In conclusion, the charmonium model remains an attractive hypothesis for the newly discovered particles. A crucial test of the hypothesis will be to search for the three p states of Table I, by means of the electromagnetic decays of Table II. The fine-structure corrections which we have calculated indicate that the p states will be sufficiently different in mass to allow experimental separation of them.[7] The precise values of the masses will of course be different for different forms of the binding potential $v(r)$. Confirmation of the model discussed here would not point to any specific interpretation for the quantum numbers of the new quark. The SU(4)-charm picture, for example, which is consistent with the leptonic decays of $\psi/J(3.1)$ and $\psi(3.7)$,[8] would require the discovery of charmed *particles*.

Note added.—After submitting this paper for publication, we received a manuscript by H. J. Schnitzer,[9] in which the p-state splittings are calculated by a similar method. He finds splittings which are somewhat smaller than those shown in Table I, due to a different choice of parameters. Like ours, however, they are considerably larger than the previous estimate of De Rújula, Georgi, and Glashow.[7]

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p States of Charmonium and the Forces that Confine Quarks*

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A measurement of the energy differences of the $n=2$, ^3P_J states of charmonium may give significant information on the dynamics of the spin-dependent quark-antiquark ($q\bar{q}$) forces, since the more rapidly these forces increase with $q\bar{q}$ separation, the larger the ratios

$$R_1=[E(^3P_2)-E(^3P_1)]/[E(^3P_1)-E(^3P_0)] \text{ and } R_2=[E(^3P_1)-E(^1P_1)]/[E(^3P_1)-E(^3P_0)]$$

become if the forces are of vector character.

The interpretation[1] of the newly discovered boson resonances[2] as bound states (charmonium) of charmed quarks and antiquarks ($c\bar{c}$) receives considerable support from the recently reported[3] decays $\psi' \to X+\gamma$ and $\psi'' \to \psi+2\gamma$. In the context of the charmonium picture[1] these decays are expected to be $\psi' \to {}^3P_J+\gamma$ and ${}^3P_J \to \psi+\gamma$, since the assignment[1] of $\psi(3100)$, $\psi'(3700)$, and $\psi''(4200)$ as the lowest $^3S_1$ states of charmonium also requires P states in the energy interval between ψ and ψ'. These features are compatible with a number of linear potential models,[4] which strongly indicate that the low-lying levels of the ψ system are nonrelativistic and that the charmed quark mass is heavy, i.e., in the 1.5–2-GeV range. In addition to the obvious interest in the gross features of ψ spectroscopy, certain finer details may be particularly useful in revealing important features of quark dynamics. As such, it is the purpose of this paper to draw attention to the fact that careful measurements of the energy *differences* among these P states may serve to resolve the controversy between two opposing views as to the dynamical origin of the spin dependence in the bound states of quark-antiquark ($q\bar{q}$) pairs. This is important since the spin-dependent forces are intimately related to the mechanism which confines quarks.

The energy difference among the P states is presumably due to relativistic spin-dependent corrections to the static spin-independent forces which confine $q\bar{q}$ pairs in color singlets. However, the detailed dynamics of these spin-dependent corrections is the subject of controversy. One view[5] holds that the spin dependence comes *only* from short-range (Coulomb-like) gluon interactions. This particular view is abstracted from lattice gauge models,[6] where spin-dependent forces originating from the confinement mechanism are exponentially damped relative to the leading spin-independent force which confines quarks. An alternative position holds[7] that since the $c\bar{c}$ system is nonrelativistic, the skeleton expansion of the Bethe-Salpeter kernel, describing low-lying bound states of color singlets, might be truncated at a single *dressed* (effective) Abelian gluon ladder. In this case, the same mechanism which gives rise to quark confinement also leads to the leading spin-dependent corrections to the nonrelativistic $c\bar{c}$ system. Here the spin-dependent interactions are short ranged, but by no means exponentially damped. We now develop an argument to show that these two points of view can be distinguished by the measurement of the energy differences of the lowest P states.

In contrast to the dynamics of positronium, a systematic derivation of an effective Hamiltonian for the lowest-order relativistic corrections for charmonium cannot as yet be given from first principles, so that some *ad hoc* hypotheses must be made in order to proceed. We assume[7] that (1) after the formation of color singlets, the Bethe-Salpeter kernel for the $c\bar{c}$ system can be approximated by a single dressed ladder, with an *effective* Abelian gluon propagator

$$D_{\mu\nu}(q)=g_{\mu\nu}d(q^2) \tag{1}$$

in Feynman gauge; (2) the nonrelativistic limit of the system is obtained by treating the kernel in the instantaneous (single-time) approximation; and (3) the effective gluon-quark vertex, to lowest order in the average quark velocity v/c, is given by

$$\Gamma_\mu=\text{const}\gamma_\mu. \tag{2}$$

It is a straightforward matter to obtain the effective Hamiltonian, to leading order in $(v/c)^2$, from these hypotheses. The static Fourier transform of $d(q^2)$ is

$$\int d^3r\, e^{-i\vec{q}\cdot\vec{r}}d(\vec{q}^2)=\text{const}V(r), \tag{3}$$

where the constants in (2) and (3) are chosen so that $V(r)$ is the static potential of the nonrelativistic Schrödinger equation. The effective Hamiltonian in the center of mass of the $c\bar{c}$ system (with relative coordinate r), to leading order to $(v/c)^2$, is

$$H=2m+\frac{p^2}{2m}+V(r)+\frac{3}{2m^2}\frac{1}{r}\frac{dV(r)}{dr}\vec{L}\cdot\vec{S}+\frac{1}{6m^2}\vec{\sigma}_1\cdot\vec{\sigma}_2\nabla^2V(r)-\frac{1}{12m^2}S_{12}\left(\frac{d^2V(r)}{dr^2}-\frac{1}{r}\frac{dV(r)}{dr}\right), \tag{4}$$

where spin-*independent* corrections have been omitted. Here m is the mass of the charmed quark, $\vec{L}$ the orbital angular momentum operator, $\vec{S}=\frac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2)$, and S_{12} the standard tensor operator. A reasonable assumption for the potential,[4] based on popular speculations, is

$$V(r)=ar+C-\alpha_s/r, \tag{5}$$

which includes a long-range force to confine quarks, a short-ranged Coulomb-like term as expected in asymptotically free gauge theories, and a constant C which subsumes those spin-independent forces not explicity included in (5). Combining (5) with (4), we obtain the effective Hamiltonian for the lowest-order spin-dependent corrections,

$$H_S=\tfrac{3}{2}m^{-2}[ar^{-1}+\alpha_S r^{-3}]\vec{L}\cdot\vec{S}+\tfrac{1}{6}m^{-2}[2ar^{-1}+4\pi\alpha_S\delta^3(\vec{r})]\vec{\sigma}_1\cdot\vec{\sigma}_2+\tfrac{1}{12}m^{-2}[ar^{-1}+3\alpha_S r^{-3}]S_{12}. \tag{6}$$

The $n=2$ P states are particularly advantageous for the study of quark dynamics. The P states in the 3400–3500-MeV region are probably sufficiently removed from the apparent threshold at 4 GeV to enable us to study them without undue complications due to the neglect of (virtual) charmed-meson intermediate states, so that these states are probably not too contaminated by virtual $q\bar{q}$ pairs. Furthermore, the tensor coupling to F states can be neglected because of large energy separations. Therefore, we treat (6) as a perturbation, retaining diagonal matrix elements only, obtaining

$$\langle {}^3P_2|H_S|{}^3P_2\rangle = \tfrac{1}{5}[9a\langle r^{-1}\rangle + 7\alpha_S\langle r^{-3}\rangle]m^{-2}, \tag{7a}$$

$$\langle {}^3P_1|H_S|{}^3P_1\rangle = -[a\langle r^{-1}\rangle + \alpha_S\langle r^{-3}\rangle]m^{-2}, \tag{7b}$$

$$\langle {}^3P_0|H_S|{}^3P_0\rangle = -[3a\langle r^{-1}\rangle + 4\alpha_S\langle r^{-3}\rangle]m^{-2}, \tag{7c}$$

and

$$\langle {}^1P_1|H_S|{}^1P_1\rangle = -a\langle r^{-1}\rangle m^{-2}, \tag{7d}$$

where the (positive definite) matrix elements $\langle r^{-1}\rangle$ and $\langle r^{-3}\rangle$ are to be evaluated with the $2P$ eigenfunction of the Hamiltonian

$$H_0 = 2m + p^2/2m + V(r). \tag{8}$$

The ratios

$$R_1 = \left[\frac{E({}^3P_2) - E({}^3P_1)}{E({}^3P_1) - E({}^3P_0)}\right] = \frac{1}{5}\left[\frac{12\alpha_s\langle r^{-3}\rangle + 14a\langle r^{-1}\rangle}{3\alpha_s\langle r^{-3}\rangle + 2a\langle r^{-1}\rangle}\right] \tag{9}$$

and

$$R_2 = \left[\frac{E({}^3P_1) - E({}^1P_1)}{E({}^3P_1) - E({}^3P_0)}\right] = \frac{-\alpha_S\langle r^{-3}\rangle}{3\alpha_s\langle r^{-3}\rangle + 2a\langle r^{-1}\rangle} \tag{10}$$

are restricted to

$$0.8 \le R_1 \le 1.4, \quad -\tfrac{1}{3} \le R_2 \le 0, \tag{11}$$

because of the positivity of the matrix elements, α_s, and a. The lower (upper) limit is attained if a purely Coulomb-like (linear) potential is the *sole* contributor to the *spin-dependent* forces between $c\bar{c}$ pairs. Thus, if the workers[5] who abstract the spin-dependent forces from lattice gauge models[6] are correct, then

$$(R_1)_{\text{Coulomb}} = 0.8 \quad \text{and} \quad (R_2)_{\text{Coulomb}} = -\tfrac{1}{3} \tag{12}$$

uniquely, exactly as in positronium. On the other hand, for typical charmonium models[4] with spin-dependence derived from the Bethe-Salpeter kernel,[7] R_1 and R_2 are close to the upper limits in (11). By way of contrast, a pure harmonic oscillator potential inserted in (4) gives $(R_1)_{\text{osc}} = 2$ and $(R_2)_{\text{osc}} = \frac{1}{3}$. In general, the more rapidly the spin-dependent forces increase with r, the larger the ratios R_1 and R_2.

In addition to the ratios R_1 and R_2, the absolute scale of the P-state energy differences will provide useful constraints for charmonium model builders. An estimate of the matrix elements in (7) can be made with the additional assumption that the Coulomb-like term in (6) can also be treated as a perturbation, as justified by the level spacings between ψ, ψ', and ψ''. We have evaluated $\langle r^{-1}\rangle$ and $\langle r^{-3}\rangle$ by using the $2P$-state wave function of a three-dimensional harmonic oscillator, and a variational principle to optimize the oscillator strength. This reduces the matrix elements to simple quadratures,[8] with

$$am^{-3}\langle r^{-1}\rangle = \tfrac{8}{3}(15\pi^2)^{-1/3}\beta^4 \tag{13}$$

and

$$\alpha_s m^{-3}\langle r^{-3}\rangle = \frac{32}{45\pi}\alpha_s\beta^3, \tag{14}$$

where the dimensionless parameter

$$\beta = (a/m^2)^{1/3} \simeq v/c \tag{15}$$

is the natural expansion parameter of the problem. The dependence of (13) and (14) on α_s and β follows from dimensional analysis alone, and the overall normalization has been estimated to be good to 10–15%. [This calculation neglects terms of order β^5, $\alpha_s\beta^4$, and α_s^2 in (13) and (14).][9]

Two typical linear potential models describing charmonium spectroscopy are those of Eichten *et al.*[4] [$\alpha_s = 0.2$, $\beta = 0.42$, and $m = 1.6$ GeV], and Kang and Schnitzer[4] [$\alpha_s = 0$, $\beta = 0.42$, and $m = 2$ GeV]. For the model of Eichten *et al.*[4] we find

$$\Delta E = E({}^3P_2) - E({}^3P_1) = (72 + 13)\text{ MeV} = 85\text{ MeV}, \tag{16}$$

$$R_1 = 1.2, \quad R_2 = -0.13,$$

where ΔE receives contributions of 72 and 13 MeV from $\langle r^{-1}\rangle$ and $\langle r^{-3}\rangle$ respectively. In the model of Kang and Schnitzer,[4]

$$\Delta E = 88 \text{ MeV}, \quad R_1 = 1.4, \quad R_2 = 0. \tag{17}$$

Note that the absolute scale of the P-state mass splittings is almost a factor of 5 larger than that predicted by De Rújula, Georgi, and Glashow.[5] Therefore, measurements of the energy scale appropriate to the P-state splittings will distinguish between models.

The establishment of the P-state analogs of ψ, ψ', and ψ'' in the 3400–3500-MeV region will give strong support to the simple potential picture of bound $c\bar{c}$ pairs. In this note we have emphasized how careful measurement of the P-state energy differences may serve to distinguish between competing pictures of quark confinement, as revealed by the spin-dependence of the $c\bar{c}$ forces. If in fact the γ rays observed in the decay of ψ' are correctly interpreted as $\psi' \to {}^3P_J + \gamma$, then a measurement of the ratio R_1 may be possible in the near future. A determination of R_1 to 10% accuracy might be sufficient to give us an experimental handle on an important issue of quark dynamics.

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Note added.—After this work was submitted, I became aware that similar calculations had been performed by Pumplin, Repko, and Sato.[10] The results seem to be in reasonable agreement.

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Comment on Direct Lepton Production*

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Recent data on observations of direct leptons produced in proton-proton collisions are analyzed. It is found that the large bulk of the data can be accounted for if the origin of the leptons is a low-mass, ~(1 to 5)m_π, vector meson which decays weakly into $\mu+\nu$ and $e+\nu$ symmetrically. The proposed object is produced and decays with a cross section times branching ratio of 10^{-3} of the pion production cross section.

Direct production of leptons in hadronic collisions has now been observed in a wide variety of conditions. Early results[1-3] strongly suggested a remarkable parallelism between leptons and pions. A rigorous invariance of the lepton-to-pion ratio would clearly imply something fundamentally new, perhaps about pions. Although the data are uncertain to factors of 50% or so, the very large domain of observations is impressive. However, very recent data[4-7] appear to show clear variations of the lepton-to-pion ratio as a function of $P_\perp$ and $\sqrt{s}$.