Decay Modes of the Meson $\eta'(958)$ and Chiral-Symmetry Breaking

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Using the simple model that the amplitude for $\eta' \to \eta\pi\pi$ is dominated by the $\delta(970)$ resonance we find a satisfactory fit to the data. The quadratic mass formula for η-η' mixing with a positive mixing angle (i.e. reduction of $s\bar{s}$ content in the physical η state) is preferred in agreement with the requirement for $\eta' \to 2\gamma$. Failure of the current-algebra calculation for $\eta' \to \eta\pi\pi$ is explained and is shown *not* to be evidence against the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ model of chiral-symmetry breaking.

Considerable interest attaches to the study of the properties[1] of the meson $\eta'(958)$ discovered in 1964.[2] Experimentally the Dalitz-plot analyses[3] of the decays $\eta' \to \eta\pi\pi$ and $\eta' \to \pi^+\pi^-\gamma$ favor the assignment $J^P = 0^-$ but cannot rule out[4] spin 2. Further, although the branching ratios for the three principal modes[1] $\eta' \to \gamma\gamma$ [$(1.9 \pm 0.3)\%$], $\eta' \to \pi^+\pi^-\gamma$ [$(27.4 \pm 2.2)\%$], and $\eta' \to \eta\pi\pi$ [$(70.6 \pm 2.5)\%$] are known, at present only an upper limit of $\Gamma_{\rm tot} < 0.8$ MeV is given for the total width.[5] Theoretically a great deal of effort has gone into the study[6] of the mode $\eta\pi\pi$, for it is supposed to be a testing ground for our ideas on chiral-symmetry breaking.[7] Doubts have been cast on the validity of the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ model of Gell-Mann, Oakes, and Renner[8] and of Glashow and Weinberg[9] as it gives too low a value[10] for the width, ≈ 2 keV. This has led to the introduction of complicated schemes of chiral-symmetry breaking[6] or abandoning the assignment of $\eta'(958)$ to the pseudoscalar nonet. It is not infrequently then that one sees in the literature the assignment of $E(1420)$ to the nonet.[11] The process $\eta' \to 2\gamma$ is related[12] to $\eta \to 2\gamma$ and $\pi^0 \to 2\gamma$ by SU(3) symmetry and its value is sensitively dependent on the sign of the mixing angle between η and η'. The upper limit of 0.8 MeV on the total width of η' favors the positive sign (i.e. the one that reduces the $s\bar{s}$ content of the physical η state). This is also the sign favored by our model for $\eta' \to \eta\pi\pi$. We first describe the model and the results. Its relation to chiral-symmetry considerations is discussed later.

It is known experimentally[1] that there is an $I = 1$ $(\eta\pi)$ resonance with a mass 970 MeV and a full width of approximately 50 MeV. Since $J^P = 0^+$ is the preferred spin-parity assignment we regard δ as a scalar meson. We take the amplitude for $\eta' \to \eta\pi\pi$ to be dominated by $\eta' \to \delta\pi$ followed by $\delta \to \eta\pi$. The matrix element for $\eta'(P) \to \eta(q) + \pi^+(k_1) + \pi^-(k_2)$ is then written as

$$M = g_{\delta\eta'\pi} g_{\delta\eta\pi} \left[\frac{1}{(P-k_1)^2 - m_\delta^2 - i m_\delta \Gamma_\delta} + \frac{1}{(P-k_2)^2 - m_\delta^2 - i m_\delta \Gamma_\delta} \right], \quad (1)$$

where P, q, k_1, and k_2 are the momenta of the particles as indicated and we take $m_\delta = 970$ and Γ_δ to be 50 MeV. The constant $g_{\delta\eta\pi}$ is readily fixed from the partial width of $\delta \to \eta\pi$,

$$\Gamma(\delta \to \eta\pi) = (g_{\delta\eta\pi}^2/4\pi) k / 2m_\delta^2. \quad (2)$$

The constant $g_{\delta\eta'\pi}$ is related to $g_{\delta\eta\pi}$ by Zweig's rule[13]; i.e., we assume that the $s\bar{s}$ part of the η' and η wave functions does not contribute to the transition to $\delta\pi$ (which has only u and d quarks). We take the bare states[14] to be $\eta_8 = (1/\sqrt{6})(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $\eta_1 = (1/\sqrt{3})(u\bar{u} + d\bar{d} + s\bar{s})$ with the physical states $\eta = \cos\theta\, \eta_8 + \sin\theta\, \eta_1$ and $\eta' = -\sin\theta\, \eta_8 + \cos\theta\, \eta_1$. This gives

$$g_{\delta\eta'\pi}/g_{\delta\eta\pi} = (\sqrt{2} - \tan\theta)/(1 + \sqrt{2}\tan\theta). \quad (3)$$

From the experimental masses of π, K, η, and η' we get[15] $|\theta| = 10°$ for the quadratic mass formula and $|\theta| = 24°$ for the linear mass formula. A positive sign for the mixing angle means a reduction in the $s\bar{s}$ content of η as compared to the bare state. We have computed the dacay rate by using (1)-(3) as a function of $\Gamma(\delta \to \eta\pi)$ and for $\theta = \pm 10°$ and $\pm 24°$ and the results are displayed in Table I. For comparison we have also given the $\eta' \to 2\gamma$ rate as computed[17] from the $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$ rates,

$$\Gamma(\eta' \to 2\gamma) = \frac{m_{\eta'}^3}{\sin^2\theta} \left[\left(\frac{\Gamma(\pi^0 \to 2\gamma)}{3m_\pi^3} \right)^{1/2} \mp \cos\theta \left(\frac{\Gamma(\eta \to 2\gamma)}{m_\eta^3} \right)^{1/2} \right]^2,$$

where the minus sign is to be taken for positive

TABLE I. Computed values of $\Gamma(\eta' \to \eta\pi\pi)$ for various values of the partial width of $\delta \to \eta\pi$. Part of the table for $\eta' \to 2\gamma$ is reproduced from Ref. 12.

Decay mode	Input	Γ_{theory} $\theta = 10°$	$\theta = 24°$	$\theta = -10°$	$\theta = -24°$	Γ_{exp} [a]
$\eta' \to \eta\pi\pi$	$\Gamma(\delta \to \eta\pi) = 25$ MeV	0.28 MeV	0.1 MeV	1.28 MeV	7.17 MeV	<0.59 MeV
	$\Gamma(\delta \to \eta\pi) = 30$ MeV	0.4 MeV	0.15 MeV	1.84 MeV	10.32 MeV	
	$\Gamma(\delta \to \eta\pi) = 35$ MeV	0.56 MeV	0.2 MeV	2.51 MeV	14.06 MeV	
	$\Gamma(\delta \to \eta\pi) = 40$ MeV	0.72 MeV	0.26 MeV	3.27 MeV	18.3 MeV	
$\eta' \to 2\gamma$	$\Gamma(\pi^0 \to 2\gamma) = 7.3 \pm 0.8$ eV[b] $\Gamma(\eta \to 2\gamma) = 324 \pm 46$ eV	4.0 ± 2.4 keV	450 ± 35 eV	150 ± 20 keV	28 ± 4 keV	<17 keV

[a]Refs. 1 and 5. [b]Ref. 16.

mixing angle and the plus sign for negative mixing angle. It will be seen that the negative mixing angle is definitely excluded. Our calculations for $\eta' \to \eta\pi\pi$ are then an additional independent confirmation of this result. Although the precise value of the partial width $\Gamma(\delta \to \eta\pi)$ has not yet been measured it is known to be the dominant decay mode.[1] It seems reasonable therefore to assume that it may be between[18] 25 and 40 MeV, so that the quadratic mass formula which we prefer gives $\Gamma(\eta' \to \eta\pi\pi)$ between 280 and 720 keV. This is compatible with the experimental branching ratio $(\eta' \to \eta\pi\pi)/(\eta' \to 2\gamma) = [(70.6 \pm 2.5)\%]/[(1.9 \pm 0.3)\%]$ and the width predicted for $\eta' \to 2\gamma$ of ≈ 9 keV from quark-model calculations[12] or within a factor of 2 if we use the SU(3) prediction of 4 ± 2.4 keV[12] given in Table I.

An independent test of our model is provided by the study of decay distributions. In Fig. 1 we have plotted the mass spectrum of the pion pairs and compared it with the data of Danburg *et al.*[3] There are two sets of events, $\eta' \to \pi^+\pi^-\eta_N$ and $\eta' \to \pi^+\pi^-\eta_C$, where the subscripts N and C refer to the η decay modes $\eta \to$ neutrals and $\eta \to \pi^+\pi^-\pi^0$, $\pi^+\pi^-\gamma$, respectively. A complication arises from the fact that a substantial fraction of the events from $\eta' \to \pi^0\pi^0\eta_C$ are included in $\eta' \to \pi^+\pi^-\eta_N$. Figure 1(b) corresponds to the subtracted data when this is taken into account while Fig. 1(a) gives the combined data.

Understanding the ratio $(\eta' \to \rho\gamma)/(\eta' \to 2\gamma)$ causes no problem as usual vector-dominance arguments adequately account for its value.

Current algebra for $\eta' \to \eta\pi\pi$.—Consider the matrix element (1) and take the soft-pion limit, say $k_{1\mu} \to 0$ in the first term corresponding to the $\delta^-(970)$ pole. Partial conservation of axial-vector current demands that the $\pi\delta\eta'$ vertex must

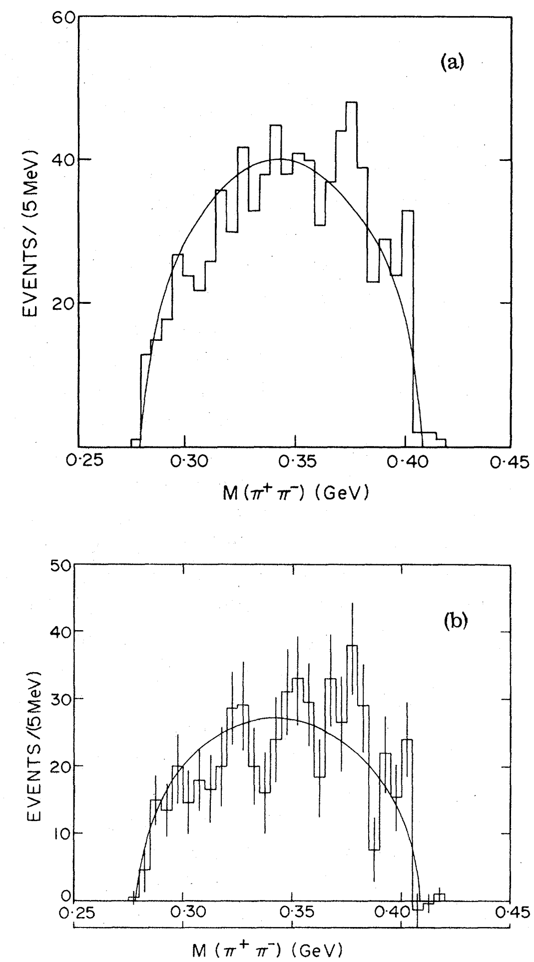

FIG. 1. (a) Mass spectrum of the pion pairs. Histogram is the combined data (Ref. 3) of $\eta' \to \pi^+\pi^-\eta_N$ and $\eta' \to \pi^+\pi^-\eta_C$. The curve is the spectrum calculated from the matrix element, Eq. (1). (b) Mass spectrum of the pion pairs. Subtracted data (cf. text and Ref. 3). The curve is the spectrum calculated from the matrix element, Eq. (1).

vanish as the four-vector $k_{1\mu}$ goes to 0, so that we can write the matrix element as

$$\lim_{k_{1\mu}\to 0} \frac{(P+P-k_1)\cdot k_1}{m_\delta^2 - m_{\eta'}^2 - k_1^2 + 2P\cdot k_1 + i m_\delta \Gamma_\delta} \frac{g_{\delta\eta\pi} g_{\delta\eta'}{}^A}{F_\pi},$$

where

$$(2\pi)^3 (4q_{10}q_{20})^{1/2} \langle \delta, q_1 | A_\mu | \eta', q_2 \rangle = g_{\delta\eta'}{}^A (q_1 + q_2)_\mu + \dots$$

defines the axial-vector coupling constant $g_{\delta\eta'}{}^A$ and F_π is the pion decay constant. Now let $\vec{k}_1 = 0$ and insert the values of m_δ, $m_{\eta'}$, and Γ_δ to get

$$\text{const}[k_{10}/(12 + k_{10} + i25)]$$

which is zero for $k_{10} = 0$ as demanded by the Adler condition but is almost unity at the edge of the Dalitz plot $k_{10} = m_\pi = 140$. In other words the proximity in mass of δ(970) and η′(958) makes the matrix element vary rapidly around the soft-pion limit. We cannot therefore find the physical-region matrix element by smooth extrapolation from the Adler point. The calculations in Ref. 6 are based on linear extrapolation. We must then disregard the criticism against the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ model for chiral-symmetry breaking.

Our conclusions are as follows: (1) η′ is the ninth member of the pseudoscalar nonet. (2) The mixing angle is positive and the quadratic mass formula is preferred. (3) If the total width of η′ turns out to be a few hundred keV as indicated by our model, it will mean that there are no violent departures from SU(3) symmetry even when it is used for dimensional coupling constants. (4) There is no evidence against the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ model of chiral-symmetry breaking from η′ decays.

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[13]G. Zweig, unpublished; cf. Ref. 12 for a discussion.

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[18]The calculated value of the decay rate $\eta' \to \eta\pi\pi$ is sensitive to variations in m_δ and Γ_δ (total) by 10 MeV.

Measurement of Elastic Scattering of Hadrons on Protons from 50 to 175 GeV/c*

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Differential cross sections have been measured at Fermilab with a focusing spectrometer for $\pi^\pm p$, $K^\pm p$, and $p^\pm p$ elastic scattering at 50-, 70-, 100-, 140-, and 175-GeV/c incident momentum over the $|t|$ range 0.03 to 0.8 GeV^2. The results are smooth in t and are parametrized by quadratic exponential fits.

This paper describes measurements of $\pi^\pm p$, $K^\pm p$, and $p^\pm p$ elastic scattering made at the Fermi National Accelerator Laboratory. The reactions were studied for momentum transfers $|t|$ from 0.03 to 0.08 GeV^2 and incident momenta of 50, 70, 100, 140, and 175 GeV/c, with typically