TABLE I. Elasticities and fitted slopes.

| | 50 | 100 | 200 |
|---|---|---|---|
| | | $b$ at $-t=0.2$ (GeV/$c$)$^2$ | |
| $\pi^+p$ | 7.74 ± 0.10 | 8.16 ± 0.11 | 8.46 ± 0.15 |
| $\pi^-p$ | 7.92 ± 0.10 | 8.22 ± 0.11 | 8.46 ± 0.10 |
| $K^+p$ | 6.86 ± 0.45 | 7.38 ± 0.21 | 7.54 ± 0.40 |
| $K^-p$ | 7.26 ± 0.40 | 7.77 ± 0.20 | 7.91 ± 0.55 |
| $pp$ | 9.61 ± 0.20 | 10.06 ± 0.12 | 10.40 ± 0.15 |
| $\bar{p}p$ | | 11.13 ± 0.25 | |
| | | $\sigma_{el}/\sigma_{tot}$ | |
| $\pi^+p$ | 0.155 ± 0.012 | 0.149 ± 0.012 | 0.146 ± 0.012 |
| $\pi^-p$ | 0.156 ± 0.012 | 0.149 ± 0.012 | 0.145 ± 0.012 |
| $K^+p$ | 0.139 ± 0.012 | 0.134 ± 0.012 | 0.141 ± 0.012 |
| $K^-p$ | 0.147 ± 0.012 | 0.139 ± 0.012 | 0.142 ± 0.012 |
| $pp$ | 0.209 ± 0.015 | 0.197 ± 0.015 | 0.192 ± 0.015 |
| $\bar{p}p$ | 0.207 ± 0.015 | 0.196 ± 0.015 | 0.193 ± 0.015 |

at 100 GeV/$c$. However, comparison of our 100 GeV/$c$ data with the data of Antipov *et al.*[3] leads one to conclude that the $\bar{p}p$ slope continues to decrease slightly with energy.

The ratio of elastic cross section to total cross sections is shown in Table I. The errors are essentially all due to the absolute-normalization uncertainty. The ratios are remarkably similar for all the mesons but differ markedly from those for $pp$ and $\bar{p}p$ scattering.

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[1]R. Rubinstein *et al.*, Phys. Rev. Lett. 30, 1010 (1973).

[2]J. Pumplin and G. L. Kane, Phys. Rev. D 11, 1183 (1975).

[3]Y. M. Antipov *et al.*, Nucl. Phys. B57, 333 (1973).

# Chiral-Symmetry Breaking and the Ambiguity of Alternative Soft-Pion Approaches to Threshold $\pi N \to \pi\pi N$

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The alternative soft-pion approaches to threshold $\pi N \to \pi\pi N$ provided by the phenomenological-Lagrangian and current-commutator theories are shown to differ only in their pion-pole contributions. Since the data for the reaction $\pi^-p \to \pi^-\pi^+n$ seem well fitted by the former theory for $\xi=0$, and by the latter for $\xi=-\frac{1}{2}$, where $\xi$ is the chiral-symmetry-breaking parameter introduced by Olsson and Turner, threshold total-cross-section data for this charge state will not be able to determine a unique $\xi$ value.

It is well known[1-3] that chiral symmetry alone is insufficient to fix the $\pi$-$\pi$ scattering lengths $a_0$ and $a_2$ without some further assumption as to how the symmetry is broken. Of all those reactions[3] which could serve to discriminate experimentally among the various chiral-symmetry-breaking models[2-4] that have been suggested, threshold $\pi N \to \pi\pi N$ has long been most favored.[3,5-7] In addressing themselves to the subject of this predictive ambiguity and its possible removal through the study of single-pion production, Olsson and Turner,[5] working in the framework of the phenomenological Lagrangian,[8] were able to show that the most general Lagrangian derived in accordance with current algebra and the hypothesis of partially conserved axial-vector current introduces a single symmetry-breaking parameter $\xi$ into the $\pi$-$\pi$ scattering lengths which can then be determined only through additional assumptions. Moreover, $\xi$ is the *only* parameter[5] which enters the threshold one-pion production amplitude with external pions on the mass shell. In their[5] comparison with the best data then available near threshold[9] for the reaction $\pi^-p \to \pi^-\pi^+n$, Olsson and Turner[5] find that the data seem to converge to the predicted threshold curve specified by $\xi=0$ (Weinberg's model[1]) or 4.5, with the latter value apparently ruled out after comparison with *nonthreshold* data for the other charge-state reactions, $\pi^+p \to \pi^+\pi^+n$ and $\pi^-p \to \pi^0\pi^0n$. These results for $\xi = 0$ were observed to agree with those obtained by Chang,[7] who used Weinberg's current-commutator theory of multiple-pion production.[10] Possibly because of Chang's overdetailed presentation, however, Olsson and Turner did not attempt to explore the points of difference between his current-algebra approach and theirs, especially in the case of $\xi \neq 0$, which Chang did not treat.

The advent of meson factories has made threshold-pion-production experiments practicable,[8] and in fact, such an experiment,[11] in the case of the charge state $\pi^-p \to \pi^-\pi^+n$, is presently in progress at the Clinton P. Anderson Meson Physics Facility (LAMPF) with total-cross-section measurements to be pushed below pion laboratory kinetic energy $T_\pi = 250$ MeV when the beam intensity reaches 100 $\mu$A. This significant improvement in the experimental outlook has sparked the present reexamination of the relationship of these alternative current-algebra approaches to threshold pion production. As I show in the following, the two theories of threshold production, the phenomenological-Lagrangian theory and the current-commutator theory, with the latter theory now generalized for arbitrary $\xi$, can now make rather different predictions of the threshold cross sections for the various charge states. The apparent agreement of the latter theory[7] with the former[5] for the "magic" value $\xi = 0$ noted earlier by Olsson and Turner[5] must be accounted as spurious.[12]

It is generally accepted[7,13] that the $N^*$ contributions to pion production may be neglected in the energy region near threshold,[14] so that the threshold world consists of only pions and nucleons. Moreover, both current-algebra theories, the phenomenological-Lagrangian theory and the current-commutator theory, may be analyzed in terms of three-point (nucleon-pole terms with three soft-pion–nucleon vertices), two-point (nucleon-pole terms with two soft-pion–nucleon vertices), and one-point (pion-pole and three-pion contact terms)[15] tree graphs.[16] One might then be puzzled on referring to the effective Lagrangian[12,17] relevant to single-pion production,

$$\mathfrak{L} = \sum_{j=1}^{3} \mathfrak{L}_{\pi N}{}^{(j)} + \mathfrak{L}_{\pi\pi}, \tag{1}$$

$$\mathfrak{L}_{\pi N}{}^{(1)} = (g/2M)\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\varphi}, \tag{2a}$$

$$\mathfrak{L}_{\pi N}{}^{(2)} = -(g/2M)^2(g_V/g_A)^2\bar{\psi}\gamma_\mu\vec{\tau}\psi\cdot\vec{\varphi}\times\partial^\mu\vec{\varphi}, \tag{2b}$$

$$\mathfrak{L}_{\pi N}{}^{(3)} = -(g/2M)^3(g_V/g_A)^2\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\varphi}\varphi^2, \tag{2c}$$

$$\mathfrak{L}_{\pi\pi} = (g/2M)^2(g_V/g_A)^2[-\varphi^2(\partial^\mu\varphi)^2 + \tfrac{1}{2}(1-\tfrac{1}{2}\xi)m_\pi{}^2(\varphi^2)^2], \tag{2d}$$

by the remark of Olsson and Turner[5] that they "calculate the contributions from all diagrams to two charge-state amplitudes" since their results go as $(g/2M)^3(g_V/g_A)^2$. However, it is straightforward to calculate the "apparently" omitted threshold contribution from three-point graphs [which must go as $(g/2M)^3$]. These terms, with[18]

$$\langle N(p_f)\pi^\alpha(q_1)\pi^\beta(q_2)\pi^\gamma(q_3)|T|N(p_i)\rangle_{(3\text{-pt.})} \propto \sum_{\text{perm}(\alpha 1,\,\beta 2,\,\gamma 3)} \bar{u}(p_f)\tau^\alpha\tau^\beta\tau^\gamma\left(1-\frac{2M\not{q}_1}{2p_f\cdot q_1+q_1{}^2}\right)\gamma_5\not{q}_2\left(1-\frac{2M\not{q}_3}{2p_i\cdot q_3-q_3{}^2}\right)u(p_i), \tag{3}$$

make contributions to the various charge states of order $m_\pi{}^2/M^2$, so that the results of Ref. 5 are correct to order $m_\pi/M$. Precisely the same three-point terms, e.g.,

$$(g/M)^3(g_V/g_A)^3\int d^4x\, d^4y\, d^4z\, \exp(iq_1\cdot x+iq_2\cdot y+iq_3\cdot z)iq_1{}^\mu q_2{}^\nu q_3{}^\lambda\langle N(p_f)|T[A_\mu{}^\alpha(x),A_\nu{}^\beta(y),A_\lambda{}^\gamma(z)]|N(p_i)\rangle,$$

occur in the Bose-symmetric reduction[7] of the $T$ matrix in the current-commutator approach.[7] For the two-point contributions, we again find identical results to order $m_\pi/M$ at threshold.[19] [Note that Chang[7] includes the magnetic contribution of

$$\langle N(p')|V_\mu{}^\alpha(0)|N(p)\rangle = \bar{u}'\tau^\alpha[F_1(t)\gamma_\mu + iF_2(t)\sigma_{\mu\nu}q^\nu/2M]u,$$

in his calculation, while Olsson and Turner,[5] as is customary in effective-Lagrangian calculations,[8] omit this. However, such terms can be shown to contribute only to order $m_\pi{}^2/M^2$.] In the case of the one-point contributions, one finds the contact term, which is $O(m_\pi/M)$, identical in both approaches because of the explicit Bose symmetrization of the three external pions there [note the *absence* of a mediating pion in this term]; however, one does *not* expect the pion-pole term to be the same in both approaches, since only the *external* pions participate in the Bose symmetrization in the current-commutator theory, while that symmetry of the effective $\pi$-$\pi$ Lagrangian includes the virtual exchanged pion as well.[20] Specifically, in the phenomenological-Lagrangian (PL) theory, the one-point contribution is

$$T^{\mathrm{PL(1-pt.)}}(N(p_i)\to N(p_f)+\pi^\alpha(q_1)+\pi^\beta(q_2)+\pi^\gamma(q_3))$$
$$=-i(2\pi)^4\delta(Q-p_i+p_f)2\left(\frac{g}{2M}\right)^3\left(\frac{g_V}{g_A}\right)^2\bar{u}(p_f)$$
$$\times\left[\tau_\alpha\gamma_5\left(\not{q}_1-\frac{4M}{Q^2-m_\pi^2}[Q\cdot q_1-q_2\cdot q_3-m_\pi^2(1-\tfrac{1}{2}\xi)]\right)\delta_{\gamma\beta}+\text{cyclic-perm}(\alpha1,\beta2,\gamma3)\right]u(p_i), \quad (4)$$

while, since to first order in $\varphi$ one has

$$[Q_5^\gamma,[Q_5^\alpha,\partial\cdot A^\beta]]=\delta^{\alpha\beta}f_\pi m_\pi^2\varphi^\gamma(1-\tfrac{1}{2}\xi)-(\delta^{\alpha\gamma}\varphi^\beta+\delta^{\beta\gamma}\varphi^\alpha)f_\pi m_\pi^2\xi/2, \quad (5)$$

the $\xi=0$ current-commutator (CC) calculation of Chang[7,12] generalizes for arbitrary $\xi$ to the analogous, but different, expression

$$T^{\mathrm{CC(1-pt.)}}(N(p_i)\to N(p_f)+\pi^\alpha(q_1)+\pi^\beta(q_2)+\pi^\gamma(q_3))$$
$$=-i(2\pi)^4\delta(Q-p_i+p_f)2\left(\frac{g}{2M}\right)^3\left(\frac{g_V}{g_A}\right)^2\bar{u}(p_f)$$
$$\times\left[\tau_\alpha\gamma_5\left(\not{q}_1-\frac{2M}{Q^2-m_\pi^2}[Q\cdot q_1+m_\pi^2(1-2\xi)]\right)\delta_{\gamma\beta}+\text{cyclic-perm}(\alpha1,\beta2,\gamma3)\right]u(p_i). \quad (6)$$

In Fig. 1 I plot the threshold cross-section curves for the two theories calculated according to the prescription of Ref. 5 against data near threshold for the reaction $\pi^-p\to\pi^-\pi^+n$. Note that while data seem well fitted by the phenomenological-Lagrangian theory for $\xi=0$, an equally viable fit is obtained in the current-commutator theory, but now for $\xi=-\frac{1}{2}$. Thus, *threshold total-cross-section data for this charge will not be able to determine a unique* $\xi$ *value.* The following summarizes the present results for the charge states of interest in the two approaches[6,21]:

$$\sigma(\pi^-p\to\pi^-\pi^+n)=|a(-+n)|^2k^2\times(\text{phase space}),$$
$$\sigma(\pi^+p\to\pi^+\pi^+n)=|a(++n)|^2\tfrac{1}{2}k^2\times(\text{phase space}), \quad (7)$$
$$\sigma(\pi^-p\to\pi^0\pi^0n)=|a(00n)|^2\tfrac{1}{2}k^2\times(\text{phase space}),$$

with (I take[6] $f_\pi=82$ MeV)

$$a_{\mathrm{PL}}(-+n)=-1.36+0.6\xi,$$
$$a_{\mathrm{CC}}(-+n)=0.69-1.2\xi,$$
$$a_{\mathrm{PL}}(++n)=1.51+0.6\xi,$$
$$a_{\mathrm{CC}}(++n)=1.02-1.2\xi, \quad (8)$$
$$a_{\mathrm{PL}}(00n)=2.11-0.3\xi,$$
$$a_{\mathrm{CC}}(00n)=-0.18+0.6\xi.$$

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[1]S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).
[2]P. Chang and F. Gürsey, Phys. Rev. 164, 1752

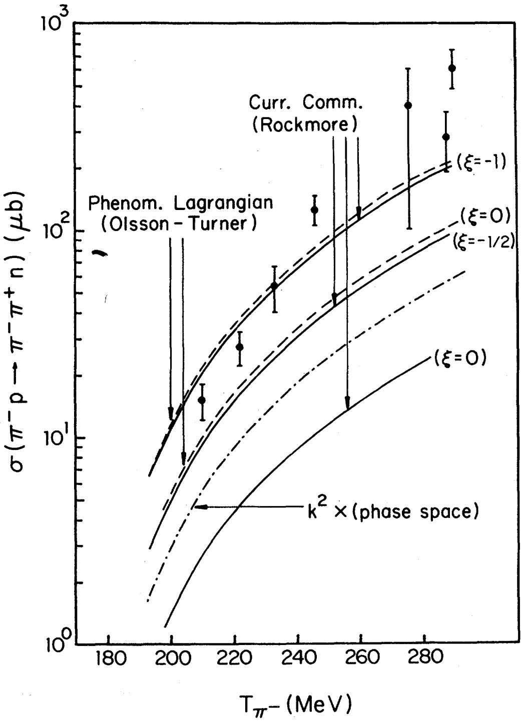


FIG. 1. Scatter of data for the reaction $\pi^- p \to \pi^-\pi^+ n$ near threshold (cf. Ref. 9). The curves drawn represent predictions for different values of the symmetry-breaking parameter $\xi$ in the two current-algebra approaches. Above threshold the cross-section prediction is made by multiplying the square of the threshold amplitude by the physical phase space (also plotted). The data appear equally well fitted by the curves labeled $\xi = 0$ in the phenomenological-Lagrangian theory and $\xi = -\frac{1}{2}$ in the current-commutator theory.

(1967).

[3]S. Weinberg, Phys. Rev. 166, 1568 (1968).

[4]J. Schwinger, Phys. Lett. 24B, 473 (1967).

[5]M. G. Olsson and L. Turner, Phys. Rev. Lett. 20, 1127 (1968).

[6]M. G. Olsson and L. Turner, Phys. Rev. 181, 2141 (1969).

[7]L. N. Chang, Phys. Rev. 162, 1497 (1967).

[8]S. Weinberg, Phys. Rev. Lett. 18, 188 (1967); R. Dashen and M. Weinstein, Phys. Rev. 183, 126 (1969).

[9]Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, Yad. Fiz. 1, 526 (1965) [Sov. J. Nucl. Phys. 1, 374 (1965)]; M. G. Olsson and G. B. Yodh, Phys. Rev. 145, 1309 (1966).

[10]S. Weinberg, Phys. Rev. Lett. 16, 879 (1966).

[11]I refer to Experiment No. 99 which is currently involved in total-cross-section measurements of $\pi^- p \to \pi^-\pi^+ n$ at $T_\pi = 300$ and 350 MeV (P.A.M. Gram, private communication).

[12]R. Rockmore, Phys. Rev. C 11, 1953 (1975). I have already remarked there on the paradoxically poor fit to threshold behavior provided by the local approximant (pion-pole plus contact term) derived from Chang's current-commutator calculation as compared to that derived from the phenomenological-Lagrangian theory in the Weinberg model ($\xi = 0$).

[13]W. F. Long and J. S. Kovacs, Phys. Rev. D 1, 1333 (1970).

[14]Unfortunately I can find no precise numerical estimate of the resonance contribution in this energy region in Ref. 7. However, there is an explicit, confirmatory calculation of the contribution to $\sigma_{\text{threshold}}(\pi^- p \to \pi^-\pi^+ n)$ associated with the $N^{*+}$ tail given in Ref. 12.

[15]The pion-pole and three-pion contact terms are shown, for example, in Fig. 1 of Ref. 5.

[16]*All* the tree graphs which contribute to the process $\pi N \to \pi\pi N$ at threshold are shown in Fig. 1 of Ref. 13. Figure 1(e), *loc. cit.*, is an example of a three-point graph, Figs. 1(c) and 1(d) are examples of two-point graphs, and Figs. 1(a) and 1(b) are the one-point graphs consisting of pion-pole and three-pion contact terms.

[17]I use the somewhat simpler effective Lagrangian of Ref. 12 here.

[18]For convenience I "cross" the incident pion $\pi^\alpha(q_1)$ here so that for calculational purposes, one has $q_1{}^\mu = (-\vec{k}, -\omega_k)$ in the center-of-mass frame.

[19]Note the neglect of two-point terms involving the scalar density in the current-commutator calculation of Ref. 7.

[20]Cf. the discussion in Appendix B of Ref. 7.

[21]One sees that this matching of the two theories persists for the charge state $\pi^+ p \to \pi^+\pi^+ n$, while for $\pi^- p \to \pi^0\pi^0 n$ one finds $\sigma_{\rm CC}(\xi = -\frac{1}{2})/\sigma_{\rm PL}(\xi = 0) \lesssim \frac{1}{16}$. Note that in the CC theory, the value $\xi \simeq 1.8$, which is close to the $\xi$ value for the "minimum-coupling model," also provides a viable fit to the $(-+n)$ data; moreover the $(++n)$ and $(00n)$ predictions in this case are down from those of the PL theory ($\xi = 0$) by factors of 2 and 4, respectively, so that they appear in better accord with the experimental data in the above-threshold region at $T_\pi = 357$ and 400 MeV.