Berkeley Laboratory Report No. LBL-3650, 1975 (to be published).

[3]J. Lestringuez, G. M. Raisbeck, F. Yiou, and R. Bernas, Phys. Lett., 36B, 331 (1971).

[4]G. M. Raisbeck, J. Lestringuez, and F. Yiou, Phys. Rev. C 6, 685 (1972).

[5]J. Radin, A. Smith, and N. Little, Phys. Rev. C 9, 1781 (1974).

[6]J. Radin, H. Quechon, G. Raisbeck, and F. Yiou, to be published. This experiment uses the technique of measuring the number of incident particles with a counter telescope and the ^{11}C produced by direct counting of a scintillator target on a photomultiplier tube, as described by J. B. Cumming, Annu. Rev. Nucl. Sci. 13, 261 (1963).

[7]P. Fontes, C. Perron, J. Lestringuez, F. Yiou, and R. Bernas, Nucl. Phys. A165, 405 (1971).

[8]Cumming, Ref. 6.

[9]F. Yiou, Ann. Phys. (Paris) 3, 169 (1968).

[10]G. M. Raisbeck and F. Yiou, unpublished.

[11]F. Yiou, G. Raisbeck, C. Perron, and P. Fontes, in *Proceedings of the Thirteenth International Cosmic Ray Conference, Denver, Colorado, 1973* (Univ. of Colorado, Denver, Colo., 1973), Vol. 1, p. 512.

[12]R. Hofstadter and H. R. Collard, in *Landolt-Börnstein: Numerical Data and Fundamental Relationships in Science and Technology,* edited by K.-H. Hellwege (Springer, Berlin, 1967), Group 1, Vol. 2, p. 21.

[13]S. Barshay, C. B. Dover, and J. P. Vary, Phys. Rev. C 11, 360 (1975).
[14]H. H. Heckman, private communication.
[15]A. M. Zebelman, A. M. Poskanzer, J. D. Bowman, R. G. Sextro, and V. E. Viola, Phys. Rev. C 11, 1280 (1975).

Observation of Electric Quadrupole Transitions in Multiphoton Ionization*

Melissa Lambropoulos, Stephen E. Moody, S. J. Smith,† and W. C. Lineberger ‡
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado 80302
(Received 31 March 1975)

Two flashlamp-pumped, tunable, dye lasers have been utilized to study three-photon ionization in atomic sodium. The resulting ion yield shows large peaks at laser frequencies which correspond to electric quadrupole transitions. This is the first direct observation of electric quadrupole effects in multiphoton ionization. The $3p\ {}^2P_{3/2} \rightarrow 4f$ matrix element is determined, and the fine-structure splitting of the $5p$ state is measured.

Previous work on tunable-laser excitation of atoms, with one exception,[1] has dealt exclusively with electric dipole processes. Various experiments have looked at both single-photon and two-photon resonances, and have monitored fluorescence,[2-6] stimulated emission,[7,8] or ionization.[9-13] Experiments on multiphoton ionization with high-intensity fixed-frequency lasers[14-16] have heretofore included only electric dipole transitions in the models for interpretation.[17] Recently, however, calculations of Lambropoulos, Doolan, and Rountree[18] on two-photon ionization in lithium have indicated that electric quadrupole transitions are dominant over the nonresonant dipole background for certain frequencies. Using two tunable dye lasers, we have measured the three-photon ionization rate of sodium as a function of laser frequencies, and have observed large resonances due to such transitions. Our results emphasize the necessity of including weak dipole-forbidden transitions in the interpretation of experiments using strong narrow-band sources.

For the results shown here, one laser is tuned to one of the $3s \rightarrow 3p$ transitions, while the second laser frequency is scanned to map out a higher transition, such as $3p \rightarrow 4f$, and ionization from this upper state is measured. The partial energy-level diagram in Fig. 1 shows the transitions which we excite. Resonances due to the $3p \rightarrow 5p$ and $3p \rightarrow 4f$ electric quadrupole transitions rise more than 2 orders of magnitude above the instrumental background (see Fig. 2) and an estimated 6 to 8 orders of magnitude above the nonresonant dipole background, depending upon the line in question. Careful measurements of the $3p \rightarrow 4d$ resonance provide a well-understood electric dipole comparison process.

In sodium, since the $3s \rightarrow 3p$ transitions have large oscillator strengths, it is necessary to pump them very weakly to avoid line-shape problems due to stimulated emission.[19,20] The quadrupole resonance, in contrast, may be excited with a much more intense laser before the onset of saturation. In this case, the ion yield is given

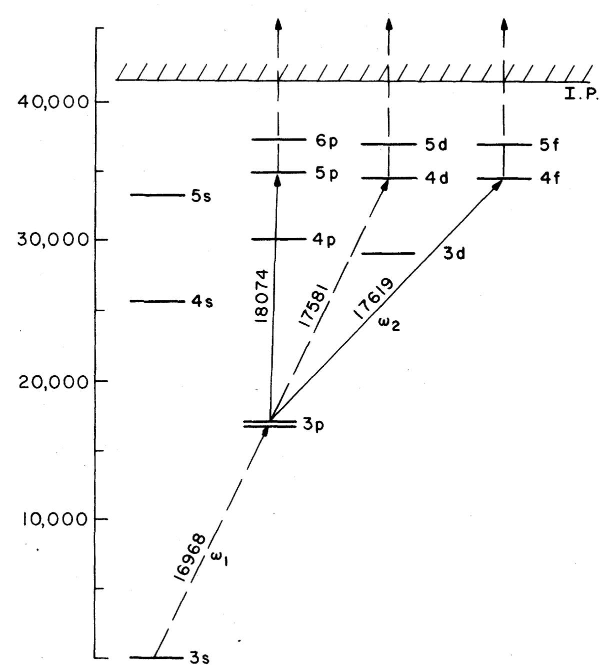

FIG. 1. Partial energy-level diagram of sodium, showing dipole transitions by broken lines and quadrupole transitions by solid lines. The vertical scale is energy in units of inverse centimeters.