Limit on the Photon Mass Deduced from Pioneer-10 Observations of Jupiter's Magnetic Field*

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We report an analysis of the Pioneer-10 data on Jupiter's magnetic field, in which the mass μ of the photon was treated as a free parameter. We set a limit of $\mu \leq 2 \times 10^{-11}$ cm^{-1} $\equiv 6 \times 10^{-16}$ eV $\equiv 8 \times 10^{-49}$ g. This is the smallest limit so far obtained from direct measurements. We compare our result with other published limits.

The Jupiter magnetic-field data[1-3] obtained on the recent Pioneer 10 mission[4] have been reanalyzed by Davis[5] to include the possibility of a massive photon. We report here the conclusion of this analysis, that a new limit on the photon mass can be set,

$$\mu \leq 2\times10^{-11}\ \mathrm{cm}^{-1} \equiv 6\times10^{-16}\ \mathrm{eV} \equiv 8\times10^{-49}\ \mathrm{g}. \tag{1}$$

We first describe how the data were obtained and analyzed in the Maxwell-equation ($\mu = 0$) case.[1-3]

The standard least-squares procedure was used to determine the best-fit coefficients in a spherical-harmonic expansion for the Pioneer-10 observations of Jupiter's magnetic field. The fit was made to 134 vector-field values, each an average of all data taken in 10 min, usually an average of 3200 observations. All data available while the spacecraft was nearer than 13.1 Jupiter radii ($R_J = 71\,372$ km) and farther than $2.84R_J$ (periapsis) were used except for six averages whose residuals were very much larger than normal. The data taken during occultation were not available.

Fits to the data were made with formulas containing three coefficients (for an internal dipole source), eight coefficients (internal dipole plus quadrupole sources), eleven coefficients (internal dipole and quadrupole plus "external dipole," i.e., a uniform field), and sixteen coefficients (the previous eleven plus five "external" quadrupole terms). At each point, the residual is defined as the square of the difference of the vector field observed and the vector field computed using the best values of the coefficients that were determined. The root mean residual (RMR) is the square root of the mean of the residuals over the 134 vector-field values. By the "best" values of the coefficients is meant the set that gives the smallest RMR.

The fits gave reasonable values of the coefficients and satisfactorily small values of the RMR as shown in the first four columns, $\mu = 0$ row, of Table I. The highest value of the field strength was 18 380 gamma (1 gamma $= 10^{-5}$ G), the smallest value was 147 gamma, and half of the averages were larger than 725 gamma. Inspection of the vector residuals shows that they are not mainly due to Gaussian fluctuations in the data, but rather to systematic effects that are not well described by the spherical harmonics used.

Other variable parameters may be added to the formulas that connect the position of the spacecraft and the three components of the magnetic field predicted by the expansion in spherical harmonics. These other parameters may be varied to reduce further the RMR. The essential question is, what significance should be given to the values of the parameters determined in this way?

On the basis of the analysis[3] of other variables, such as the field rotation rate, one can conclude that there are a variety of other systematic effects which if properly included in the analysis could lower the RMR for the sixteen-coefficient case to perhaps as low as 5 to 15 gamma. When one has sixteen spherical-harmonic coefficients plus any other single parameter to adjust, it would be surprising if some part of an ignored systematic effect could not be mimicked, with a

TABLE I. Values of RMR in gamma for various values of μ and various trajectory segments. The underlined RMR are minima.

Column number		1	2	3	4	5	6	7	8
Start of data (inbound)		$13.09R_J$	$13.09R_J$	$13.09R_J$	$13.09R_J$	$8.97R_J$	$8.97R_J$	$11.37R_J$	$11.37R_J$
End of data (outbound)		$13.06R_J$	$13.06R_J$	$13.06R_J$	$13.06R_J$	$8.93R_J$	$8.93R_J$	$2.93R_J$	2.93R
No. of S.H. Coefs.		3	8	11	16	11	16	11	16
μ in units of 10^{-11}cm^{-1}	R_J^{-1}								
-0.42	-.03	164.24	97.60	45.75	33.79				
-0.14	-.01	149.42	82.62	43.66	$\underline{33.68}$				
-0.04	-.003	148.23	81.31	$\underline{43.51}$	33.78				
0	0	$\underline{148.12}$	$\underline{81.19}$	$\underline{43.51}$	33.79	35.77	34.94	$\underline{39.59}$	$\underline{23.43}$
+0.03	+.002	148.16	81.24	$\underline{43.51}$	33.78				
0.07	.005	148.39	81.49	43.54	33.75				
0.14	.01	149.17	82.32	43.63	33.65				
0.28	.02	151.99	85.26	44.00	33.28	35.18	34.23	40.16	23.47
0.35	.025	153.97	87.25	44.30	33.04	34.92	33.88	40.45	23.49
0.42	.03	156.28	89.52	44.72	32.81	34.68	33.51	40.79	23.50
0.49	.035	158.90	92.03	45.28	32.62	34.49	33.14	41.19	23.52
0.56	.04	161.80	94.76	46.00	$\underline{32.54}$	$\underline{34.42}$	32.84	41.63	23.53
0.63	.045	164.96	97.69	46.92	32.61	34.52	32.62	42.11	23.54
0.70	.05	168.35	100.80	48.07	32.91	34.84	$\underline{32.57}$	42.65	23.56
0.84	.06	175.72	107.52	51.13	34.44	36.34	33.13	43.85	23.60
0.98	.07	183.72	114.87	55.35	37.58	39.25	34.95	45.26	23.64
1.40	.1	209.94	140.77	75.24	57.81				

reduction of RMR by several gamma.

There are a variety of plausible sources of systematic effects. Higher-order spherical harmonics could be present. When fields of the order of 10 000 gamma are being measured, it is unlikely that the calibrations are more accurate than 25 gamma, and this could produce systematic effects. There could be genuine time variations in the field near the planet, since such changes were observed far out. There could be small errors in the trajectory values used and the magnetometer data taken from a preliminary tape. There could be currents in the plasma in the region between $(2.84 \text{ and } 13.1)R_J$; this would require modifications in the spherical-harmonic analysis. On the basis of observations at larger distances, one can be virtually certain that there are such currents, but whether or not they are large enough to explain the RMR of Table I is not evident.

The kind of systematic effect that we are concerned with would occur if the photon had a nonzero rest mass. Then Maxwell's equations would be replaced by the Proca equations.[6] The most appreciable effect for a small, finite photon mass is that by a known theorem[7,8] the fractional change in the average field will be of order $(\mu D)^2$, where D is the dimension of the system under study. (This is the advantage of using Jupiter. Since the size of the system is so large, a given field variation can measure a small μ.) In particular, it is straightforward to obtain the modification of the field from any multipole due to a photon mass, and then use these new formulas to analyze the data.[5]

Analyses were made[5] for the three different data sets shown in Table I; the differences in RMR and in the coefficients obtained in going from one set to the other are very similar to those found with $\mu = 0$ for these and other data sets. The values for μ in the fits are given in units of 10^{-11} inverse centimeters and R_J^{-1}. Table I can be used to attempt to answer two questions regarding the parameter μ. The first is: What is the best value of μ if we have no other information? The second, a rather different question, is: How large a value of μ could be consistent with Pioneer-10 magnetometer observations?

Looking at the RMR minima in the columns of Table I, one would conclude that 0 was the best value for μ if one considered only columns 1, 2, 3, 7, and 8. However, if one considered only[9] column 4, derived by fitting the maximum number of coefficients to the most extensive run of data, the most likely value of μ would be 0.56×10^{-11} cm^{-1}.

This discord between the various analyses must mean that the RMR are dominated by some other systematic effects, and that there is no indication from these data that μ is not zero, since the precise locations of such shallow minima are unlikely to be significant.

To obtain a limit we note that for $\mu = 0.98 \times 10^{-11}$ cm^{-1}, the values of the residuals have begun to rise significantly from their minima in the various columns, including the first four columns that use the most data. By the value $\mu = 1.4 \times 10^{-11}$ cm^{-1} the residuals are rising steeply, and in particular, for the second to fourth columns, the $\mu = 1.4$ residual in any column is comparable with the minimum residual in the previous column, where fewer spherical harmonics were used to fit the data. Taking account of the possible systematic effects discussed below, we feel that the realistic 3-standard-deviation limit is $\mu \leqslant 2 \times 10^{-11}$ cm^{-1}.

To get a lower "secure" limit for the photon mass on the basis of the Pioneer-10 data would require a very careful study of the upper limits of other systematic effects. For example, suppose that actually μ is not zero, but there are plasma forces and currents in the shell $1.1R_J < r < 15R_J$ which comouflage the nonzero μ, so that the entire observed field could be fitted with $\mu = 0$ and sources outside this shell. Since the main part of the field is due[1-3] to a dipole of moment $M = (4\ \mathrm{G})R_J^3$, the vector potential should be of the order of the classical value $\vec{A} = (\vec{e}_\varphi M/r^2)\sin\theta$. Since $\vec{B}$ is consistent with a classical description, $\mathrm{curl}\vec{B} = 0$ and, by Eq. (2.19) of Ref. 6, $\vec{J} = (c\mu^2/4\pi)\vec{A}$. The order of magnitude of the resulting force density is given by the interaction of this current with the dipole part of the field. Hence, the force per unit volume exerted by the currents on the plasma is of order

$$\begin{aligned} \vec{F}_J &= \vec{J} \times \vec{B}/c \\ &= (\mu^2 M^2 \sin\theta/4\pi r^5)(-\sin\theta\,\vec{e}_r + 2\cos\theta\,\vec{e}_\theta) \\ &= 9.09 \times 10^9 \mu^2 (R_J/r)^5 \\ &\qquad \times(-\sin\theta\,\vec{e}_r + 2\cos\theta\,\vec{e}_\theta)\ \mathrm{dyn/cm^3}. \end{aligned} \tag{2}$$

For $\mu \neq 0$, the component along $\vec{e}_r$ is radially inward and the component along $\vec{e}_\theta$ is toward the equatorial plane.

For a steady state, this force must be balanced by some other forces on the plasma. Gravity is not in the right direction. Centrifugal force outside of $2.3R_J$, where it overbalances gravity, and a gradient in plasma pressure are possible. If we consider only orders of magnitude and ignore the problem of directions, we find that for a proton number density N_H, a hydrogen atom mass M_H, and an angular velocity due to corotation of $\Omega = 1.76 \times 10^{-4}$ rad/sec, the centrifugal force per unit volume near the equator is

$$\begin{aligned} F_c &= M_H N_H \Omega^2 r \\ &= 3.7 \times 10^{-22} N_H r/R_J\ \mathrm{dyn/cm^3}. \end{aligned} \tag{3}$$

Thus, F_c and F_J will be comparable for $N_H = 2.5 \times 10^{31}\mu^2(R_J/r)^6$ cm^{-3}. For $\mu = 2 \times 10^{-11}$ cm^{-1} and $r = 5R_J$, this requires $N_H = 6 \times 10^5$ cm^{-3} which is much too large.

If the temperature of the plasma is T and the gradient in the pressure is

$$F_P \simeq N_H kT/r = 1.93 \times 10^{-26} N_H T R_J/r \quad \mathrm{dyn/cm^3}, \tag{4}$$

we see by comparison with Eq. (2) that this force can be as large as F_J only if the thermal and magnetic energy densities of the plasma are comparable: $N_H kT 8\pi/B^2 \simeq 2\mu^2 r^2 \simeq 1$ for $\mu = 2 \times 10^{-11}$ cm^{-1} and $r = 5R_J$. This is not compatible with reasonable models of the Jovian magnetosphere; it would necessitate $N_H T = 3 \times 10^{11}$ cm^{-3} K.

Thus, if low enough limits can be placed on the plasma density and temperature, it should be possible to set a limit on μ lower than our mass limit of Eq. (1), or the number indicated as the best value in column 4; or perhaps even much closer to the zero value obtained in the fit of column 3.

Finally, we compare this photon mass limit with other limits. The best laboratory limit,[10] a test of Coulomb's law, is $2 \times 10^{-47}\ \mathrm{g} \equiv 5 \times 10^{-10}$ cm^{-1}. The limit[11] from fits to the geomagnetic field, a method devised by Schrödinger,[12] is 4×10^{-48} g $\equiv 10^{-10}$ cm^{-1}. In fact, the method of the present paper was advocated[13] as a direct extension of the geomagnetic method if such a mission[4] as Pioneer 10 could successfully measure the magnetic field of Jupiter. Recently Hollweg[14] has proposed a "reliable" limit of 1.3×10^{-48} g $\equiv 3.6\times10^{-11}$ cm^{-1} based on observations of Alfvén waves propagating in the interplanetary medium. However, this limit is proportional to the lowest observed Alfvén frequency, which, in the author's words, "is probably reliable to within a factor of 2." This means that, to compare with our value, one should multiply Hollweg's limit by at least a factor 2, and probably 3 or 4. In any event, the present limit supersedes all others obtained from direct measurements.

It should also be pointed out that less direct limits have been discussed[15-17] using known and speculated properties of the galactic magnetic field. These numbers range from[17] 10^{-14} cm^{-1} to[16] 10^{-18} cm^{-1}, and even the number 10^{-21} cm^{-1} has been suggested.[15] Confirmation of such limits[18] might be achieved either with new and better data on the galactic plasma and magnetic field, or by a new analysis of existing data, evaluating the forces required to keep the plasma in equilibrium for $\mu \neq 0$.

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