served in other non-Ohmically-heated stellarator[9] and stellarator-type devices.[10] It is believed that these currents result from the imposition of the boundary condition that fluxes are conserved inside the separatrix and that it is not allowed to move. Such a boundary condition might be reasonable for those stellarators that have a conducting wall surrounding and close to the separatrix, which is the case in both the Uragon and Scyllac experiments. In Proto-Cleo, however, the helical windings are well inside the vacuum chamber, and thus the separatrix can be distorted more easily than in other devices.

As further evidence that MHD equilibrium exists, Fig. 4 also shows the time gradient of the potential energy, ∇E, as a function of the number of iterations. It is seen to increase monotonically toward zero from a negative value. Hence the potential energy itself approaches a minimum. It should be noted that analysis of the full torus must ultimately be done in order to examine the effects of the longer-wavelength modes, but it is not expected to produce significantly different results.

The computations for the $l=2$, 6 field-period configuration were similar to those for the $l=3$, 7 field-period configuration, i.e., stable equilibria corresponding to experimental measurements appear to exist.

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Rigid-Rotor Equilibria of Nonneutral Plasmas*

A. J. Theiss, R. A. Mahaffey, and A. W. Trivelpiece
University of Maryland, Department of Physics and Astronomy, College Park, Maryland 20742
(Received 14 August 1975)

A cold, constant-density, cylindrical, nonneutral plasma column confined by a uniform axial magnetic field has equilibria characterized by rigid-rotation frequencies given by $\dot{\theta}=\frac{1}{2}\omega_c[1\pm(1-2\omega_p^2/\omega_c^2)^{1/2}]$. Measurements of these fast (+) and slow (−) equilibria are compared with a cold-fluid-theory model of nonneutral plasmas.

A nonneutral plasma is a many-body collection of charged particles in which there is not overall charge neutrality.[1] The recent interest in the equilibrium, stability, and other properties of nonneutral plasmas has been stimulated by research in several areas such as collective-effect accelerators, intense relativistic electron beams, and highly stripped heavy-ion production. One of the simpler configurations to study nonneutral plasmas is that of a cold, uniform-density, cylindrical column of electrons confined by a uniform, axial, magnetic field. Such a nonneutral-plasma column has a rigid-rotation equilibrium frequency given by

$$\dot{\theta}=\tfrac{1}{2}\omega_c[1\pm(1-2\omega_p^2/\omega_c^2)^{1/2}], \qquad (1)$$

where ω_c $(=eB/m_0)$ is the cyclotron frequency, and ω_p $[=(ne^2/\epsilon_0 m_0)^{1/2}]$ is the plasma frequency. There are two rotation modes associated with Eq. (1), the fast mode (+) and the slow mode (−).

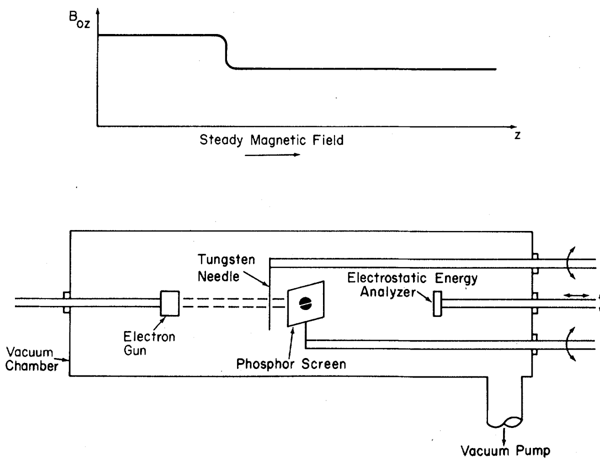

FIG. 1. Schematic of experiment used to investigate nonneutral-plasma equilibria.

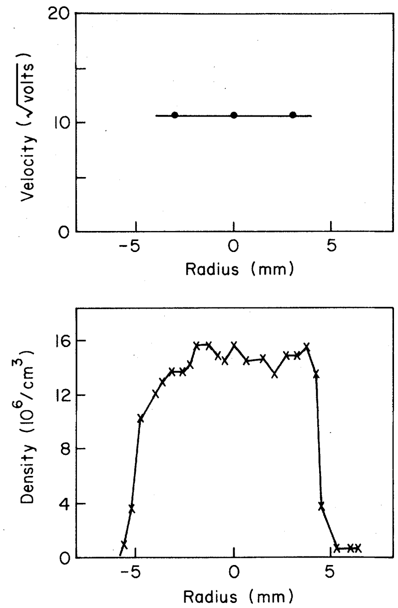

FIG. 2. Axial velocity and electron density as a function of radius for a nonneutral plasma with a normalized average rotation frequency of $\dot{\theta}/\omega_c = 0.024$. (Obtained from the electrostatic energy analyzer.)

For the case of a drifting electron beam, the high-density limit, $2\omega_p{}^2/\omega_c{}^2 = 1$, is known as Brillouin flow.[2]

This Letter reports on an investigation using an experiment that creates a nonneutral plasma that differs negligibly from these idealized equilibrium conditions. The main difference is that the nonneutral-plasma columns used in this experiment are created with a small surface ripple. The magnitude of the ripples associated with the experiments reported here is negligible and the behavior of a smooth column is duplicated closely enough to explore the features of the equilibrium.

Figure 1 is a schematic of this experiment. The steady magnetic field confines the electron cloud, and the step shown in the figure is used to create the desired equilibrium conditions. The electron gun injects a 1-cm-diam, 100–450-V, 1–10-mA, 30-μsec-pulse electron beam into the steady magnetic field (20–50 G) through the magnetic step that establishes the desired rigid-rotor frequency. Rotation in the slow mode is obtained when the magnetic field at the electron gun is in the same direction as that after the step (like that shown in Fig. 1). Rotation in the fast mode results when a reverse field step or cusp field is used. The final rotation velocity is the sum of the rotation velocity resulting from expansion of the electron column before the field step, and the rotation velocity resulting from the field step.[3] The base pressure in the system is maintained at less than 10^{-7} Torr so that there is not sufficient time for neutralization to take place during the 30-μsec beam pulse. The center, movable, electrostatic energy analyzer is used to measure the total current as well as the parallel energy distribution as a function of axial position and radius.

Figure 2 shows the axial velocity and electron density as a function of radius for a case where the averaged normalized beam rotation velocity was determined to be $\dot{\theta}/\omega_c = 0.024$. The needle and phosphor screen are used in combination to measure the average rotation of the nonneutral-plasma column. That is, the tungsten needle creates a shadow on the phosphor screen. As the screen is moved axially the shadow rotates. This average rotation is used to determine the mean angular velocity of the beam, i.e., $\langle\dot{\theta}\rangle = (\Delta\theta/\Delta z)\langle V_z\rangle$. Figure 3 shows this rotation of the nonneutral plasma column as a function of axial position for the same set of parameters, i.e., $\dot{\theta}/\omega_c = 0.024$. Data taken for several values of average rotation velocity were obtained using this method.

These results are displayed in Fig. 4 where the average rotation frequency and the average density for both fast- and slow-rotation-mode beams are shown in relation to the cold-fluid theoretical model [Eq. (1)]. As can be seen, the measured rotation velocities agree with those

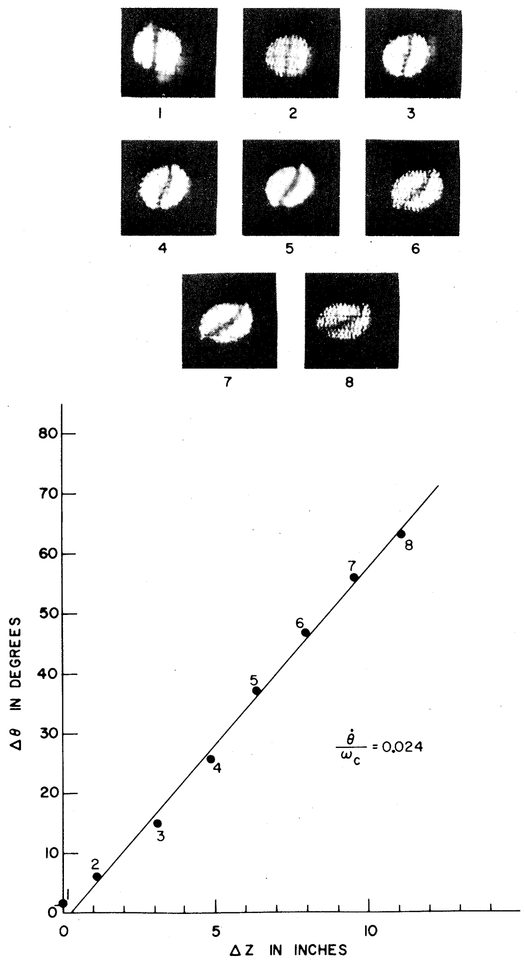

FIG. 3. Rotation of nonneutral plasma column as a function of axial position for the case $\dot{\theta}/\omega_c = 0.024$. Photographs of the phosphor screen showing the shadow of the tungsten needle are also shown.

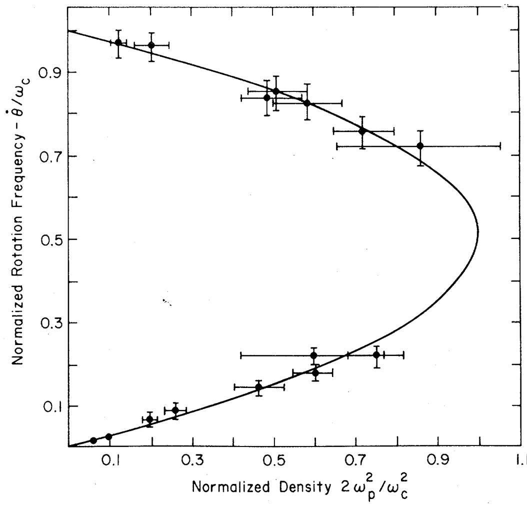

FIG. 4. Measured values of the normalized mean rotation frequency and the corresponding mean density are shown in relation to the theoretical rotation frequency as a function of density obtained using the cold-fluid nonneutral-plasma model.

predicted using cold-fluid theory. The rotation rates as obtained using the Vlasov-Maxwell theory of nonneutral plasmas show thermal corrections,[3] but for parameters used in the work reported here these corrections are negligible.

In conclusion, nonneutral-plasma equilibria are dynamic. The constant-density, nonneutral plasmas described here have a simple rigid-rotor equilibrium. The experiments reported here confirm some of their basic features. Additional experiments have verified many of the wave-propagation properties associated with these equilibria,[4,5] as well as the effect of rippling on the rotation velocity.[6]

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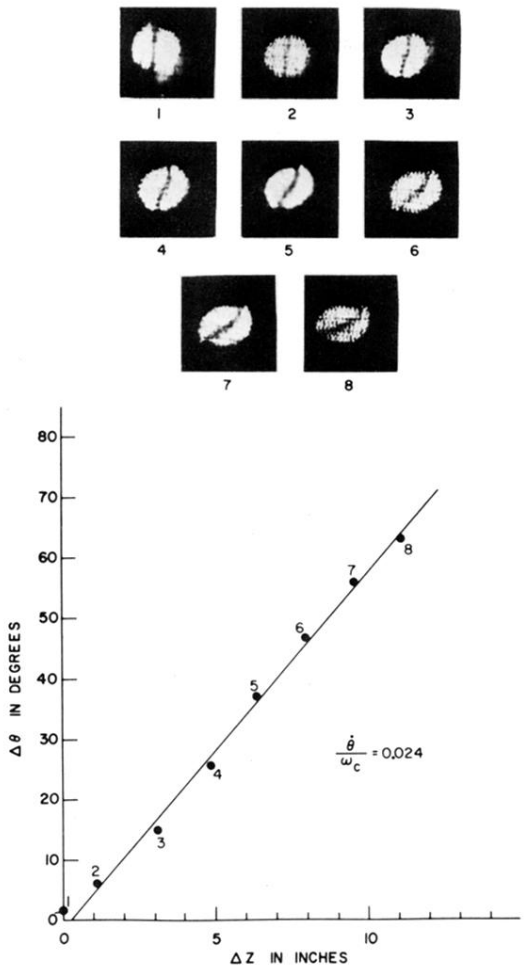

FIG. 3. Rotation of noneutral plasma column as a function of axial position for the case $\dot{\theta}/\omega_c = 0.024$. Photographs of the phosphor screen showing the shadow of the tungsten needle are also shown.