diation reaction[4]), $\omega - \omega_p \sim 10^8\ \mathrm{sec}^{-1}$ (i.e., $n_0 \sim 10^{20}\ \mathrm{cm}^{-3}$) and $eE_0/m_e c\omega \sim 10^{-1}$ (i.e., incident radiation power density $\sim 10^{16}\ \mathrm{W/cm}^2$ or $E_0 \sim 3 \times 10^9$ V/cm), and $E_z(z=0)$ is therefore of the order $10^{-7}E_0 \sim 300$ V/cm. If the close Coulomb collision rate is larger than the effective collision frequency because of the radiation reaction, we have to take into account the former contribution, thus ν_c ($\sim \omega_p \ln\Lambda/\Lambda$, where Λ is the number of electrons in a Debye sphere) may be several orders higher than $10^6\ \mathrm{sec}^{-1}$, depending on the temperature and the density of the plasma. For instance, if $\nu_c \sim 10^{11}\ \mathrm{sec}^{-1}$ and $\omega - \omega_p \sim 10^{13}\ \mathrm{sec}^{-1}$ while ω and $eE_0/m_e c\omega$ remain the same as before, we have $E_z \sim 3\times 10^5$ V/cm. Note that this dc longitudinal electric field exactly cancels the effect of the forward Lorentz force, rendering the electrons with only transverse motions. As a result, there would be no Doppler shift of the wave frequency and, furthermore, the evaluation of the radiation losses can be made in a way similar to that used by Steiger and Woods,[4] since now each electron is exactly circulating in a plane.

Finally, we want to discuss why it is appropriate to assume $v_z = 0$, an assumption which is crucial in reaching the dc longitudinal electric field. One may wonder, although a net forward motion is prohibited by the induced return force, if there are longitudinal oscillations of the electrons. To answer this, let us consider a circularly polarized plane radiation propagating towards a bounded electronic cold plasma. The front part or the amplitude-increasing part of the radiation which corresponds to the rise of the laser intensity will first interact with the plasma. The portion of the plasma which interacts with this front part will experience a forward force which is proportional to the rate of change of the intensity.[6] This force vanishes whenever the front part passes through. However, there is another forward force which is the Lorentz force mentioned in the very beginning of this article. The second force is proportional to the intensity and is always present with the radiation. If the intensity of the incident radiation increases smoothly from zero to a final constant value in such a way that the rise time is much longer than the inverse of the plasma frequency, we should expect that the total forward force does not induce longitudinal oscillations; this is in analogy to the case of a spring being stretched down by gradual adding of small weights. On the other hand, if the rise time is not sufficiently long, we should expect transient longitudinal oscillations which die out in a time of the order of the inverse of the effective collision frequency.

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Seeded Megagauss Turbulence in Dense Fusion-Target Plasmas*

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A collision-dominated inhomogeneous plasma seeded with a distribution of impurity grains is calculated to fill out rapidly with random megagauss fields for temperatures of about a kilovolt. Possible applications of this phenomenon (i) to enhance relativistic-electron-beam coupling to low-Z target plasmas, (ii) to influence transport coefficients, or (iii) to localize α-particle deposition in a reacting DT plasma are discussed.

It was recently shown[1,2] that spatial fluctuations in the chemical composition of a dense plasma give rise to "frozen-in" electron density fluctuations, δN_e, which in turn are expected to produce megagauss field fluctuations, $\delta\vec{B}$. These fields derive from a source term[3] $\nabla\,\delta N_e \times \nabla T$, where ∇T is the background temperature gradient.[4,5] This phenomenon may provide a controlled way of seeding magnetic turbulence with a predetermined k spectrum in plasmas in the solid-state density range produced by relativistic electron beams (REB's), ion beams, or lasers. In this Letter I first derive a more general solution for the development of a seeded-turbulence spectrum and then discuss several applications.

Consider a rapidly heated grainy plasma for which $T(t)$ is a given function of time [$T(t)$ derives either from local heating or from traversal by a thermal wave, etc.]. As T increases, magnetic flux is produced and diffuses within the plasma. Simultaneously (once T exceeds a few keV) ion diffusive mixing occurs and removes the source, after which the field tends to persist because of the high plasma conductivity. The Fourier component, $\delta\vec{B}_k$, of the field is derived from δN_{ek} by a generalization of Eq. (1) in Ref. 1,

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau_B}\right)\delta\vec{B}_k=\left(\frac{ick}{e}\right)\vec{k}\times\nabla T\left(\frac{\delta N_{ek}}{\langle N_e\rangle}\right)\exp\left(-\int_0^t\frac{dt'}{\tau_M}\right), \tag{1}$$

where the characteristic field (or decay) time, τ_B, and grain mixing time,[6] τ_M, are

$$\tau_M=5\pi^2 10^{-5}\left(\frac{N_h}{N_s}\right)\frac{Z_h^2Z_l^2}{T^{5/2}}\;\frac{\ln\Lambda}{k^2}\left(\frac{m_l}{m_p}\right)^{1/2},\qquad \tau_B=4\pi\sigma/c^2k^2=7.8\times10^{-2}T^{3/2}/\bar{Z}k^2\ln\Lambda\,, \tag{2}$$

where $N_s=4.5\times10^{22}\ \mathrm{cm}^{-3}$, T is in keV, Z_h and Z_l are the heavy- and light-ion charges, N_h is the heavy-ion number density in the heavy-ion regions in the unmixed plasma state, $\langle N_e\rangle$ is the space-averaged electron density, and ∇T is the locally constant temperature gradient. On the composition fluctuation scale, local pressure equilibrium is assumed between unmixed regions, i.e., $N_l(1+Z_l)=N_h(1+Z_h)$.

Now assuming that $\delta\vec{B}_k(t=0)=0$, and choosing for example a heating model

$$T=T_0(t/t_0)^{2/3}, \tag{3}$$

Eq. (1) gives

$$\delta\vec{B}_k=\frac{ick}{e}\left(\frac{\delta N_{ek}}{\langle N_e\rangle}\right)\vec{k}\times\nabla T_0\left(\frac{t_0}{t}\right)^R\int_0^{t/t_0}t_0\,dx\,x^{2/3+R}\exp\left(-\frac{3t_0}{8\tau_{M0}}x^{8/3}\right), \tag{4}$$

where $R=t_0/\tau_{B0}$ with $\tau_{B0}=\tau_B(T_0)$, and $\tau_{M0}=\tau_M(T_0)$.

The behavior of $\delta B_k=|\delta\vec{B}_k|$ is shown in Fig. 1. It increases as $t^{5/3}$ until reaching a maximum (by which time the source has mixed away), after which it decays slowly as t^{-R}. This maximum is approximately

$$\delta B_{k,\max}\approx10^6\left|\frac{\delta N_{ek}}{\langle N_e\rangle}\right|\frac{l^{1/4}t_0^{3/8}Z_h^{5/4}Z_l^{5/4}}{T_0^{9/16}l_T}\left(\frac{m_l}{m_p}\right)^{5/16}\left(\frac{N_h}{N_s}\right)^{5/8}\left(\frac{\ln\Lambda}{7}\right)^{5/8}\left[1+\frac{5.32\times10^{-7}\bar{Z}t_0}{T_0^{3/2}l_2}\left(\frac{\ln\Lambda}{7}\right)\right]^{-1}, \tag{5}$$

and is reached in a time

$$t_m\approx173t_0^{5/8}\left(\frac{N_h}{N_s}\right)^{3/8}\frac{Z_h^{3/4}Z_l^{3/4}l^{3/4}}{T_0^{15/16}}\left(\frac{m_l}{m_p}\right)^{3/16}\left(\frac{\ln\Lambda}{7}\right)^{3/8}\ \mathrm{nsec}, \tag{6}$$

where t_0 and t_m are in nanoseconds, $\ln\Lambda/7\approx1$, T_0 is in keV, and I have introduced a characteristic grain-size scale l and temperature-gradient scale l_T, where $l=\pi/k$ and $l_T=T_0/|\nabla T_0|$.

The amplitude of the "initial" electron density fluctuations to be used in (5) can be shown[1] to be

$$\left|\frac{\delta N_e}{\langle N_e\rangle}\right|=\frac{(0.5-|0.5-\alpha|)\,|Z_h-Z_l|}{Z_l(1-\alpha)(1+Z_h)+\alpha Z_h(1+Z_l)}, \tag{7}$$

where α is the volume fraction occupied by heavy-ion plasma in the unmixed state. We also note that the space-averaged ion density, $\langle N_i\rangle$, is related to the ion density, N_h, in the heavy regions by

$$N_h=\langle N_i\rangle[\alpha+(1-\alpha)(1+Z_h)/(1+Z_l)]^{-1}. \tag{8}$$

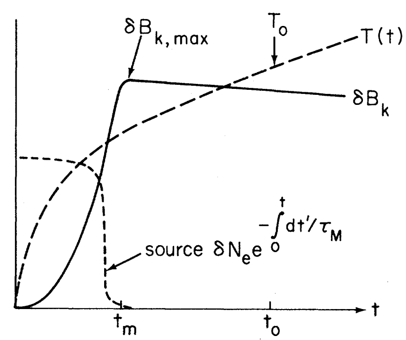

FIG. 1. Time dependence of a field Fourier amplitude, $\delta\vec{B}_k$, for a given plasma heating model, $T(t)$. The field reaches a maximum as the electron-density-fluctuation source mixes away.

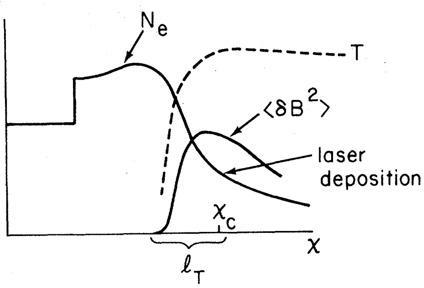

FIG. 2. Variation of electron density, N_e, temperature, T, and field fluctuation energy density produced by a laser pulse incident on a grainy target. The field originates in the thermal wave beyond the critical depth, x_c.

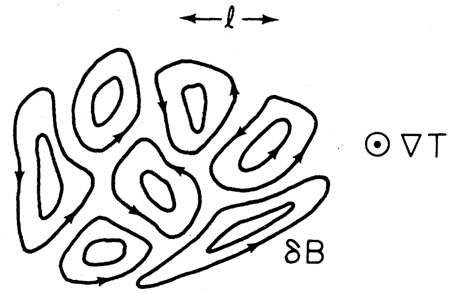

FIG. 3. Field fluctuations produced by a grainy plasma in a thermal gradient. The field flux loops lie in planes perpendicular to ∇T and their spatial distribution varies in the ∇T direction, so that they overlap to form a dense barrier.

A typical laser-driven implosion profile is shown in Fig. 2. A thermal wave penetrates ahead of the critical depth, which moves in as material is ablated. Suppose that the ablator plasma consists of a 50-50 C-Li mixture with a grain scale $l = 10^{-3}$ cm, and that the thermal wave has a thickness $l_T = 10^{-2}$ cm and moves in at 10^7 cm sec^{-1}. Plasma traversed by this wave experiences a temperature increase to ~10 keV in $t_0 \approx 1$ nsec before being ejected. For these numbers Eqs. (5) and (6) give

$$\delta B_{k,\max} \approx 2.2\times 10^7 (N_h/N_s)^{5/8},$$

and

$$t_m \approx 1.4 (N_h/N_s)^{3/8} \text{ nsec};$$

i.e., fields above a megagauss would be reached in the thermal-wave transit time, even for $N_h/N_s = 10^{-2}$. This creates a layer of dense overlapping flux loops with $\delta\vec{B}$ in planes perpendicular to the radial direction, ∇T (Fig. 3).

Now an REB pulse incident on this layer would encounter a barrier against penetration into the pellet interior (as would suprathermal electrons generated at the laser critical depth). An energetic electron in the layer has an anisotropic scattering mean free path,[1]

$$\lambda_s = \frac{2}{3\sqrt{\pi}\,\ln(2r_B/l)}\left(\frac{r_B}{l}\right)\frac{r_B}{\cos^2\theta}, \tag{9}$$

where r_B is its cyclotron radius in the field $\langle \delta B^2\rangle^{1/2}$, and θ is the angle between ∇T and the electron velocity. After partial radial diffusion into the layer, electrons tend to be trapped (note the $\cos^2\theta$ factor) and deposit their energy in shells perpendicular to ∇T (∇T is in the pellet's radial direction, x).

This suggests the possibility of a *hybrid* REB-laser compression scheme in which a laser pulse (with its high deposition power density) first creates the thermal wave needed to produce the field layer, so that an REB pulse can then couple into (and maintain) the layer through its enhanced deposition power density.

Thermal conductivity is reduced by the field fluctuations and would tend to result in steeper thermal fronts (e.g., beyond the laser critical depth). Regarding $\delta\vec{B}$ as a random field in planes perpendicular to ∇T (Fig. 3), and averaging, we find that only the heat flux along ∇T survives and is $\langle\vec{q}\rangle = -\langle K_{\perp e}\rangle \nabla T$. Using Braginskii's notation[5] for the thermal conductivity $K_{\perp e}$, and assuming a Gaussian field-magnitude distribution, gives for the averaged conductivity

$$\langle K_{\perp e}\rangle = \frac{N_e T \tau_e}{m}\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{dx}{x_0}\exp\left(-\frac{x^2}{x_0^2}\right)\frac{\gamma_1' x^2 + \gamma_0'}{x^4 + \delta_1 x^2 + \delta_0} \approx \frac{N_e T \tau_e}{m x_0}\frac{\sqrt{\pi}}{\sqrt{y_1}+\sqrt{y_2}}\left(\gamma_1' + \frac{y_0'}{(y_1 y_2)^{1/2}}\right), \tag{10}$$

where $2y_{1,2} = \delta_1 \pm (\delta_1^2 - 4\delta_0)^{1/2}$, $x_0 = \tau_e e \langle \delta B^2\rangle^{1/2}/mc$, and the approximation is for $x_0 \gg 1$.

If fields were created in the fuel region of a pellet capable of scattering 3.5-MeV α particles sufficiently to localize their energy deposition, the resulting critical ignition radius would be reduced for a given density, $N_{\rm DT}$. This occurs because the minimum radius for ignition (at the minimum of $rN_{\rm DT}$ as a function of T) is of the order of the α-particle mean free path at the ignition temperature.

To examine this consider a grainy DT-Li mixture with the Li fraction, α, small so that $|\delta N_e/\langle N_e\rangle| \approx \alpha/2$ (for $\alpha < \frac{1}{6}$, Li bremsstrahlung also becomes small). The growth of $\delta\vec{B}$ in the course of ignition of a spherical region of compressed radius r and density $\langle N_i\rangle \approx N_{DT}$ can be estimated from (5) by choosing the temperature scale $l_T = r$, grain scale $l = r/10$, $T_0 \approx 20$ keV, and the inertial time for the field growth, $t_0 \approx r/v_s \approx 10r$ nsec. This gives

$$\delta B_{k,\max} = 4.2\times10^5 \alpha r^{-3/8} (\langle N_i\rangle/N_s)^{5/8}.$$

Noting that the α-particle cyclotron radius r_α is $2.7\times10^5/\delta B$, and writing for the compressed density $\langle N_i\rangle/N_s = (r_0/r)^3$, where r_0 is the initial *ignition-sphere* radius (which may be less than the initial fuel radius), gives

$$r_\alpha/r = (0.64/\alpha r_0^{5/8})(r/r_0)^{5/4}$$

and

$$t_m/t_0 = 1.63 r_0^{3/8}(r_0/r)^{3/4}.$$

For $\alpha = 0.25$, $r_0 = 5\times10^{-2}$ cm, and a compression $r/r_0 = 0.1$, we find $r_\alpha/r = 0.94$ and $t_m/t_0 = 2.98$. The situation is borderline in that δB is not quite strong enough to retain α particles in the ignition sphere. Note however that the above fields may be underestimated because I did not include the effect of compressional amplification[2] of δB's in the last stages of compression, so that this application can not be ruled out. A more complete analysis of these effects will be given later.

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[4]Temperature fluctuations, δT_e, associated with δN_e, tend to be smoothed out by electron thermal conduction. However they may contribute an additional source term in some situations and this is currently being examined, together with the thermal-force term which is nonlinear in $\delta\vec{B}$ [see S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), p. 249].

[5]Braginskii, Ref. 4.

[6]τ_M is the time for a light ion to diffuse a composition fluctuation scale length $l = \pi/k$. The formula applies provided l exceeds the light-ion mean free path, $3.7\times10^{-4}T^2(N_s/N_h)/Z_h^2 Z_l^2 \ln\Lambda$.

"Crystal Field" in Liquid Rare-Earth Metals*

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Neutron-inelastic-scattering measurements have been performed on liquid La, Ce, and Pr. The results demonstrate that a well-defined "crystal field" exists in liquid rare-earth metals and that the scattering can be understood if it is assumed that the crystal-field potential is uniaxial.

Recently, experimental and theoretical evidence has been presented that the "crystal field"—the electric field due to the ions surrounding each atom—is well defined in amorphous metals and alloys.[1] In model calculations[2] it was assumed that the dominant term of the crystal-field potential in the amorphous alloys is uniaxial, i.e., of the form

$$V_c = -DJ_z^2, \tag{1}$$

where D is the axial crystal-field strength, and the direction z varies randomly from atom to atom in the material. Amorphous materials are topologically disordered, but measured radial distribution functions[3] support the simple picture of a dense random packing of atomic spheres as in liquids. This analogy suggests that the crystal field might also exist in liquid metals and alloys. Elastic-neutron-scattering measurements on liquid La and Ce[4-6] have indeed shown that the