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Josephson Junctions in Transverse Magnetic Fields

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Recent observations by Rosenstein and Chen of dc Josephson tunnel-current diffraction patterns occurring in a transverse magnetic field are ascribed to surface demagnetizing currents feeding the interior of the junction.

Recently Rosenstein and Chen[1] have reported the observation of a diffraction-pattern modulation of the critical current I_c of a Josephson tunnel junction caused by a magnetic field H applied perpendicular to the plane of the thin films forming the junction. We suggest that this effect is a straightforward consequence of the pattern of surface currents induced on the films by the magnetic field, and that the phenomenological description of Ref. 1 invoking a characteristic "edge penetration length" is not required.

The films, with the geometry shown in the inset of Fig. 1, lie in the x-y plane with $H_\perp$ ($\theta = 90°$) applied in the z direction. Label the upper and lower surfaces of the upper film by 1 and 2 and those of the lower film by 3 and 4. Denote the overlapping portions of surfaces 2 and 3 which form the junction as interior surfaces and the remaining visible portions as exterior surfaces. We suppose that the films are thick compared to the London penetration depth λ but thin compared

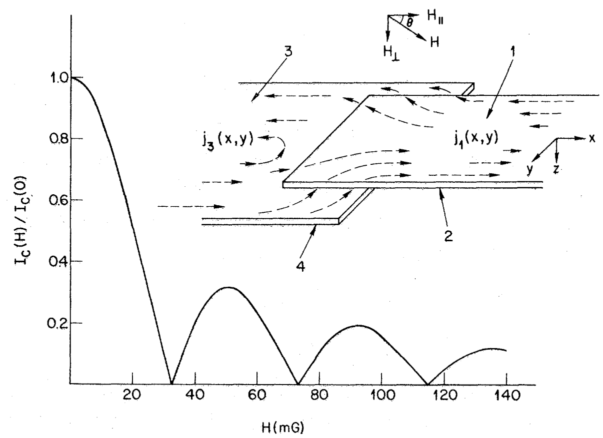

FIG. 1. Calculated diffraction pattern in a transverse field ($\theta = 90°$) for the in-line geometry with equal-width electrodes. The inset shows the actual geometry of the sample used in Ref. 1 which had unequal film widths of 0.74 and 1.00 mm and a common overlay region 0.35 mm in length. Note that the current streamlines appear continuous across the junction boundary, as discussed in the text.

to their other dimensions.

The application of H will induce a surface current per unit length $\vec{j}_i(x,y)$ on surfaces $i=1,4$. These currents satisfy a simple condition, namely that the pattern of $\vec{j}_i$ on the exterior surfaces is very nearly the same as would occur on a single piece of superconducting film of the same geometry, i.e., as if the interior surfaces of 2 and 3 were fused together. (This rule holds generally for closely spaced overlapping superconducting films, whether insulated or with Josephson coupling.) The fact that $\vec{j}_2=-\vec{j}_3$ on the interior surfaces provides the needed equality of the components of $\vec{j}_1$ and $\vec{j}_3$ (and also $\vec{j}_2$ and $\vec{j}_4$) across the boundary. It is this current configuration which minimizes the external magnetic field energy, and since the field energy inside the junction is comparatively small (they are roughly proportional to the relative volumes), the exterior patterns of $\vec{j}_i$ can be taken as identical for single and multiple films of the same geometry.

The values of $\vec{j}_2=-\vec{j}_3$ inside the junction are related to the phase differences $\varphi(x,y)$ relevant to the Josephson supercurrent flow and to the in-plane magnetic field h in the oxide layer by $\vec{j}_3=\hat{z}\times\vec{h}=-\{\Phi_0/[2\pi\mu_0(2\lambda+d)]\}\nabla\varphi$, where Φ_0 is the flux quantum and d is the oxide thickness. For a junction with lateral dimensions small compared to the Josephson penetration length, as is the case in Ref. 1, φ obeys $\nabla^2\varphi=0$. The solutions are determined by the values of the normal component of $\nabla\varphi$ at the boundaries, or equivalently, by the values of $\vec{j}_i$ at the boundaries. We have investigated the properties of φ for the more tractable geometry in which the width $2w$ of each film is the same. In this case, to a good approximation, we have[2] $\vec{j}_i(x,y)=(Hy/\mu_0 w)[1-(y/w)^2]^{-1/2}\hat{x}$ on all four surfaces.

The solution for the phase difference $\varphi(x,y)$ corresponding to this boundary condition was obtained numerically for the parameters of Ref. 1 with a relaxation technique on a 20×40-point square grid. Because of symmetry in the current flow pattern, the computation was done in only one quadrant of the junction area. The resulting dependence of the critical current $I_c(H)/I_c(0)$ on transverse magnetic field H ($\theta=90°$) is shown in Fig. 1 and the current flow pattern on surface 3 is shown in Fig. 2. It can be seen that the exterior surface currents in the x direction create a nonuniform interior surface current distribution predominantly in the y direction, thus causing the tendency[1] of the flux to thread the junction in the x direction. The period of modulation resulting from our calculation is roughly twice that observed experimentally in Ref. 1 and the angle θ at which a horizontal field tends to cancel the induced channeling field turns out to be $-32°$ compared with the experimental angle of $-8°$. We expect that these differences arise from the unequal widths of the films in the experiment,[3] a feature which should tend to enhance the flux in the x direction substantially as shown schematically in the inset of Fig. 1, but we have not attempted to determine the $\vec{j}_i$ quantitatively for this geometry.

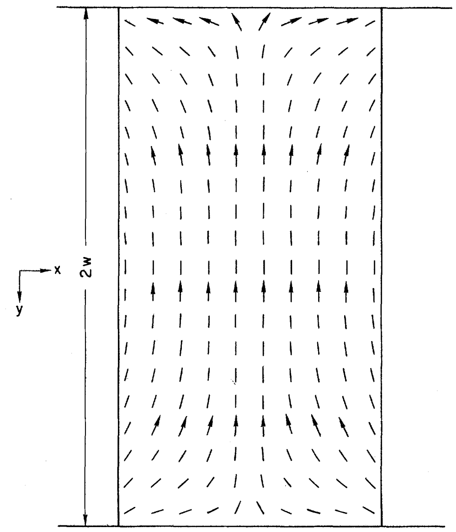

FIG. 2. Grid of calculated unit current vectors on surface 3 showing the manner in which the channeling field is created by the applied transverse field.

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