vanish as the four-vector $k_{1\mu}$ goes to 0, so that we can write the matrix element as

$$\lim_{k_{1\mu}\to 0} \frac{(P+P-k_1)\cdot k_1}{m_\delta^2 - m_{\eta'}^2 - k_1^2 + 2P\cdot k_1 + i m_\delta \Gamma_\delta} \frac{g_{\delta\eta\pi} g_{\delta\eta'}{}^A}{F_\pi},$$

where

$$(2\pi)^3(4q_{10}q_{20})^{1/2}\langle \delta, q_1 | A_\mu | \eta', q_2\rangle = g_{\delta\eta'}{}^A (q_1+q_2)_\mu + \ldots$$

defines the axial-vector coupling constant $g_{\delta\eta'}{}^A$ and F_π is the pion decay constant. Now let $\vec{k}_1 = 0$ and insert the values of m_δ, $m_{\eta'}$, and Γ_δ to get

$$\text{const}[k_{10}/(12 + k_{10} + i25)]$$

which is zero for $k_{10} = 0$ as demanded by the Adler condition but is almost unity at the edge of the Dalitz plot $k_{10} = m_\pi = 140$. In other words the proximity in mass of δ(970) and η′(958) makes the matrix element vary rapidly around the soft-pion limit. We cannot therefore find the physical-region matrix element by smooth extrapolation from the Adler point. The calculations in Ref. 6 are based on linear extrapolation. We must then disregard the criticism against the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ model for chiral-symmetry breaking.

Our conclusions are as follows: (1) η′ is the ninth member of the pseudoscalar nonet. (2) The mixing angle is positive and the quadratic mass formula is preferred. (3) If the total width of η′ turns out to be a few hundred keV as indicated by our model, it will mean that there are no violent departures from SU(3) symmetry even when it is used for dimensional coupling constants. (4) There is no evidence against the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ model of chiral-symmetry breaking from η′ decays.

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Measurement of Elastic Scattering of Hadrons on Protons from 50 to 175 GeV/c*

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Differential cross sections have been measured at Fermilab with a focusing spectrometer for $\pi^\pm p$, $K^\pm p$, and $p^\pm p$ elastic scattering at 50-, 70-, 100-, 140-, and 175-GeV/c incident momentum over the $|t|$ range 0.03 to 0.8 GeV^2. The results are smooth in t and are parametrized by quadratic exponential fits.

This paper describes measurements of $\pi^\pm p$, $K^\pm p$, and $p^\pm p$ elastic scattering made at the Fermi National Accelerator Laboratory. The reactions were studied for momentum transfers $|t|$ from 0.03 to 0.08 GeV^2 and incident momenta of 50, 70, 100, 140, and 175 GeV/c, with typically

3×10^5 events total at each energy.

The experiment was carried out using the single-arm spectrometer in M6E, a high-resolution unseparated secondary-particle beam. Collimators defined the solid-angle and momentum acceptances of the beam to maximum values of 1.75 μsr and 1.2%, respectively. The beam was momentum recombined and focused to a small spot at the hydrogen target, typically 6 mm by 3 mm. Five scintillation counter hodoscopes, a threshold Cherenkov counter, a differential Cherenkov counter, and a DISC counter[1] provided full tagging of the incident beam momentum, trajectory, and particle type. Beam fluxes of typically 2×10^6/burst were used.

The spectrometer had point-to-parallel-to-point optics. Scintillation counters and/or proportional-wire-chamber cuts defined the spectrometer acceptance regions of typically 5 μsr, ± 1.5 mrad scattering angle, and ± 2% $\Delta p/p$. Ten proportional wire chambers, a hodoscope, three threshold Cherenkov counters, and a differential Cherenkov counter[2] were used for full tagging of the trajectory and particle type. For straight-through beam tracks, the rms differences between the beam and spectrometer measurements were typically ± 0.07% in momentum and ± 0.13 mrad in scattering angle.

Three bending magnets located just upstream of the liquid hydrogen target were used to vary the momentum transfer by pitching the incident beam in the vertical plane to a maximum angle of ± 25 mrad. The target cells were 1-in.-diam Mylar tubes, 10 and 20 in. long. The fields of the angle-varying magnets, as well as the beam and spectrometer momentum values, were continuously monitored with Rawson probes located in 3-ft-long monitor magnets connected in series with the appropriate bending magnets. Measurements were made with both positive and negative deflection angles to average out any systematic errors caused by small misalignments. The t values were determined directly in terms of the magnetic field calibration to a precision of about 0.3%.

Scattered events mixed with an unbiased sample of beam events were used as triggers. The sample of beam events permitted determinations of beam-line detector efficiencies, the beam phase-space distribution, and the effects of any fiducial cuts on incoming particle fluxes. A PDP-11/45 computer operating under the SPEX multitask system[3] recorded up to 200 events per beam burst and provided on-line analysis and monitoring.

The square of the missing mass (M_x^2), t, and particle type were determined for each event. Figure 1 is a typical M_x^2 distribution showing the separation of the elastic peak from inelastic background. A fit made to the M_x^2 distribution determined the inelastic contamination under the elastic peak, typically (3± 1)%. Under most running conditions the contamination through misidentification of the incoming particle type was small and well determined. Muon and electron contaminations were measured independently and were each ~ 1%.

The total detection efficiency of the spectrometer system including the wire chambers was typically 95% and the absolute efficiency was known to about ± 1%. The presence of two simultaneous incoming particles was virtually eliminated by requiring only one particle in the beam-line hodoscopes.

Corrections for decay in flight and attenuation in the target were calculated, and also measured from the transmission of the straight-through unscattered beam into the spectrometer. Small corrections were applied for double nuclear scattering and Coulomb interference effects. Radiative corrections, made according to the prescription of Sogard,[4] were a few percent for pions and considerably less for kaons and pro-

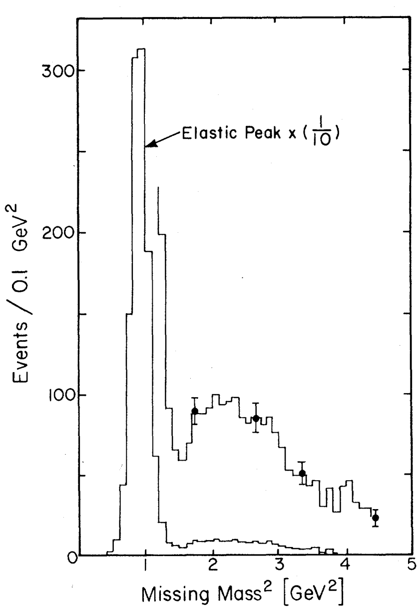

FIG. 1. Missing-mass-squared distribution showing the elastic peak for π^+p at 70 GeV/c for $0.04\leq|t|\leq0.18$ GeV^2.

tons.

The spectrometer solid angle was calculated from the known magnetic and geometric properties of the spectrometer and verified by direct measurement at each momentum. An overall check of the normalization was made by comparing our pp results with the data of Bartenev *et al.*[5] at $|t|=0.09$ GeV^2. Agreement was found within 3%.

The pion and proton events were restricted by cuts to a well-understood region of the spectrometer to give absolute cross sections. The low-rate particles, $K^\pm$ and $\bar{p}$, were accepted over the entire aperture of the spectrometer and the corresponding solid angle obtained from the pion and proton fluxes.

The overall normalization uncertainty is ± 3%, independent of t. In addition, there is an uncertainty increasing from ± 1% at small t to ± 6% at 0.8 GeV^2 due to uncertainties in the inelastic contamination, the double-scattering corrections, and the determination of the scattering angle. Decay-in-flight corrections were only significant for kaons, and at 50 GeV/c (where 25% decay in the spectrometer) there is an additional 3% systematic uncertainty.

Our differential cross section data in the range $0.03 \leq |t| \leq 0.8$ GeV^2 cannot be fitted with a simple exponential, but may be represented by

$$d\sigma/dt = A\exp(Bt + Ct^2). \quad (1)$$

This form gives a good representation of the data

TABLE I. Results of fitting the differential cross sections by Eq. (1). A, B, and C are from fits to data out to $|t|=0.8$ GeV^2; only statistical errors were used for the data points, while the optical theorem point (Opt. Pt., Ref. 8) was included with a ±3% error to account for the uncertainty in absolute normalization. The values of the logarithmic slope at $|t|=0.2$ GeV^2 were found from Eq. (2) using fits to data with $|t|<0.4$ GeV^2; to take into account the t-dependent uncertainties, a systematic error of ± 0.15 GeV^{-2} was added in quadrature to the statistical errors. The total elastic cross sections were obtained by integrating fits (with A fixed to the optical point) out to $|t|=0.8$ GeV^2; a small correction was made for contributions at larger t by assuming a constant logarithmic slope at $|t| \geq 0.8$ GeV^2.

	(GeV)	Opt. Pt. (mb/GeV^2)	A (mb/GeV^2)	B (GeV^{-2})	C (GeV^{-4})	b(0.2) (GeV^{-2})	σ_{elas} (mb)
π^+p	50	27.2	27.8 ± 0.4	8.85 ± 0.12	1.92 ± 0.18	8.07 ± 0.17	3.33 ± 0.07
	70	27.5	27.7 ± 0.5	8.98 ± 0.14	2.35 ± 0.23	8.08 ± 0.17	3.31 ± 0.08
	100	27.7	27.5 ± 0.8	8.80 ± 0.18	2.12 ± 0.23	7.92 ± 0.20	3.33 ± 0.09
	140	28.1	29.1 ± 0.6	9.11 ± 0.17	2.36 ± 0.28	8.18 ± 0.17	3.42 ± 0.08
	175	28.5	29.2 ± 0.4	9.02 ± 0.10	2.26 ± 0.15	8.16 ± 0.16	3.46 ± 0.07
π^-p	50	29.5	31.7 ± 0.4	9.74 ± 0.11	3.07 ± 0.15	8.61 ± 0.17	3.53 ± 0.07
	70	29.4	28.3 ± 0.5	8.92 ± 0.20	2.07 ± 0.47	7.97 ± 0.17	3.37 ± 0.10
	100	29.3	27.4 ± 0.5	9.04 ± 0.14	2.38 ± 0.20	8.09 ± 0.17	3.25 ± 0.07
	140	29.6	29.0 ± 0.6	9.20 ± 0.18	2.40 ± 0.29	8.17 ± 0.18	3.37 ± 0.09
	175	29.9	29.2 ± 0.8	9.66 ± 0.18	2.88 ± 0.24	8.59 ± 0.19	3.26 ± 0.08
K^+p	50	16.6	17.1 ± 0.4	6.95 ± 0.29	0.42 ± 0.59	6.94 ± 0.31	2.51 ± 0.12
	70	17.3	17.8 ± 0.4	7.65 ± 0.26	1.31 ± 0.54	7.30 ± 0.23	2.44 ± 0.10
	100	18.2	18.0 ± 0.5	7.79 ± 0.32	1.89 ± 0.54	6.98 ± 0.24	2.50 ± 0.11
	140	19.1	19.0 ± 0.7	8.50 ± 0.28	2.39 ± 0.43	7.61 ± 0.28	2.43 ± 0.09
	175	19.6	19.2 ± 0.4	8.52 ± 0.16	2.20 ± 0.28	7.63 ± 0.19	2.42 ± 0.06
K^-p	50	21.0	21.1 ± 0.7	8.73 ± 0.34	2.30 ± 0.61	8.28 ± 0.43	2.60 ± 0.11
	70	21.0	19.9 ± 0.7	8.11 ± 0.59	1.05 ± 1.63	7.52 ± 0.48	2.55 ± 0.19
	100	21.3	21.0 ± 0.7	9.16 ± 0.40	3.20 ± 0.71	7.48 ± 0.35	2.54 ± 0.12
	140	21.6	21.8 ± 0.5	9.03 ± 0.27	2.49 ± 0.58	8.03 ± 0.18	2.61 ± 0.09
	175	21.9	22.2 ± 0.7	9.46 ± 0.31	3.00 ± 0.63	8.37 ± 0.27	2.55 ± 0.09
pp	50	76.2	75.3 ± 0.7	10.28 ± 0.11	1.42 ± 0.23	9.67 ± 0.16	7.55 ± 0.14
	70	75.8	72.9 ± 1.2	10.73 ± 0.17	1.91 ± 0.30	10.03 ± 0.17	7.05 ± 0.15
	100	76.0	75.6 ± 1.8	10.95 ± 0.21	2.05 ± 0.36	10.19 ± 0.18	7.17 ± 0.17
	140	76.2	75.5 ± 1.4	11.30 ± 0.13	2.49 ± 0.20	10.37 ± 0.17	6.99 ± 0.12
	175	76.5	72.1 ± 1.5	10.95 ± 0.16	2.31 ± 0.29	10.00 ± 0.16	6.88 ± 0.14
$\bar{p}p$	50	98.3	99.5 ± 1.9	12.66 ± 0.25	3.13 ± 0.51	11.82 ± 0.31	8.22 ± 0.19
	70	94.0	88.0 ± 1.7	12.47 ± 0.38	2.43 ± 1.19	11.51 ± 0.33	7.31 ± 0.24
	100	90.3	92.0 ± 2.4	12.39 ± 0.34	3.97 ± 0.67	10.91 ± 0.36	7.90 ± 0.24
	140	88.9	88.3 ± 2.2	12.57 ± 0.40	3.60 ± 0.88	10.99 ± 0.28	7.41 ± 0.25
	175	88.4	85.4 ± 3.0	13.19 ± 0.43	4.38 ± 1.04	11.30 ± 0.28	6.87 ± 0.24

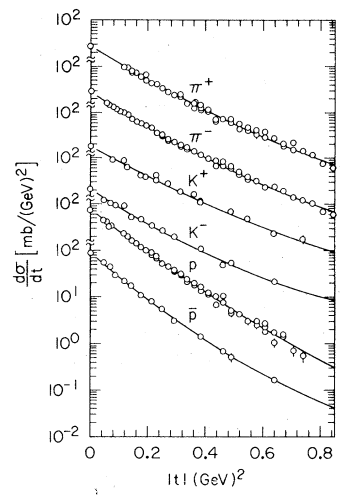

FIG. 2. Differential cross sections for elastic scattering at 100 GeV/c. The optical-theorem point is shown at $t=0$. The error bars for most points fall within the symbols. The curves are best fits by Eq. (1) with the parameters listed in Table I.

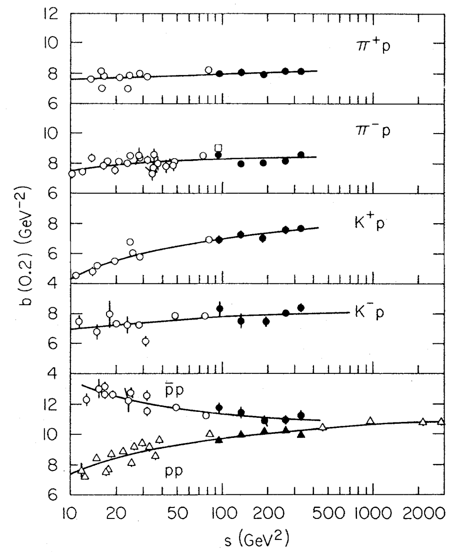

FIG. 3. Logarithmic slopes at $|t|=0.2$ GeV^2 as a function of s, the total energy squared in the center of mass. The lines are drawn only to guide the eye. Data sources: □, Ref. 6; ○,△, Ref. 7; ●,▲, this experiment.

and a typical set of fits is shown in Fig. 2. Table I lists the parameters of these fits. Our results at 50 GeV/c are in good agreement with those from Derevchekov *et al.*[6] and Antipov *et al.* and Nurushev[7] at 40 and 50 GeV/c, and with the optical points derived from total-cross-section measurements.[8]

Table I also includes the logarithmic slope parameters defined as

$$b(|t|)=\frac{d}{dt}\left(\ln\frac{d\sigma}{dt}\right)=B-2C|t|, \qquad (2)$$

evaluated at 0.2 GeV^2. Figure 3 shows that our values for $b(0.2)$ connect smoothly to previous results[6,7] at other energies. While the data for all reactions studied are well described by the fits with Eq. (1), other forms such as piecewise linear exponentials, as suggested by Carrigan[9] and Amaldi *et al.* and Barbiellini *et al.*,[10] are also possible. The values of C listed in Table I indicate a decrease in the logarithmic slope $\Delta b = 2C\Delta t \approx 2$ GeV^{-2} in going from $|t|=0.1$ to 0.5 GeV^2.

The crossover points, where particle and antiparticle have equal cross sections, were calculated from the fits to Eq. (1). No significant energy dependence was observed. Averaging over the five incident momenta and including systematic errors gave $|t|=0.17\pm0.04$ and 0.12 ± 0.02 GeV^2 for the kaon and proton crossovers, respectively. These results are similar to the values of 0.19 and 0.16 GeV^2 previously observed[11] in the region 3 to 6 GeV/c.

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Further Observation of Dimuon Production by Neutrinos*

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Using a quadrupole focused neutrino beam, 61 events with two muons in the final state have been observed at Fermilab. These include seven $\mu^-\mu^-$ events. A comparison of the event rate in two targets of different hadron absorption length indicates that attributing the events to π or K leptonic decay is ruled out by 4.0 standard deviations. No trimuon events were observed which, combined with lepton conservation, indicates an unobserved neutral lepton is present in most of the events.

We have reported previously evidence for dimuon production by neutrinos.[1,2] In order to confirm the existence of dimuon events a new experiment was carried out and the results are reported here.[3]

The calorimeter–magnetic spectrometer[4] was exposed to a very-high-energy neutrino beam obtained from quadrupole focusing of the parent hadrons. The quadrupole triplet was set to focus 200-GeV charged hadrons and the primary proton energy was 380 GeV. This beam is predominantly composed of neutrinos with a small admixture of antineutrinos ($\sim \frac{1}{7}$). A beam spill of ~ 1 msec was used to minimize accidental coincidences. Events were detected in two separate targets, the liquid scintillation calorimeter and a large block of iron adjacent to the calorimeter (hadron filter). The trigger requirement for all events was either a single muon that penetrated the entire magnetic spectrometer or an energy deposition in the calorimeter.

A total of 114 dimuon candidates were observed and the events were distributed as 58 and 56 for the iron and liquid targets, respectively. In about 30% of the events one of the muons failed reconstruction because of chamber topology. The final number of reconstructed events from each target was 41 and 36, respectively. No events with three muons were observed.

The momentum and angle of each muon was measured and extrapolated back into the appropriate target. The distance (Δ) between the extrapolated rays at the interaction point, determined by appropriate counters, is shown in Fig. 1(a). Events with $\Delta < 50$ cm were accepted in the sample provided they passed additional requirements such as correct timing and correct position of the muon tracks in a sixteen-element hodoscope located in the magnetic spectrometer. Figure 1(b) shows the resulting z (along the beam direction) distribution for the events. After applying a fiducial volume cut to the data, we re-