(11) and (12)]. Notice that when $E=0$ in (2) the evolution equation (4) contains one derivative in y, whereas if $E\neq 0$ two derivatives in y appear in the evolution equation [see (7) and (10)]. Higher derivatives work in a similar way. This is analogous to the role of the dispersion relation in the one-dimensional problem.[6]

Note added.—We have become aware that Zakharov and Shabat[12] have obtained, by a different approach, the two- (spatial) dimensional three-wave interaction equations and Dryuma's result in Ref. 8.

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Theories of Gravity with Structure-Dependent γ's

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A gauge theory for the Lorentz connection of a space-time describes geometries of stars with variable parametrized post-Newtonian parameter γ; its value depends explicitly on the structure of a star: The value of γ for a neutron star is different from that for the solar system ($|\gamma_\odot - 1| \leq 10^{-2}$). Because of the absence of a "local parity invariance" γ is positive for ordinary stars and γ is negative for sources of gravity consisting of antimatter.

Several recent Letters[1-3] present various aspects of two-parameter static and spherically symmetric space-time geometries, generated by Yang's gravitational field equations $R_{ab;c}-R_{ac;b}=0$.[4] The geometric background of these field equations has partially been discussed by Kilmister and Newman,[5] they have been used by Lichnerowicz[6] in his quantization of the gravitational field, and they appear in Bel's investigation of the super-energy-momentum tensor of the gravitational field[7]; the Lagrangian for Yang's equations has been noted by Eddington[8] as an alternate choice for gravitation. The space-time connections discussed in Refs. 1–3 may be classified by means of the family of vacuum solutions of the curvature dynamical equations,[9,10] which are the natural gauge equations of second order for the connection of the Lorentz-frame bundle of a static and spherically symmetric space-time (here given in terms of Schwarzschild's coordinates, $g_{00}=e^{2\mu}$, $g_{11}=e^{-2\lambda}\equiv\Delta^{-2}$),

$$r^{-2}(r^2f')'-2r^{-2}f = -e^{-1\lambda}f'\chi(\Delta',f)+\kappa j^{(1)0}e^{2(\mu-\lambda)}, \tag{1}$$

$$\tfrac{1}{2}\Delta^{2\prime\prime}-r^{-2}(\Delta^2-1) = -e^{-\mu}f\chi(\Delta',f)+\kappa e^{-\lambda}r^2S^{(3)2}, \tag{2}$$

$$\chi(\Delta',f)\equiv\Delta'-e^{-\mu}f. \tag{3}$$

$f=(e^\mu)'\Delta$ represents the gravitational force measured in the static frame of reference; $j^{(1)0}(r)$ and $S^{(3)2}(r)$ are the only nonvanishing components of the external Lorentz current $\mathcal{J}^Q$ which couples matter to geometry. Any static and spherically symmetric connection which is regular in the asymptotic region and a solution of Eqs. (1) and (2) with $j^{(1)0}=0$ and $S^{(3)2}=0$ has the following form parametrized by the corresponding mass M, parametrized post-Newtonian parameter $\gamma \in R$, and parity parameter $\beta_0=\pm 1$:

$$e^{2\mu}=1-2x+\sum_{n=2}^{\infty} A_n(\gamma,\beta_0)x^n, \quad x\equiv M/r, \tag{4}$$

$$\Delta^2=1-2\beta_0\gamma x+\sum_{n=2}^{\infty} R_n(\gamma,\beta_0)x^n, \tag{5}$$

$$f=\beta_0(M/r^2)N(x;\gamma,\beta_0), \tag{6}$$

$$N=1-\tfrac{1}{2}(1-\beta_0\gamma)x+\sum_{n=2}^{\infty} Q_n(\gamma,\beta_0)x^n. \tag{7}$$

The coefficients A_n, R_n, and Q_n are polynomials of order n in γ; they all have a zero at $\beta_0\gamma=1$; i.e., $\beta_0=+1$ and $\gamma=1$ reproduce the exterior Schwarzschild connection.[11] The expansion (4)–(7) is only valid in the asymptotic region defined by $x<\epsilon$, where $\epsilon \simeq 1$. The usual parametrized post-Newtonian parameter γ, β, and λ_p,[12] which characterize the light deflection $\theta=\theta_E(\gamma+1)/2$, the nonlinearity in the gravitational potential, and the perihelion shift $\delta\Phi=(\delta\Phi)_E\lambda_p$,[13] are completely determined by γ itself,

$$\beta=\beta(\gamma)=\tfrac{1}{4}(3+\gamma), \quad \gamma\in R, \tag{8}$$

$$\lambda_p=\lambda_p(\gamma)=\tfrac{1}{12}(5+7\gamma). \tag{9}$$

$\gamma=-1$ [which corresponds to solution (2) in Ref. 3] would produce a vanishing light deflection, and $\gamma<-\frac{5}{7}$ a retrograde perihelion shift; in particular $\gamma=-1$ generates a retrograde perihelion shift of $\frac{1}{6}$ the Einsteinian value.[3] Geometries with $\beta_0=+1$ are expressed in terms of a right-handed tetrad, and those having $\beta_0=-1$ in terms of a left-handed one; if $\gamma>0$, the corresponding geometries (4)–(7) have positively oriented 2-spheres (t = const, r = const), and for $\gamma<0$ we obtain negatively oriented 2-spheres. Finally, a theory of gravity described by Eqs. (1) and (2) is no longer β_0 invariant (i.e., not "locally parity invariant"); however it is invariant under the combined transformation $\beta_0 \to -\beta_0$ and $\gamma \to -\gamma$.[10] A further distinction to the Schwarzschild geometry is important: The geometries (4)–(7) are not able to form a regular event horizon unless $\gamma=1$ and $\beta_0=+1$; if $r\to r_0$, where $e^{2\mu}(r_0)=0$ and $r_0\simeq 2M$, infinite tidal forces would arise[11]; in other words, the Schwarzschild geometry cannot be continuously embedded into the family (4)–(7).

At various places[1-3] it has been pointed out that most of the geometries (4)–(7) are unphysical since their γ's are in harsh contrast to the values of γ measured in the solar system. However, theories of the type (1) and (2) characterize the state of static sources of gravity by means of *two* parameters: their mass M and their structure parameter γ; for curvature dynamical theories of gravity γ is a source-related parameter, while in any theory of the Einsteinian type γ appears definitely fixed by coupling constants. If, therefore, a curvature dynamical theory of gravity has to be viable, it must predict for the solar system $\gamma_\odot=1-\epsilon_\odot$ with $|\epsilon_\odot|\lesssim 10^{-2}$; in general, however, different types of stars have different γ's, and so, e.g., $\gamma_{n\,\mathrm{star}}\neq\gamma_\odot$. It is therefore of primary experimental importance to measure the values of γ for different types of sources of gravity, particularly for neutron stars. Quite general physical arguments show[14] that γ depends on the mass-to-radius ratio (M/R) of a star; it was found that the relation $\gamma=1-\epsilon$ with $\epsilon\simeq M/R$ (and $\epsilon_\odot=2\times 10^{-6}$!). For stars consisting of normal matter γ would always be positive ($\gamma_{n\,\mathrm{star}}\simeq 0.8$); whether geometries with negative γ's might be attributed to stars consisting of antimatter is an open question, but seems, however, to be a very promising interpretation. Stars consisting of antimatter would then generate a retrograde periastron shift.

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