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Superfluid Density in Porous Vycor Glass*

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The superfluid density of ^{4}He confined to porous Vycor glass has been determined from fourth-sound measurements over a temperature range from below 0.1 K to within 10 mK of the transition. For temperatures below 1.4 K the normal-fluid fraction can be described in terms of a roton contribution with gap $\Delta/k_B = 5.85$ K and a "one-dimensional," T^2, phonon contribution. Near the transition temperature, $T_c = 1.955$ K, the superfluid density is found to vary as $(T_c - T)^\zeta$ with $\zeta = 0.65 \pm 0.03$.

The study of helium confined to the narrow (approximately 60 Å diameter) channels of porous Vycor glass[1] provides an excellent opportunity to examine the effect of restricted geometry on the excitations of the liquid and on the character of the superfluid transition. In the work reported here we consider both of these questions.

In the past, extensive determinations have been made of the superfluid density using fourth-sound[2] and persistent-current[3] techniques in both packed-powder and Vycor systems. The temperature for which superflow is first seen, the "onset" temperature, is observed to decrease with decreasing pore size. For onset temperatures on the order of 2 K the fourth-sound measurements in packed powders indicate that the superfluid density approaches zero in a nearly linear fashion.[4] Both the depression of the onset temperature[5] and the temperature dependence of the superfluid density near onset[6] have been discussed in terms of the temperature-dependent healing length of the Ginzburg-Pitaevskii-Mamaladze theory.[7] In this theory the healing length, ξ, is given by $\xi = \xi_0(1 - T/T_\lambda)^{-2/3}$. The experiments indicate that ξ_0 is in the range of 1 to 2 Å. A key assumption for this analysis is the requirement that the order parameter obey a zero-value boundary condition. Unfortunately there exists neither rigorous theoretical justification nor direct experimental evidence for such a boundary condition in superfluid helium.

Earlier fourth-sound measurements in Vycor by Fraser and Rudnick[9] and by Gregory and Lim[10] have indicated that the superfluid transition may be sharper in Vycor than for a packed powder of comparable pore size. Our work confirms this indication. We believe that the relatively sharp transition seen in the Vycor case is a result of greater homogeneity in pore size distribution as compared to packed powders.

In the present experiment we have observed fourth-sound resonances using an oscillating-cavity technique.[11] The experimental details are described later in the paper. The fourth-sound velocity, C_4, is obtained from the observed resonance frequencies and the known cavity length. An effective superfluid density ratio, ρ_s/ρ, is given by the usual expression,[2]

$$\rho_s/\rho = [C_4(T)/C_4(0)]^2, \quad (1)$$

where $C_4(0)$ is the zero-temperature extrapolation of the fourth-sound velocity. In Fig. 1, the superfluid density given by Eq. (1) is plotted against the temperature. The solid line gives the behavior of ρ_s/ρ for bulk helium at saturated vapor pressure. For temperatures below 1 K, the temperature dependence of the superfluid density for helium in Vycor clearly differs from that of bulk helium. However, apart from a shift in transition temperature, the data near T_c closely resemble data for bulk helium.

We have made a power-law fit of the form $\rho_s/\rho = A(1-T/T_c)^\zeta$ using only the data lying within 0.1 K of the transition temperature. (The best fit is given for $\zeta = 0.65 \pm 0.03$ and $T_c = 1.955 \pm 0.002$ K.) In Fig. 2 we show the conventional log-log plot of the superfluid density against reduced temperature, $\epsilon = 1 - T/T_c$. It is clear that the superfluid density follows the same "two-thirds" power-law in porous Vycor as it does in bulk helium, in marked contrast to the results obtained in packed powders[4] and the expectation based on the Ginzburg-Pitaevskii-Mamaladze[7] healing-length model.

Josephson[12] has pointed out a scaling relationship between the superfluid exponent, ζ, the heat-capacity exponent, α, and the dimensionality of the system, d. Thus, $\zeta = [(d-2)/d](2-\alpha)$. For three-dimensional bulk helium, the heat-capacity exponent is always near zero[13]; therefore ζ is close to two-thirds as observed.[14] Since we find the same superfluid exponent for helium in Vycor, we can conclude that in spite of the small dimensions of the channels in the porous glass the channels are sufficiently interconnected that the superfluid can behave as a three-dimensional system. If this point of view is correct, then one would expect the heat-capacity exponent, α, to be near zero and a sharp cusp would be expected in the heat capacity at the transition. Published measurements of full-pore heat capacity[15] by the University of Sussex group, however, show a

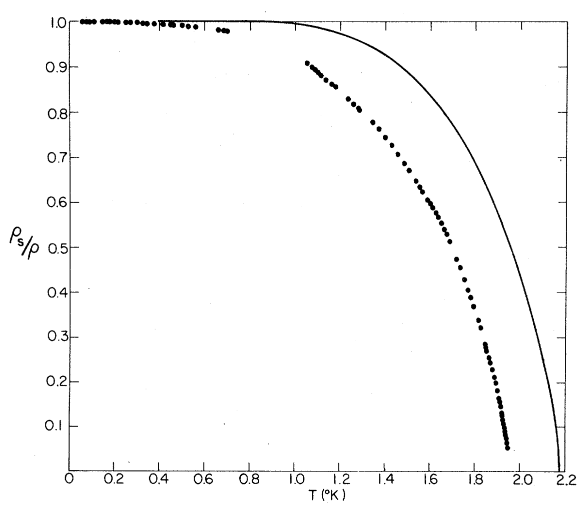

FIG. 1. Superfluid density $\rho_s/\rho = [C_4(T)/C_4(0)]^2$ versus temperature for liquid ^{4}He in Vycor. The solid line gives ρ_s/ρ for bulk helium at saturated vapor pressure.

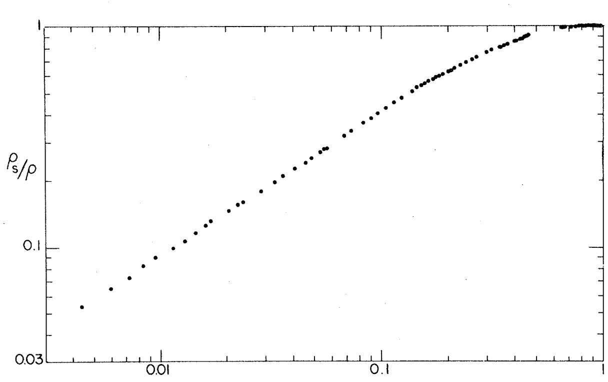

FIG. 2. Superfluid density, ρ_s/ρ, for liquid ^{4}He in Vycor plotted on a log-log scale versus reduced temperature, $\epsilon = 1 - T/T_c$, for $T_c = 1.955$ K.

rounding of the peak over a range of about 50 mK. It would be interesting to repeat this heat-capacity measurement on a Vycor sample which has shown the sharp behavior in superfluid density.

At temperatures well below the transition temperature, particularly below 1 K, it is clear that the superfluid density decreases more rapidly with increasing temperature for helium within the restricted geometry of the Vycor than for bulk liquid. In Fig. 3 we have plotted the normal-fluid fraction $\rho_n/\rho = 1-\rho_s/\rho$ against temperature on a log-log scale. For temperatures below 0.5 K, the data can be fitted with a T^2 dependence, characteristic of a "one-dimensional" phonon contribution to the normal fluid. Similar departures from the T^4 phonon contribution expected for bulk helium have been reported previously. Pobell *et al.*[16] observed a linear "zero-dimensional" phonon contribution for helium confined to interstices of tightly packed powder while more recently Washburn, Rutledge, and Mochel[17] have reported a "two-dimensional" T^3 contribution for a thin helium film on a flat substrate.

A theoretical discussion of this problem was first given by Padmore,[18] who pointed out that an increase in the phonon contribution to the normal-fluid density is to be expected in helium systems of less than three dimensions. More recently Saam and Cole[19] have treated the problem of films and full pores for the case of cylindrical channels. They find a T^2 dependence for the normal-fluid density in channels with diameters comparable to the pore diameters of Vycor glass. These discussions have provided a qualitative description conforming to the experimental findings, but they underestimate the size of the observed effect by at least an order of magnitude.

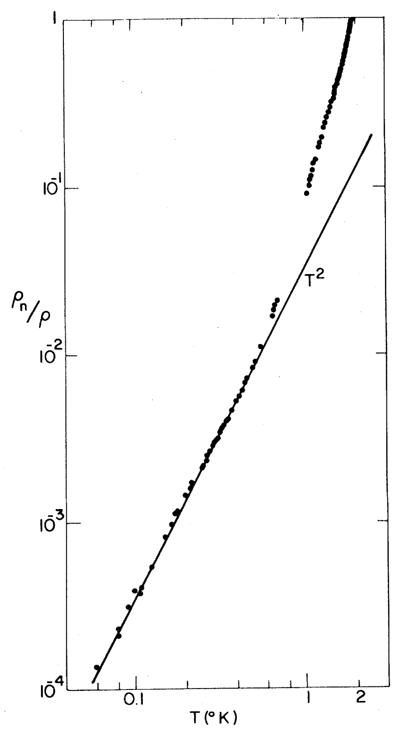

FIG. 3. The normal-fluid fraction, $\rho_n/\rho = 1-[C_4(T)/C_4(0)]^2$, versus temperature on log-log scale.

For temperatures above 0.5 K the data in Fig. 3 show a departure from the T^2 dependence seen at lower temperatures. This departure can be fitted up to a temperature of at least 1.4 K with a roton-type excitation. Thus the superfluid density may be expressed by $\rho_s/\rho = 1 - AT^2 + BT^2 \times \exp(-\Delta/k_B T)$ for $T \leq 1.4$ K. The value of the roton energy gap, Δ/k_B, is found to be 5.85 K. This value may be compared to the value of 6.2 K derived by the University of Sussex group[20] from heat-capacity measurements. They attribute the roton contribution to the second statistical layer. The observed reduction in the roton gap of about 3 K below the bulk value corresponds to the decrease found by Padmore[21] in his Feynman-Cohen calculation for two-dimensional rotons.

A newly developed technique for the generation and detection of fourth sound has been employed in this work. The apparatus is similar in configuration to that used by Yanof and Reppy[11] in their fourth-sound investigations of superfluid ^{3}He. A cylinder of porous Vycor glass, 2.2 cm long by 0.68 cm in diameter, was sealed with epoxy in a thin copper sleeve. The cavity is suspended in a vacuum from a flexible, 0.5-mm-diam, stainless-steel tube. This tube also serves as a fill line. A thin copper wire acts as a thermal link between the Vycor cavity and a ^{3}He-^{4}He dilution refrigerator. The cavity is driven along the longitudinal axis of the cylinder.

The forces acting on the cavity are those of the drive, the elastic restoring force of the support, and that due to the acceleration of the center of mass of the helium confined within the cell. At low frequencies (~ 100 Hz) the system displays a resonance on the support system. Changes in the support resonance frequency are useful in determining the mass of helium condensed into the cell. In the present experiment the fourth-sound resonance frequencies lie well above the frequency of the support resonance; therefore, we shall neglect the influence of the support resonance on the fourth-sound resonances. We shall also neglect any effect of fourth-sound damping; the equation of motion given by Yanof and Reppy[11] for the driven longitudinal motion of the Vycor cell is then considerably simplified. Then, for a drive force $F_0 \exp(-i\omega t)$, the amplitude, X_0, of the longitudinal motion is given by

$$X_0 = -F_0\omega^{-2}\left[M - \tilde{m}_s\left(1 - \frac{2C_4}{\omega L}\tan\frac{\omega L}{2C_4}\right)\right]^{-1}, \quad (2)$$

where M is the mass of the cavity including the fluid, and $\tilde{m}_s$ is the effectice superfluid mass.

The motional response of the cavity has several interesting features. For frequencies corresponding to odd half-wave modes, i.e., odd multiples of $\omega_0 = 2\pi C_4 L^{-1}$, the pressure field associated with the fourth sound can balance the drive force, and the displacement of the cavity, X_0, approaches zero. A determination of the zero-motion point then serves to determine the fourth-sound velocity, $C_4(T)$. At a somewhat higher frequency, ω_1, the amplitude of the cavity motion diverges. In practice this motion will be damped by dissipation in the support even in the case of an infinite fourth-sound quality factor, Q_4. Measurement of the two frequencies, ω_0 and ω_1, allows us to determine the effective superfluid mass directly without reference to the fourth-sound velocity. For $\tilde{m}_s \ll M$ (in this experiment $\tilde{m}_s \sim 10^{-3}M$), we have $\tilde{m}_s \cong (\pi/2)^2[(\omega_1 - \omega_0)/\omega_0]M$. In the case where the fourth-sound damping is too large to neglect, the effective superfluid mass, $\tilde{m}_s$, can still be obtained by use of the equation of motion given in Ref. 11.

In the present experiment we find that at the lowest temperature the effective superfluid mass is only 10% of the total helium mass, m, required to fill the Vycor pores. The elastic properties of the helium in the porous Vycor can be characterized by a sound velocity, C, through the relation $C_4^2 = (\tilde{m}_s/m)C^2$. Taking the low-temperature values of C_4 and $\tilde{m}_s/m$, we obtain a value for C of about 330 m/sec. This velocity corresponds to the first-sound velocity in bulk helium at low temperatures for a density of 0.165 g/cm^3. It is interesting to note that this value is close to the measured average density of 0.168 g/cm^3 for ^{4}He in porous Vycor glass.[15]

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