Pion and Nucleon Structure Functions near $x = 1$*

Glennys R. Farrar† and Darrell R. Jackson
California Insitute of Technology, Pasadena, California 91125
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In a colored-quark and vector-gluon model of hadrons we show that a quark carrying nearly all the momentum of a nucleon ($x \approx 1$) must have the same helicity as the nucleon; consequently $\nu W_2^n/\nu W_2^p \to \frac{3}{7}$ as $x \to 1$, not $\frac{2}{3}$ as might naively have been expected. Furthermore as $x \to 1$, $\nu W_2^\pi \sim (1-x)^2$ and $(\sigma_L/\sigma_T)^\pi \sim \mu^2 Q^{-2}(1-x)^{-2} + O(g^2)$; the resulting angular dependence for $e^+e^- \to h^\pm + X$ is consistent with present data and has a distinctive form which can be easily tested when better data are available.

There have been two significant paradoxes associated with the interpretation of electron-hadron scattering at large q^2 in terms of quarks: While the threshold dependence of $\nu W_2^p \sim (1-x)^3$, where $x = -q^2/(2q\cdot p)$, appears to reflect the underlying three-quark structure of the proton at short distances,[1] that same three-quark structure with the simple dynamics that controls short-distance processes would naively appear to lead to the prediction $\nu W_2^n/\nu W_2^p \to \frac{2}{3}$ as $x \to 1$, contrary to observation.[2] Furthermore, while for $Q^2 \lesssim 15$ GeV^2 the colored-quark model gives the correct value of the famous ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, that same model has been thought to predict a $(1+\cos^2\theta)$ angular distribution for hadrons having large $x = 2|p|/Q$. Instead, in that same Q^2 range the distribution is roughly isotropic.[3] We show here that the above paradoxes are products of the naive arguments: In fact, when the actual dynamics of the quark-gluon interaction are considered, the predictions are in good agreement with observations.

The problem of calculating properties of structure functions is tractable, we believe, for large Q^2 and $x \approx 1$ because in that kinematic configuration the quark which couples to the electromagnetic current necessarily has a very large invariant mass, even in the scaling limit.[4] A wave function in which one quark has very large invariant mass can be generated from the "normal" wave function (in which the invariant mass of each quark is limited) by an interaction of the sort shown in Fig. 1, where the incoming quark lines are understood to be convoluted with the normal wave function. Since each propagator marked with a cross has a large invariant mass $[p^2 \sim m^2/(1-x)$, where m^2 is some characteristic mass or $p_\perp$ scale for the quarks], it is reasonable to imagine that the effective quark-gluon couplings displayed in Fig. 1 are small.[5] Thus we can use lowest-order perturbation theory to go from the normal to "exceptional" (one quark having large p^2) wave functions. We assume that (a) the normal wave function is sufficiently damped at large p^2's that the convolution is dominated by the region in which the p^2's of the incoming quarks are finite, and (b) the spin and SU(3) structure of the normal wave function are what one would have in a nonrelativistic quark model. With these two assumptions,[6] the $x \to 1$ properties of hadron structure functions are given to $O(m^2/q^2)$ by lowest-order perturbation theory in which the incoming quarks can be treated as free (Fig. 1),[7] the convolution with the wave function having no effect other than fixing the overall normalization.

The results of direct calculation of nucleon diagrams in the limit of $Q^2 \to \infty$, $1-x$ fixed but very small, are that $\nu W_2^p \sim \kappa^4(1-x)^3$,[8] where $\kappa = g^2/4\pi$; also $(\sigma_L/\sigma_T)^p \sim m^2/Q^2 + O(\kappa)$.[9] Most interestingly, the quark which is struck by the virtual photon must, to leading order, have the same helicity as the nucleon itself. We verified

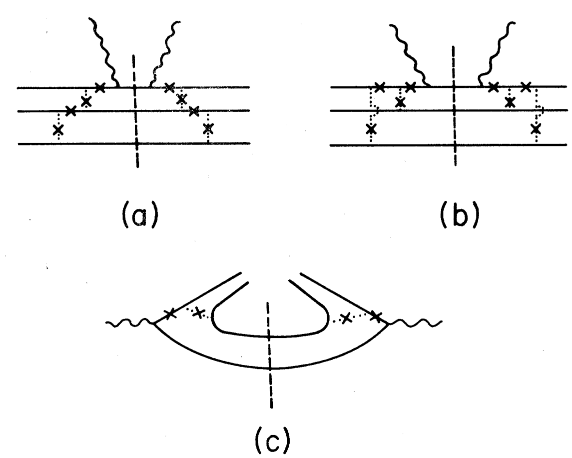

FIG. 1. Typical graphs for (a), (b) $eN \to e + X$, and (c) $e^+e^- \to \pi + X$.

this by direct computation but the physics can be understood by the following argument: Consider, for instance, the nucleon diagram shown in Fig. 1(a) and focus attention on the lower two incoming quarks. In the case where their spins are opposite they can exchange a transverse gluon and flip spins. In the case where their spins are aligned, angular momentum conservation implies that they can only exchange a longitudinal gluon. However, the coupling of a large-k^2 $[\sim m^2/(1-x)]$ longitudinal gluon to small-p^2 quarks, as on the bottom quark line, is suppressed by $(p^2/k^2)^{1/2} \sim (1-x)^{1/2}$ relative to the transverse coupling. Since chirality and angular momentum conservation introduce no additional suppression in the upper lines of the diagram, the helicities-aligned configuration of the quarks which give up their momentum gives a contribution to νW_2 and W_1 suppressed by $1-x$ relative to the antialigned configuration. The same phenomenon occurs in all other leading diagrams, such as in Fig. 1(b).

This result has two important measurable consequences. First, in polarized e-p scattering[10] at x near 1 the leading contribution comes only from the helicity configuration in which (in the proton-photon c.m. frame) the proton and photon have antialigned helicities. That is, $\sigma_{1/2}$ (total γ,p spin projection $\frac{1}{2}$) dominates $\sigma_{3/2}$ by at least a factor $1-x$ in the limit $x\to 1$. Second, it implies that $\nu W_2{}^n/\nu W_2{}^p = \frac{3}{7}$ for large x, as follows. The initial proton wave function which is perturbed by gluon exchange to give the wave function near $x=1$ has the isospin and helicity structure

$$2u(\uparrow)u(\downarrow)d(\uparrow) - u(\uparrow)d(\uparrow)u(\downarrow) + \text{permutations}.$$

Thus the probability that an up quark in a proton has the same helicity as the proton ($\uparrow$ in this case) is 5 times the probability that a down quark has the same helicity as the proton. Hence for x near 1, $\nu W_2{}^n/\nu W_2{}^p = \frac{3}{7}$. Since to leading order in Q^2, W_1 goes like νW_2 for nucleons, one has $\sigma^{\gamma n}/\sigma^{\gamma p} = \frac{3}{7}$, which is in agreement with the data (shown in Fig. 2 with only statistical errors) for x near 0.7.[11] For $x \gtrsim 0.7$–0.8, the deuteron smearing corrections are very large, making reliable determination of $\sigma^{\gamma n}/\sigma^{\gamma p}$ essentially impossible.[12]

For the pion, we find that $W_1 \sim \kappa^2(1-x)^2 + O[\kappa^2 m^2(1-x)/Q^2]$ and $\sigma_L/\sigma_T = \mu^2/Q^2 + O(\kappa)$.[8] This is consistent (as are our proton results) with the inclusive-exclusive connection between structure functions and elastic form factors[13]:

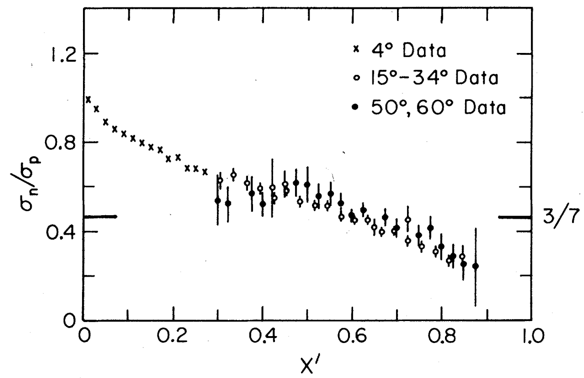

FIG. 2. $e^- N \to e^- + X$: σ_n/σ_p as a function of x', figure from Ref. 2.

$$\nu W_2{}^{T,L} \sim Q^2 |F^{T,L}(Q^2)|^2, \tag{1}$$

at fixed $W^2 \sim Q^2(1-x)$, where $\nu W_2{}^T = -[q^2/(q\cdot p)] \times W_1$, $\nu W_2{}^L = \nu W_2 + [q^2/(q\cdot p)]W_1$ are the contributions of transverse and longitudinal photons to νW_2. For the proton, $F^T \sim (Q^2)^{-2}$,[1] so Eq. (1) gives $\nu W_2{}^T \sim (1-x)^3$ at fixed W^2. For the pion, $F^L \sim (Q^2)^{-1}$[1] and the model gives $\nu W_2{}^L \sim \mu^2/Q^2$ ($F^T = 0$ for pions) so that Eq. (1) holds. Of course, $\nu W_2{}^L$ is not leading compared to νW_2; rather, $\nu W_2 = \nu W_2{}^T + \tau W_2{}^L = (1+\sigma_T/\sigma_L)\nu W_2{}^L \sim (1-x)^2 + \mu^2/Q^2$.

This behavior of σ_L/σ_T has the interesting consequence for $e^+e^- \to \pi + X$ that in addition to the $(1+\cos^2\theta)$ which should dominate the large-x angular distribution of pions at very large Q^2, there is a significant $\sin^2\theta$ component at lower values of Q^2:

$$\frac{d\sigma}{dx\, d\cos\theta} \sim (1+\cos^2\theta)\sigma_T + \sin^2\theta\,\sigma_L .$$

For comparison with experiment, we define

$$\alpha = \frac{\sigma_T - \sigma_L}{\sigma_T + \sigma_L} = \frac{Q^2(1-x)^2 - \mu^2}{Q^2(1-x)^2 + \mu^2} . \tag{2}$$

For $x \lesssim 0.6$–0.7 we do not expect Eq. (2) to be a good representation of α since most particles having $x \lesssim 0.6$ have probably come from the cascade of an object of larger x, and thus their correlation with the parent's direction has been diluted. At $x=0$, α should be zero; it should slowly increase as x increases; finally, for $x \gtrsim 0.6$–0.7, it should be given by Eq. (2): decreasing from its maximum, passing through zero at $x = 1-|\mu/Q|$, and going to -1 at $x=1$. The best available data[14] are at $\sqrt{Q^2} = 7.4$ GeV and are statistically inadequate to test this effect. We cannot predict μ^2, as it depends upon the details of the normal wave function. Taking for illustration $\mu^2=0.1$, we predict that at $\sqrt{Q^2}=7.4$ GeV, for $0.6<x<0.8$, $\alpha=0.96$, and for $0.8<x<1.0$, $\alpha=0.76$, which is consistent with the experimental values 1.0 ± 0.2 and 1.0 ± 0.25, respectively. The trend of the data, that α decreases for moderate x as Q^2 decreases, is consistent with our picture but provides no test because the statistical quality is so poor.[15] Integrating over angles gives $d\sigma/dx\sim(1-x)^2+\mu^2/2Q^2+O[(1-x)\mu^2/Q^2]$. Figure 3 shows that this behavior is consistent with the data for $\mu^2=0.1$; the constant term $\mu^2/2Q^2$ intersects the $(1-x)^2$ curve at about $x=0.97$ when $\sqrt{Q^2}=7.4$ GeV.

That $d\sigma/d\cos\theta\sim\sin^2\theta$ in the completely exclusive limit $e^+e^-\to\pi^+\pi^-$ is obvious. However our inclusive result suggests that other exclusive channels, $e^+e^-\to\pi\rho$, $e^+e^-\to\pi B$, etc., which *a priori* could have leading-order transition form factors through a transverse photon, do not. This is evident as follows. Consider the diagram of Fig. 1(c). The $q\bar{q}$ which make up the pion have opposite helicities, hence the same chiralities to order $1/Q^2$. Chirality is conserved by vector coupling, so that the other $q\bar{q}$ pair must have the same chirality and opposite helicities, hence $s_z=0$. Thus, unless they have $L\neq0$, angular momentum conservation requires that they cannot couple to a transverse photon, giving $F_{\pi\rho}{}^T(Q^2)\sim(Q^2)^{-2}$.[16] A direct calculation shows that $F_{\pi B}{}^T(Q^2)\sim(Q^2)^{-2}$ and similarly for transition form factors between other $L=0$ and $L>0$ systems.

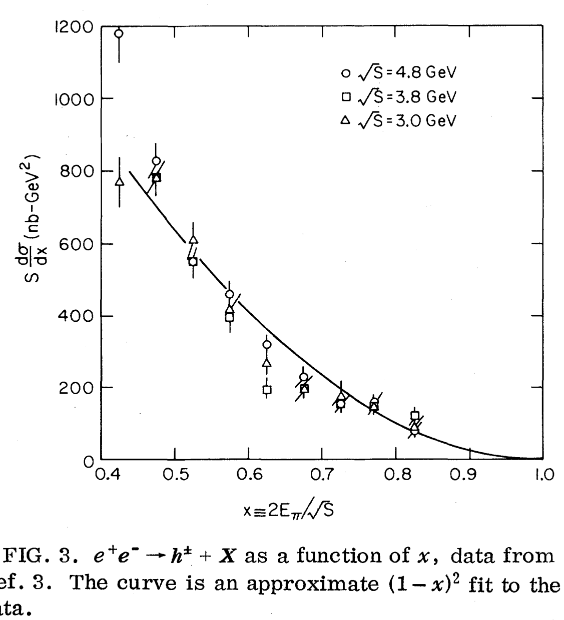

FIG. 3. $e^+e^-\to h^\pm+X$ as a function of x, data from Ref. 3. The curve is an approximate $(1-x)^2$ fit to the data.

It is interesting to note that these meson results also hold with scalar glue, since both q and $\bar{q}$ merely have their chirality reversed, but nonetheless have opposite helicities. On the other hand, the prediction that the quark having $x\approx1$ has the same helicity as the nucleon (so that $\nu W_2{}^n/\nu W_2{}^p\to\frac{3}{7}$ as $x\to1$) holds only in a theory with vector glue.

We have benefitted throughout this work from the provocative questions and perceptive observations of R. P. Feynman, who encouraged us to do the detailed calculations which show that a leading quark has the same helicity as the nucleon. Adam Schwimmer participated in an intermediate stage of this work, and provided valuable stimulation which we greatly appreciate.

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†Alfred P. Sloan Foundation Fellow.

[1]According to the "Drell-Yan-West relation" [S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970); G. B. West, Phys. Rev. Lett. 24, 1206 (1970); see also Eq. (1) above], $\nu W_2^p\sim(1-x)^3$ corresponds to $G_M{}^p\sim(Q^2)^{-2}$. However, S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973), and Phys. Rev. D 11, 1309 (1975), showed that $G_M{}^p\sim(Q^2)^{-2}$ reflects an underlying three-quark structure at short distances.

[2]See W. B. Atwood, thesis, Stanford University, SLAC Report No. SLAC-185, 1975 (unpublished).

[3]B. L. Richter, in *Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, 1974*, edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975).

[4]P. V. Landshoff, J. C. Polkinghorne, and R. D. Short, Nucl. Phys. B28, 225 (1971).

[5]For a discussion of this, and a more detailed description of our model and procedure, see G. R. Farrar, California Institute of Technology Report No. CALT-48-497 (to be published). See also Brodsky and Farrar, Ref. 1.

[6]We wish to thank P. V. Landshoff for a discussion of this point.

[7]The Abelian and non-Abelian cases differ only by an overall normalization since the only diagram which could be present for the non-Abelian but not Abelian case at this order (which involves a trilinear gluon coupling) does not contribute.

[8]The results $\nu W_2^p\sim(1-x)^3$ and $\nu W_2^\pi\sim(1-x)^2$ were obtained by Z. Ezawa, Nuovo Cimento 23A, 271 (1974), in a similar model, but differ from the results of C. Alabiso and G. Schierholz, SLAC Report No. SLAC-PUB-1588 (unpublished). Recent results [R. Taylor,

in Proceedings of the Conference on Lepton and Photon Interactions at High Energies, Stanford, California, 1975 (to be published)] show that if a new variable x_s is chosen, $x_s^{-1} \equiv x^{-1} + 1.5\ \mathrm{GeV}^2/Q^2$, then $W_1 \sim (1-x_s)^4$ for $0.2 \leq x_s \leq 0.85$. If experiment were to demonstrate conclusively that $\nu W_2 \not\sim (1-x)^3$, for x so near 1 and Q^2 so large that $\nu W_2 \sim W_1$, then this theory would be wrong. However, at present we are simply unable to test the theory with available data, since changing the scaling variable, and hence the treatment of nonscaling contributions, changes the apparent form of W_1, and even more νW_2, since x is not sufficiently close to 1 that $\nu W_2 \sim W_1$.

[9]Order κ corrections to σ_L/σ_T are expected to vanish near threshold as the first power of $1-x$; A. Zee, F. Wilczek, and S. B. Treiman, Phys. Rev. D 10, 2881 (1974).

[10]For a review of polarized e-p scattering see, e.g., F. J. Gilman, SLAC Report No. SLAC-167, 1973 (unpublished), p. 71.

[11]Our model certainly is not unique in giving $n/p \neq \frac{2}{3}$. For instance, see F. E. Close, Phys. Lett. 43B, 422 (1973), and R. Carlitz, University of Chicago Report No. EFI 75/6 (to be published). The interesting feature of the present work is that in a model with perfect SU(3) of flavor, which one would like to have if short distances are controlled by a color gauge interaction, we dynamically find $n/p \neq \frac{2}{3}$ in the $x \to 1$ limit.

[12]P. V. Landshoff points out that a consequence of $u/d = 5$ at $x \simeq 1$ would be that $\pi^+/\pi^- = 20$ for $z \simeq 1$ in the parton fragmentation region for $ep \to eX + \pi$.

[13]Drell and Yan, Ref. 1; West, Ref. 1; E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970); R. P. Feynman has emphasized the need for care in distinguishing between transverse and longitudinal contributions in the pion case. See also D. Scott, Nucl. Phys. B74, 524 (1974).

[14]R. F. Schwitters *et al.*, Phys. Rev. Lett. 35, 1320 (1975).

[15]R. Hollebeek, Lawrence Berkeley Laboratory Report No. LBL-3874 (unpublished), analyzes the SPEAR data of the Stanford Linear Accelerator Laboratory–Lawrence Berkeley Laboratory collaboration at $\sqrt{Q^2} = 3.0$, 3.8, and 4.8 GeV. It is tantalizing, but statistically insignificant, that for each Q^2, α in the largest x bin drops off.

[16]Since $F_{\pi\rho}{}^L \equiv 0$ as a result of the vanishing of the Clebsch-Gordan coefficient of $|1,0\rangle$ in $|1,0\rangle \otimes |1,0\rangle$ and $F_{\pi\rho}{}^T \sim (Q^2)^{-2}$ we should expect very little $e^+e^- \to \rho\pi$ to be seen except near the ω resonance (similarly for $e^+e^- \to \omega\pi$). This may also account for the absence of an important meson current contribution to the deuteron form factor: R. G. Arnold *et al.*, Phys. Rev. Lett. 35, 776 (1975).

Small-Angle Scattering of 7–14-MeV Neutrons by Pb and U

W. P. Bucher and C. E. Hollandsworth
U. S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland 21005
(Received 3 March 1975)

Absolute cross sections for neutrons elastically scattered through small angles by Pb and U have been accurately measured at various energies in the range 7–14 MeV. The results disagree with many previously reported measurements and especially their interpretations regarding anomalous scattering. However, optical-model predictions based on the energy-independent, nonlocal potential of Perey and Buck are, apart from normalization, in agreement with the present measurements for Pb.

Previous experimental studies[1-7] of the forward elastic scattering of fast neutrons from heavy nuclei have resulted in many reports of anomalously strong scattering at small angles, and such effects have been variously attributed to the fission process, an unexpectedly high value for the induced electric dipole moment of the neutron, and the possible existence of long-range nuclear forces. On the other hand, other investigators have reported that little, if any, anomalous behavior was indicated from their measurements but in the process of interpreting their data have also relied on a variety of different nuclear models to represent the specifically nuclear component of the scattering. These differences still are not clearly resolved. It has been suggested[6] that the discrepancies in the above results are only apparent and are solely due to differences among the nuclear models employed. However, it can be shown that the application of a more uniform model to these data would not resolve the conflict but rather widen it. It will be demonstrated that the primary difficulty with many previous measurements on Pb and U in the 7–15 MeV energy range lies not only in the nuclear models employed but more so in the data.

It is the purpose of this paper to present results which contribute to the resolution of the