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†Alfred P. Sloan Foundation Fellow.

[1]G. G. MacFarlane, T. P. McLean, J. E. Quarrington, and V. Roberts, Phys. Rev. 111, 1245 (1958).

[2]J. R. Haynes, M. Lax, and W. F. Flood, J. Phys. Chem. Solids 8, 392 (1959).

[3]P. J. Dean, Y. Yafet, and J. R. Haynes, Phys. Rev. 184, 837 (1969).

[4]K. L. Shaklee and R. E. Nahory, Phys. Rev. Lett. 24, 942 (1970).

[5]T. Nishino, M. Takeda, and Y. Hamakawa, Solid State Commun. 12, 1137 (1973).

[6]T. P. McLean and R. Loudon, J. Phys. Chem. Solids 13, 1 (1960).

[7]N. O. Lipari and A. Baldereschi, Phys. Rev. B 3, 2497 (1971).

[8]R. J. Elliott, Phys. Rev. 108, 1384 (1957).

[9]M. Lax and J. J. Hopfield, Phys. Rev. 124, 115 (1961).

[10]D. L. Smith and T. C. McGill, to be published.

[11]In deriving Eq. (3), we have neglected differences in the exciton dispersion relations for the two exciton states. See E. O. Kane, Phys. Rev. B 11, 3850 (1975).

[12]G. F. Koster, J. O. Dimmock, R. G. Wheeler, and H. Statz, *Properties of the Thirty-Two Point Groups* (M.I.T. Press, Cambridge, Mass., 1963), p. 13 ff.

[13]E. I. Blount, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1962), Vol. 13, p. 359 ff.

[14]J. R. Haynes, Phys. Rev. Lett. 17, 860 (1966).

[15]Ya. E. Pokrovsky, A. Kamisky, and K. Svistunova, in *Proceedings of the Tenth International Conference on the Physics of Semiconductors, Cambridge, Massachusetts, 1970*, edited by S. P. Keller, J. C. Hensel, and F. Stern, CONF-700801 (U. S. Atomic Energy Commission, Division of Technical Information, Springfield, Va., 1970), p. 504.

[16]R. B. Hammond, T. C. McGill, and J. W. Mayer, to be published.

COMMENTS

Fine-Structure Corrections and Electromagnetic Decays of Charmonium*

J. Pumplin, W. Repko, and A. Sato†

Department of Physics, Michigan State University, East Lansing, Michigan 48824

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We calculate fine-structure corrections to the spectrum of ψ/J particles in the charmonium model, assuming that the linear potential which confines the quarks is due to an effective vector exchange. We also calculate the electromagnetic decay rates between the various low-lying s and p states.

Recent observations of monoenergetic γ rays in colliding-beam experiments at DESY[1] and at Stanford Linear Accelerator Center[2] lend support to the charmonium model[3] of the narrow resonances $\psi(3.1)$ and $\psi(3.7)$. In particular, the prediction[4,5] of three p states, with masses around 3.5 GeV, and the associated γ-ray decay schemes, could form a basis for understanding the data. A measurement of the spacing between the $^3P_0$, $^3P_1$, and $^3P_2$ levels—assuming they are found—will yield information about the quark-antiquark ($c\bar{c}$) dynamics. Therefore, we have calculated these fine-structure splittings, under the assumption that the two-body system is essentially nonrelativistic[3] and that the binding results from an effective vector exchange which provides quark confinement.

Specifically, we assume an effective $c\bar{c}$ interaction in momentum space of the form

$$I=(\gamma_4\gamma_\mu)^{(1)}(\gamma_4\gamma_\mu)^{(2)}\bar{v}(k^2), \tag{1}$$

where k is the four-momentum transfer and $\bar{v}(k^2)$ is a confining potential. In the nonrelativistic limit, where $k_0 \to 0$, the Fourier transform of $\bar{v}(\vec{k}^2)$ is $v(r)$, the potential used in the two-body Schrödinger equation. To obtain the lowest-order relativistic corrections to $v(r)$, we expand Eq. (1) to order $(v/c)^2$ and express the result in nonrelativistic (two-component) form. We then transform to coordinate space

and obtain the effective interaction

$$V(\vec{r}) = v(r) + \frac{1}{4m_c^2}\left\{2p_l(v - rv')p_l + \tfrac{1}{2}\nabla^2(3v + rv') + 6\frac{v'}{r}\vec{L}\cdot\vec{S} + 2\frac{v'}{r}\vec{L}^2 + \left[(\nabla^2 v) - \frac{v'}{r}\right]\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)} + \left(\frac{v'}{r} - v''\right)\vec{\sigma}^{(1)}\cdot\hat{r}\vec{\sigma}^{(2)}\cdot\hat{r} + \left[\pm\frac{4\pi}{3}\delta(\vec{r})\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}\right]\right\}, \quad (2)$$

where the prime denotes differentiation with respect to r. The last term in Eq. (2) appears only when $v(r)$ contains a Coulomb term $\pm r^{-1}$. This term agrees with the known result for positronium[6] (without the annihilation potential). For a completely general $v(r)$, the first-order splittings between the three angular momentum states in the $l = 1$ multiplet have no simple relation to one another, since the coefficients of the spin-orbit and tensor terms in Eq. (2) involve different radial matrix elements. However, for potentials of the form $v(r) = r^n/a^{n+1}$ $(n \geq -1,\ n \neq 0)$ we obtain the sum rule

$$5(5-n)[M(^3P_2) - M(^3P_1)] = 2(13+n)[M(^3P_1) - M(^3P_0)]. \quad (3)$$

The $n = -1$ case of this sum rule was given in Ref. 5.

In order to make quantitative predictions for the energy levels, we assume a purely linear potential $v(r) = Ar$, in which case all correction terms in Eq. (2) behave like $1/r$. The complete Hamiltonian is $H = H_0 + K$, with

$$H_0 = \vec{p}^{\,2}/m_c + V(\vec{r}), \quad K = -(\vec{p}^{\,2})^2/4m_c^3. \quad (4)$$

Numerical calculations were performed by assuming a polynomial times exponential form for the radial wave functions, and determining the coefficients by the variational technique of minimizing the expectation value of H_0. This procedure converged well using 5–6 parameters. It is equivalent to numerically integrating the Schrödinger equation, but is technically easier. The relativistic kinetic energy correction K was included by first-order perturbation theory.

We assume $m_c = 1.5$ GeV and $A = 0.306$ (GeV)^2. These values reproduce the observed mass difference between the s states $\psi(3.1)$ and $\psi(3.7)$. The third parameter of the theory is an overall additive constant in the energy, which is allowed because of the infinite binding energy. This constant was chosen to reproduce the ground-state mass of 3.095 GeV. The masses of the other low-lying states are then predicted, as shown in Table I. The separations between these levels are on the order of 100 MeV, and it should therefore be possible to resolve them experimentally. The lowest-lying d state is nearby in mass to the first excited s state. The mixing can be calculated using Eq. (2), and is about 4%.

The widths for the decays $^3S_1 \to {}^3P_J$ are given by

$$\Gamma_{^3S_1 \to {}^3P_J} = 2(\tfrac{2}{3})^5 \alpha\omega A_0^2 (2J+1)\left(1 + 2\frac{\Delta M}{M_f}\right)^{-1/2}, \quad (5)$$

where ω is the photon energy, and the expression for the decays $^3P_J \to {}^3S_1$ is obtained by replacing $(2J+1)$ by 3. The radial integral in Eq. (5) is

$$A_0 = m_c^{-1}\int_0^\infty dr\, r^2 R_1(r) j_0(\tfrac{1}{2}\omega r) R_0'(r), \quad (6)$$

where $R_1(r)$ is the p-state radial wave function and $R_0'(r)$ is the derivative of the s-state wave function. The widths for the various decays are given in Table II. They are similar in magnitude to the spinless result.[5] However, we predict $\Gamma_{2^3S_1 \to {}^3P_J}$ to be largest for $J = 1$. Hence $^3P_1$ should be the easiest of the p states to isolate experimentally.

TABLE I. Masses of the low-lying levels.

Level	Mass (GeV)
$1\,^3S_1$	3.095
$^3P_0$	3.303
$^3P_1$	3.420
$^3P_2$	3.572
$2\,^3S_1$	3.695
$1\,^1S_0$	2.976
$2\,^1S_0$	3.603
$^3D_1$	3.677

TABLE II. Gamma-ray widths $\Gamma_{i \to f}$.

Initial state	Final state	Width (keV)
$2\,^3S_1$	$^3P_2$	88
$2\,^3S_1$	$^3P_1$	144
$2\,^3S_1$	$^3P_0$	76
$^3P_2$	$1\,^3S_1$	340
$^3P_1$	$1\,^3S_1$	274
$^3P_0$	$1\,^3S_1$	194

In conclusion, the charmonium model remains an attractive hypothesis for the newly discovered particles. A crucial test of the hypothesis will be to search for the three p states of Table I, by means of the electromagnetic decays of Table II. The fine-structure corrections which we have calculated indicate that the p states will be sufficiently different in mass to allow experimental separation of them.[7] The precise values of the masses will of course be different for different forms of the binding potential $v(r)$. Confirmation of the model discussed here would not point to any specific interpretation for the quantum numbers of the new quark. The SU(4)-charm picture, for example, which is consistent with the leptonic decays of $\psi/J(3.1)$ and $\psi(3.7)$,[8] would require the discovery of charmed *particles*.

Note added.—After submitting this paper for publication, we received a manuscript by H. J. Schnitzer,[9] in which the p-state splittings are calculated by a similar method. He finds splittings which are somewhat smaller than those shown in Table I, due to a different choice of parameters. Like ours, however, they are considerably larger than the previous estimate of De Rújula, Georgi, and Glashow.[7]

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†On leave from Department of Physics, Osaka University, Toyonaka, Osaka, Japan.

[1]W. Braunschweig *et al.*, Phys. Lett. 57B, 407 (1975).

[2]G. J. Feldman *et al.*, Phys. Rev. Lett. 35, 821 (1975).

[3]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975).

[4]T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, Phys. Rev. Lett. 34, 365 (1975).

[5]E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975).

[6]See, for example, A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, translated by G. M. Volkoff (Interscience, New York, 1965), Sect. 39.

[7]For other discussions of the p states see B. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 706 (1975); A. De Rújula, H. Georgi, and S. Glashow, Phys. Rev. D 12, 147 (1975).

[8]J. Pumplin and W. Repko, Phys. Rev. D 12, 1376 (1975).

[9]H. J. Schnitzer, following Letter [Phys. Rev. Lett. 35, 1540 (1975)].

p States of Charmonium and the Forces that Confine Quarks*

Howard J. Schnitzer
Department of Physics, Brandeis University, Waltham, Massachusetts 02154
(Received 18 August 1975)

A measurement of the energy differences of the $n=2$, ^3P_J states of charmonium may give significant information on the dynamics of the spin-dependent quark-antiquark ($q\bar{q}$) forces, since the more rapidly these forces increase with $q\bar{q}$ separation, the larger the ratios

$$R_1=[E(^3P_2)-E(^3P_1)]/[E(^3P_1)-E(^3P_0)] \text{ and } R_2=[E(^3P_1)-E(^1P_1)]/[E(^3P_1)-E(^3P_0)]$$

become if the forces are of vector character.

The interpretation[1] of the newly discovered boson resonances[2] as bound states (charmonium) of charmed quarks and antiquarks ($c\bar{c}$) receives considerable support from the recently reported[3] decays $\psi' \to X+\gamma$ and $\psi'' \to \psi+2\gamma$. In the context of the charmonium picture[1] these decays are expected to be $\psi' \to {}^3P_J+\gamma$ and ${}^3P_J \to \psi+\gamma$, since the assignment[1] of $\psi(3100)$, $\psi'(3700)$, and $\psi''(4200)$ as the lowest $^3S_1$ states of charmonium also requires P states in the energy interval between ψ and ψ'. These features are compatible with a number of linear potential models,[4] which strongly indicate that the low-lying levels of the ψ system are nonrelativistic and that the charmed quark mass is heavy, i.e., in the 1.5–2-GeV range. In addition to the obvious interest in the gross features of ψ spectroscopy, certain finer details may be particularly useful in revealing important features of quark dynamics. As such, it is the purpose of this paper to draw attention to the fact that careful measurements of the energy *differences* among these P states may serve to resolve the controversy between two opposing views as to the dynamical origin of the spin dependence in the bound states of quark-antiquark ($q\bar{q}$) pairs. This is important since the spin-dependent forces are inti-