monolayer thickness and from the evaporation rate assuming a unity sticking coefficient.

[16] J. D. Joannopoulos and M. L. Cohen, Phys. Rev. B 10, 5075 (1975).

[17] R. Dorn, H. Lüth, and G. J. Russell, Phys. Rev. B 10, 5049 (1974).

[18] A. U. MacRae and G. W. Gobeli, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic, New York, 1966), Vol. 2, p. 115.

[19] This behavior has been predicted by theoretical calculation: V. Heine, Phys. Rev. 138, A1689 (1965); and in much more detail by J. A. Appelbaum and D. R. Hamann, in *Physics of Semiconductors*, edited by M. Pilkuhn (Teubner, Stuttgart, Germany, 1974), p. 681. Although the qualitative concept of metal-induced surface states agrees with experiment, the predicted barrier height of the latter calculations is in error by a large amount, 0.6–0.7 eV. This is probably due to their assumed geometric model of substitutional replacement of Si atoms by Al atoms.

Constancy of Minimum Metallic Conductivity in Two Dimensions*

D. C. Licciardello and D. J. Thouless
Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT, England

(Received 30 June 1975)

A scaling argument is used to demonstrate the existence of a minimum metallic conductivity with a universal value for two-dimensional random lattices. We present a summary of the results of a detailed numerical experiment which supports this suggestion and indicates that the minimum metallic conductivity is $(0.12\pm0.03)e^2/\hbar$, in fair agreement with some experimental results for inversion layers.

A description of the electronic properties of disordered systems requires a firm basis for characterizing the pure homogeneous disordered state. This characterization has been hampered on the one hand by calculations which are not easily related to experimentally measurable quantities, and on the other hand by experiments in which sample preparation too often plays a determining role. This has meant that it is not easy to distinguish one theoretical system from another by the accuracy of its predictions, nor to tell which approximations provide helpful simplification and which produce misleading results.

Mott[1] argued some time ago for the existence of a minimum metallic conductivity in disordered systems, but his analysis has been subject to controversy.[2] In this note we suggest that for two-dimensional systems definite predictions can be made about this quantity, which has been measured, for example, in silicon inversion layers. In particular we present arguments not only for its existence, but to show that, for noninteracting electrons in a static two-dimensional random potential, the minimum metallic conductivity should have a universal numerical value, and we find that value to be about $0.12e^2/\hbar$. This should serve as a standard of comparison for theories which take into account further aspects of physics such as many-body effects.

Our analysis consists of two parts. We first construct a scaling argument, a rather crude version of those used in the theory of critical phenomena[3]; the steps in this argument were almost all in the paper of Edwards and Thouless,[4] but their significance was not understood. This scaling argument shows that the minimum metallic conductivity has a universal value independent of the details of the two-dimensional disordered system. This being accepted, it is possible to calculate its value using any convenient system, and the tight-binding model with random site energies seems the most suitable for calculations. We have numerically sampled small replicas of tight-binding systems, varying the size of the system to detect the positions of the mobility edges, and varying the lattice structure and degree of disorder to check the universality hypothesis. In this note we give only a brief summary of the results of these extensive calculations, which are reported in full elsewhere.[5] Before presenting our arguments we observe that the Ioffe-Regel condition,[6] the importance of which Mott[7] has repeatedly emphasized, leads to a two-dimensional minimum conductivity in fair agreement with what we obtain. According to simple kinetic theory the conductivity of a two-dimensional electron gas is given by

$$\sigma = 2ne^2\tau/m = (e^2/\hbar)k_F\lambda/2\pi,$$

where $\pi k_F^2/(2\pi)^2$ is the number of electrons n with a particular spin per unit area, τ is the relaxation time, λ the mean free path, and k_F the wave number of electrons on the Fermi surface. The Ioffe-Regel condition says that $k_F\lambda$ cannot be significantly less than unity, and if we take $k_F\lambda = 1$ as the minimum value we obtain a minimum conductivity of $0.16e^2/\hbar$. In three dimensions, however, the same argument leads to an extra factor of k_F, and so the conductivity depends on the length scale of the system.

We present the scaling argument for the tight-binding model on a square lattice. We start off with a set of atomic sites, each with a random energy ϵ_i uniformly distributed in the range $-\frac{1}{2}W$ to $\frac{1}{2}W$ coupled to its nearest neighbors by a constant matrix element $-V$. We consider a square of side L containing N of these sites, and consider the energy levels of this square with suitable boundary conditions. If we take the periodic continuation of this square, that is, the periodic system whose unit cell is this $L \times L$ square, then each level of the square will broaden into a band of width $2\Delta E$, where ΔE is the shift of energy produced by a change from periodic to antiperiodic boundary conditions along one cell edge. The next stage in the scaling argument involves the consideration of a square array of N of these $L \times L$ squares, with a different set of site energies in each. This is similar to the original tight-binding model, with each $L \times L$ square equivalent to a site. The coupling between levels in different squares is of order $V_N' = \frac{1}{4}\Delta E$ since this is the coupling between identical levels that would produce a band of width $2\Delta E$ in the case of periodic continuation. The spacing between energy levels on neighboring squares should be of order $L^{-2}(dn/dE)^{-1}$, where dn/dE is the density of states per unit area; it would be within the spirit of a scaling theory to ignore those levels which lie outside the range of a few times V_N' around a particular energy, and to identify this spacing as W_N', the energy spread in the rescaled problem. We ignore all other parameters that may be relevant and so we have replaced the tight-binding problem which was characterized by the ratio V/W by a rescaled problem in which the ratio V_N'/W_N' plays a similar role, namely the strength of the coupling between the closest energy levels on a pair of neighboring squares divided by a width characterizing their random mismatch. We continue this process by calculating V_{N^2}' and $W_{N^2}' = N^{-1}L^{-2}(dn/dE)^{-1}$ for a square containing N^2 atoms. The calculation can be repeated for N^3 atoms and so on, and at each stage the quantity V'/W' is evaluated. If the states of energy E are localized V'/W' will decrease with each iteration since the coupling V_N' will decrease as $\exp(-N^{1/2})$. Furthermore, since we suppose *a priori* the existence of a localization edge at E for some $(V/W)_c$ there should be a maximum value $(V'/W')_{\text{crit}}$ for which this exponential decrease occurs.

In previous work[4,8,9] the bandwidth for the periodically continued system has been related to $\hbar$ divided by the time it takes for an electron to diffuse across the unit cell. Specifically, from Eqs. (3.12) and (3.14) of Ref. 9, the curvature of a band generated from a particular level is statistically distributed with a distribution whose mean is zero and whose geometric mean is $2\hbar D(E)$, where $D(E)$ is the diffusion constant. If we assume, as is implicit in our analysis in the previous paragraph, that these bands are of the tight-binding form $2(\frac{1}{4}\Delta E_1)\cos(K_1 L) + 2(\frac{1}{4}\Delta E_2)\cos(K_2 L)$, where ΔE_1 and ΔE_2 each have geometric mean $\overline{\Delta E}$, then comparison of the curvature of the tight-binding band with the relation to the diffusion constant shows that $\overline{\Delta E}$ is equal to $4\hbar D(E)/L^2$. This formula should be valid when the states are extended and the mean free path is much less than L. Combining $V_N' = \frac{1}{4}\overline{\Delta E}$ with the relation between W_N' and the density of states dn/dE, one finds that V_N'/W_N' is simply $\hbar D(E)dn/dE$. The relation between the diffusion constant and the conductivity σ of a degenerate electron gas shows that V_N'/W_N' for large N is equal to $\sigma\hbar/2e^2$.

It is clear now from the argument that if V'/W' is less than its critical value the electrons must be localized, and this therefore implies the existence of a nonzero minimum conductivity given by $\sigma_m = 2(e^2/\hbar)(V_N'/W_N')_{\text{crit}}$. Furthermore when the electrons are extended and a diffusion constant exists V_N'/W_N' should scale to a constant value independent of N, and it is this constant value that is related to the conductivity.

In three dimensions V_N' should scale like $N^{-2/3}$ and W_N' like N^{-1}, so that V_N'/W_N' should *increase* as $N^{1/3}$ in the region of extended states. Thus, although we would also argue in this case that states will be localized if V_N'/W_N' is less than some critical value, the conductivity is given by $N^{-1/3}V_N'/W_N'$, and cannot be directly related to this critical value. The minimum metallic conductivity in three dimensions must, as is well known, depend on some length scale of the system.

As long as we assume that the only *relevant* parameter (in the critical-phenomena sense) is V_N'/W_N' there should be no dependence on either the

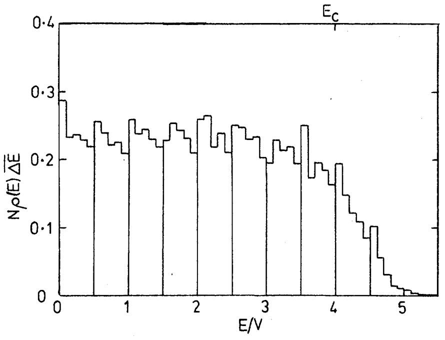

FIG. 1. Results of numerical calculations for square lattices with $W/V=5.0$. In each energy range of magnitude $0.5V$ the values of $N\rho(E)\overline{\Delta E}$ (which should be proportional to conductivity for extended states) are shown for $N=36$, 64, 100, 144, and 196.

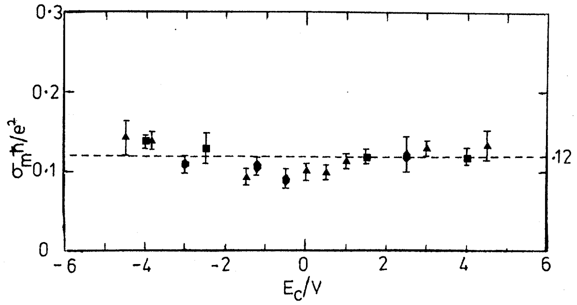

FIG. 2. The minimum metallic conductivity corresponding to each mobility edge E_c for the honeycomb, square, and triangular lattices (indicated by the appropriate symbol) for various values of W/V. The error bars indicate the spread in results as N is varied over the values 64, 100, 144, and 196.

lattice structure or the nature of the disorder since the fixed point for our scaling parameter in the region of localized states is zero. No reference is made to the nature of the lattice or the type of disorder when we go from V_N'/W_N' to V_{N^2}'/W_{N^2}'. This implies a common $(V'/W')_{\rm crit}$ for all two-dimensional systems which may be described by a random single-particle field. Since it is the rescaled V_N'/W_N' in the limit of large N that gives the conductivity, the minimum value does not depend on the nature of the system.

To test this conclusion and to establish the value of the minimum metallic conductivity we have made extensive calculations for machine-generated samples of tight-binding lattices with random site energies. These calculations are reported in detail elsewhere,[5] but because of their relevance to the scaling argument we give a brief account of them here. The method of calculation was similar to that used in Ref. 4, in that the energy shift ΔE produced by a change in boundary conditions was calculated for lattices with 36, 64, 100, 144, and 196 sites. For the smaller lattices 100 samples were used, and for the largest one at least 15 samples. The value of $N\overline{\Delta E}\rho(E)$ was found in each energy interval as a function of the number of sites N; $\overline{\Delta E}$ is the geometric mean of the energy shift in the interval and $\rho(E)$ is the density of states per site. We have already argued that this quantity should be related to the conductivity by

$$N\overline{\Delta E}\rho(E)=2\sigma\hbar/e^2$$

for extended states. Intervals outside the mobility edge were identified by the steady decrease of this quantity (which is essentially V_N'/W_N') with N, while in the region of extended states there is a more or less random variation with N. Figure 1 shows a typical graph of the results for a square lattice with $W/V=5$. In each energy interval the value of $N\overline{\Delta E}\rho(E)$ is shown for the five lattice sizes, with $N=36$ on the left-hand side and $N=196$ on the right-hand side of the interval. The two sides of the symmetrical band are lumped together, and the mobility edge can be deduced from the value of $N\overline{\Delta E}\rho(E)$ in this energy range. The calculations were performed for the honeycomb lattice ($Z=3$), the square lattice ($Z=4$), and the triangular lattice ($Z=6$) for several values of W/V. The results are shown in Fig. 2, with the conductivity at the edge plotted as a function of the position of the edge for the three types of lattice. Although there is some scatter in the results, we believe that they support the suggestion that the minimum metallic conductivity should have a constant value. From these results we deduce that the two-dimensional minimum metallic conductivity has the value $(0.12\pm0.03)e^2/\hbar$, or about 3.0×10^{-5} mho. This is a little higher than the value measured by Pepper, Pollitt, Adkins, and Oakley[10] for the minimum metallic conductivity in a metal-nitride-oxide-semiconductor field-effect transistor, but is not inconsistent with their results. It is quite incompatible with the results reported by Tsui and Allen[11] and their results may suggest the importance of many-body effects in such systems.

We would like to thank Dr. E. M. Forgan for his helpful comments on the manuscript.

---

*Work supported by the Science Research Council of Great Britain.

[1] N. F. Mott, Philos. Mag. 22, 7 (1970).

[2] M. H. Cohen and J. Jortner, Phys. Rev. Lett. 30, 699 (1973).

[3] K. G. Wilson and J. Kogut, Phys. Rep. 12C, 75 (1974).

[4] J. T. Edwards and D. J. Thouless, J. Phys. C: Solid State Phys. 5, 807 (1972).

[5] D. C. Licciardello and D. J. Thouless, to be published.

[6] A. F. Ioffe and A. R. Regel, Prog. Semicond. 4, 237 (1960).

[7] N. F. Mott, *Metal Insulator Transitions* (Taylor and Francis, London, England, 1974).

[8] J. T. Edwards, Ph.D. thesis, University of Birmingham, 1972 (unpublished).

[9] D. J. Thouless, Phys. Rep. 13, 93 (1974).

[10] M. Pepper, S. Pollitt, C. J. Adkins, and R. E. Oakley, Phys. Lett. 47A, 71 (1974); M. Pepper, S. Pollitt, and C. J. Adkins, Phys. Lett. 48A, 113 (1974), and J. Phys. C: Solid State Phys. 7, 1273 (1974).

[11] D. C. Tsui and S. J. Allen, Jr., Phys. Rev. Lett. 32, 1200 (1974), and 34, 1293 (1975).

COMMENTS

Excitation Functions of the Reaction ^{12}C + ^{13}C

E. Haindl, W. Galster, P. Dück, W. Treu, F. Siller, and H. Voit
Physikalisches Institut der Universität Erlangen-Nürnberg, Erlange, West-Germany

(Received 24 February 1975)

Excitation functions have been measured for the α-particle, proton, and elastic channels of the reaction ^{12}C + ^{13}C at several angles in the energy range $E_{\rm lab} = 16-28$ MeV. Observed structures cannot be due to molecular-type doorway states as is the case for the sub-Coulomb resonances of the reaction ^{12}C + ^{12}C.

Recently Crozier and Legg[1] have measured excitation functions for the α-particle and elastic channels of the reaction ^{12}C + ^{13}C at energies around and above the Coulomb barrier. Observed structures have been interpreted to be due to the existence of molecular-type doorway states as is the case in the reaction ^{12}C + ^{12}C at sub-Coulomb energies.[2] The signature of those doorway states is correlations among excitation functions measured for different reaction channels and different angles. In the reaction ^{12}C + ^{12}C these correlations have been seen very clearly.[2, 3] In the reaction ^{12}C + ^{13}C they are not very striking,[1] in particular since structures in the elastic channel often are due to nuclear and Coulomb interference effects at these low energies.

In order to see if distinct correlations do exist in the excitation functions of the reaction ^{12}C + ^{13}C we have measured excitation functions for the α-particle, proton, and elastic channels at several angles in the energy range $E_{\rm lab} = 16-28$ MeV. This energy range is just above the energy range for which smooth excitation functions have been found by Voit *et al.*[4] and by Halbert and Nagatani[5]; it includes those energies for which structures are reported in Ref. 1. The measurements have been performed at the Erlangen EN tandem accelerator. Alpha-particle and proton excitation functions have been measured simultaneously by use of a multidetector array. They are displayed in Fig. 1. We have summed over the first thirteen and the first 29 states in the respective residual nuclei. The excitation functions for the elastic channel have been measured simultaneously with the α-particle excitation function at $\theta_{\rm lab} = 13°$. The curves are shown in Fig. 2.

The α-particle curve measured at $\theta_{\rm lab} = 10°$ exhibits indeed pronounced structures (see Fig. 1). It agrees quite well with the α-particle function of Ref. 1. The remaining α-particle curves are, however, rather smooth. An increasing smoothness is very evident with increasing scattering angle. Among the proton curves only the one measured at the most backward angle shows some structure. The elastic curves in Fig. 2 are structureless at least in the energy range where structure is reported in Ref. 1.

The lack of any correlated structure in our data (the α-particle curves for $\theta_{\rm lab} = 10°$ and 13° are