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Random-Field Instability of the Ordered State of Continuous Symmetry*

Yoseph Imry†
Brookhaven National Laboratory, Upton, New York 11973

and

Shang-keng Ma‡
Department of Physics and Institute for Pure and Applied Physical Sciences, University of California at San Diego, La Jolla, California 92037
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We consider phase transitions in systems where the field conjugate to the order parameter is static and random. It is demonstrated that when the order parameter has a continuous symmetry, the ordered state is unstable against an arbitrarily weak random field in less than four dimensions. The borderline dimensionality above which mean-field-theory results hold is six.

In view of the recent developments in the theory of critical phenomena in ideal systems,[1] the questions of the influence of random impurities, disorder, etc., on the phase transitions become relevant. Since these nonideal effects exist in any real system, understanding them is essential for any meaningful comparison of theory and experiment. The case where the coupling constants which are responsible for the transition are random has recently attracted considerable attention.[2] Here we would like to discuss the case where the field conjugate to the order parameter is random, and we shall consider only the static (or "quenched") situation. A particular case of this model, the ideal Bose gas, was recently considered by Lacour-Gayet and Toulouse[3] who found that space dimensionalities of four and six played special roles and that some of the usual scaling laws are not obeyed in this inhomogeneous system. It is easily seen that the borderline dimensionality above which the mean-field or Gaussian results hold is six. We shall give arguments demonstrating that, for $n \geq 2$ (in the isotropic case where the order parameter has a continuous symmetry), the ordered state of a very large system is unstable against an arbitrarily small random field. We shall see this in two ways: by considering the energetics of domain formation following the fluctuations in the random field, and by using the diverging susceptibility of the ordered state for $n \geq 2$. It should be possible to formulate a more rigorous proof[4] of our statement, but we believe that our qualitative arguments bring out the essential physics of the problem.

There are a number of situations where quenched random fields appear to be physically realizable. In Ref. 3, random sources and sinks of superfluid particles (e.g., Josephson junctions connecting the system to other systems) and stray magnetic fields were suggested. The latter may also be due to local magnetic moments. A case of particular interest would be displacive transitions or the various kinds of electronic instabilities leading to lattice distortions[5] where defects, impurities, and dislocations may couple to the order parameter. The continuous order-parameter

symmetry can be obtained in this case if the lattice distortion is incommensurate with the original lattice. Another interesting case may be a classical ferromagnet with antiferromagnetic site impurities, where a redefinition of the impurity spins can make their coupling with the host ferromagnetic. However, a finite applied magnetic field will have a reversed sign on the impurity spins, thus leading to a random component of the field. One may also visualize impurities which may influence the local orientation in a liquid crystal to yield a random field, as in our model.

We consider the usual isotropic n-component model[1] for phase transitions with an order parameter $\vec{\sigma}(x)$ coupled to a random field $\vec{h}(x)$,

$$\Delta\mathcal{H} = -\int d^d x\, \vec{h}(x)\cdot\vec{\sigma}(x), \tag{1}$$

by the new term $\Delta\mathcal{H}$ in the Hamiltonian. The integration is in the d-dimensional coordinate space and the scalar product is in the order-parameter space. $\vec{h}(x)$ is a random function which we shall assume to have a zero mean and short-range spatial correlations $\hat{f}$,

$$\langle\vec{h}(x)\vec{h}(x')\rangle = \hat{f}(x-x'); \quad \langle\vec{h}_k\vec{h}_{k'}\rangle = \delta_{k,k'} f(k), \tag{2}$$

where $f(k) = \int d^d x\, \hat{f}(x)$ and $\vec{h}_k = V^{-1/2}\int d^d x\, \vec{h}(x) e^{ik\cdot x}$. One may visualize the field $\vec{h}(x)$ as being generated by random frozen centers.

Suppose one has two neighboring domains of linear sizes $\sim L$ measured in lattice-constant units. The domain wall energy is $\sim L^{d-1}$ for the Ising case and $\sim L^{d-2}$ for the continuous-symmetry case.[6] (In the latter case, the domain wall energy is optimized by a continuous rotation of the order parameter over a distance comparable to L.) This argument immediately shows[6] that for $d \leq 2$ there will always exist large enough values of L (for a large system) for which the wall energy will be low enough so that it will pay for the ordered system to break into domains of size L. This is the well-known[4,6] lack of long-range order in continuous-symmetry systems for $d \leq 2$. In the case where the random field (1) exists, the situation becomes more interesting, as can be easily seen by employing a scaling-type argument due to Brooks Harris[7] in the random-exchange problem. Although $\langle h\rangle = 0$, the value of h summed over all spins in a size L will fluctuate from one domain to another with typical values being given by $\langle\Delta h^2\rangle \sim L^d$. Thus, by splitting into domains of size L, the system will gain bulk energies of $O(L^{d/2})$ per domain, and lose a "surface" energy of $O(L^{d-2})$ per domain (for $n \geq 2$). Thus, whenever

$$d/2 \geq d-2 \text{ or } d \leq 4, \tag{3}$$

there will exist a large enough L, for an arbitrarily small random field, where it will become energetically favorable to the system to break into domains of that size. For $n=1$, the limiting dimensionality becomes $d \leq 2$. It is also straightforward to generalize the argument to the case where the random field has long-range spatial correlations.

We note that, for $d<4$, the destruction of long-range order will occur also in the ground state. Since the entropy factor always favors the creation of domains, or, for that matter, any type of disorder, it is clear that once a domain formation is energetically favorable it will have to happen even more extensively at finite temperature, assuming thermodynamic equilibrium.

One can arrive at the above result by examining the fluctuations that would follow if $\vec{m} = V^{-1} \times \int d^d x\, \vec{\sigma}(x) \neq 0$. Suppose that $\vec{m} \neq 0$ and that $\vec{h}(x)$ is very weak. For $n \geq 2$, the components of $\vec{h}(x)$ perpendicular to $\vec{m}$ will produce a magnetization whose Fourier components are, for a given $\vec{h}$,

$$m_k^{\perp} = G_{\perp}(k) h_k^{\perp} + O(h^2), \tag{4}$$

where $G_{\perp}(k)$ is the transverse susceptibility for the pure system, which, for $k \to 0$, behaves as[8]

$$G_{\perp}(k) \sim (ck^2 + H/m)^{-1}, \tag{5}$$

where c is a constant and H is the uniform applied field, which we assume to vanish. The correlation function of the transverse magnetization is then

$$\langle m^{\perp}(x) m^{\perp}(x')\rangle = (2\pi)^{-d}\int d^d k\, G_{\perp}(k)^2 \langle|h_k^{\perp}|^2\rangle e^{ik\cdot(x-x')}, \tag{6}$$

which diverges for $d \leq 4$, in view of (5), for $H \to 0$ and $\langle|h_k^{\perp}|^2\rangle \neq 0$. Since $m^{\perp}(x)$ is never infinite, we have a contradiction. The conclusion is then that m must vanish for $d \leq 4$. This argument does not constitute a rigorous proof, because only the linear part of $G_{\perp}(q)$ is used. Since the divergence is obtained because of the large number of terms and *not* because of a simple diverging susceptibility, this may not be a serious drawback. The conclusion is the same as the one obtained from the domain-wall argument, which should not be sensitive to nonlinear effects.

For $d>4$, it is possible to have an ordinary critical point, provided that the random field is sufficiently weak. (Obviously, if the random

field is stronger than the field effected by the neighboring spins, then the spin will follow the random field and no long-range order is possible at any temperature.) One easily verifies, using the renormalization-group technique,[1] that for $d > 6$ the critical exponents are those of the Gaussian fixed point. For $d = 6 - \epsilon$ and $4 < d < 6$, one finds

$$\frac{1}{\nu} = 2 - \frac{n+2}{n+8}\epsilon + O(\epsilon^2) = d - 4 + O(n^{-1}),$$
$$\eta = O(\epsilon^2) = O(n^{-1}). \tag{7}$$

We have demonstrated the instability of the ordered state to domain formation for continuous-symmetry systems below four dimensions. This agrees with the $(n \to \infty)$ results of Ref. 3, at constant volume. It is probable that our results will be immediately relevant to defect influence on displacive transitions driven by electronic instabilities,[5] in the incommsurate case. In fact, this type of mechanism was proposed by Sham and Patton[5] to explain the lack of complete long-range ordering in $K_2Pt(CN)_4Br_{0.3} \cdot 3H_2O$[9] at low temperatures. Our results support their suggestion, although other explanations based on random interactions[10] are also possible. In the magnetic case, it would be interesting to consider random anisotropy fields along the same lines as in this work.

One should emphasize that our conclusion is *not* the obvious one that a strong random field will force the local "magnetization" to point parallel to the local field. Our considerations demonstrate the more subtle point that even when the local random field is much weaker than the interactions which favor the ordered state, there will exist a large enough L where it will pay for the system to break up into domains of size L. L is determined by an interplay of the domain wall energy and the statistics of the random field.

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†On leave from the Physics Department, Tel Aviv University, Tel Aviv, Israel.

‡Alfred P. Sloan Foundation Fellow.

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