*Present and permanent address.
[1]A. J. Leggett and S. Takagi, Phys. Rev. Lett. 34, 1424 (1975).
[2]R. Combescot and H. Ebisawa, Phys. Rev. Lett. 33, 810 (1974).
[3]V. Ambegaokar, in *Quantum Statistics and the Many-Body Problem*, edited by S. B. Trickey *et al.* (Plenum, New York, 1975).
[4]R. Combescot, Phys. Rev. Lett. 35, 471 (1975).
[5]P. Bhattacharyya, C. J. Pethick, and H. Smith, Phys. Rev. Lett. 35, 473 (1975).
[6]H. M. Bozler, M. E. R. Bernier, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 32, 875 (1974).
[7]D. D. Osheroff, Phys. Rev. Lett. 33, 1009 (1974).
[8]D. D. Osheroff and W. F. Brinkman, Phys. Rev. Lett. 32, 584 (1974).
[9]K. Maki and H. Ebisawa, to be published.
[10]J. C. Wheatley, private communication.
[11]W. F. Brinkman, Phys. Lett. 49A, 411 (1974).
[12]K. Maki and C.-R. Hu, J. Low Temp. Phys. 18, 377 (1975).
[13]R. A. Webb, R. L. Kleinberg, and J. C. Wheatley, Phys. Rev. Lett. 33, 145 (1974).
[14]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).
[15]After this work was completed I received a preprint by K. Maki and H. Ebisawa which, *inter alia*, gives a more rigorous discussion of the decay of the wall-pinned mode. They use a quite general phenomenological description of the relaxation: To interpret this along the lines of Ref. 1 it seems necessary to take their τ equal to $(\lambda^{-1}-1)(\chi/\chi_0)\tau_{LT}$ [not, as they state, $(\lambda^{-1}-1)\tau_{LT}$]. If this is done their results agree with the ones given here.
[16]R. A. Webb, R. E. Sager, and J. C. Wheatley, this issue [Phys. Rev. Lett. 35, 1164 (1975)].
[17]A. J. Leggett, Ann. Phys. (N.Y.) 85, 11 (1974).
[18]A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

Positron Lifetimes in Metals*

Werner Brandt and Diane Isaacson
Department of Physics, New York University, New York, New York 10003
and
Robert Paulin
Institut National des Sciences et Techniques Nucléaires, Saclay, France
(Received 2 July 1975)

The positron lifetimes in annealed metals reported recently by MacKenzie, Jackman, and Thrane range from 0.1 to 0.4 nsec and exhibit systematic trends with the atomic number. These observations are shown to be related predominantly to the dynamic screening of the positron charge by the metal electron gas.

Based on a new series of remarkable centroid-shift measurements of the positron annihilation-time spectra in annealed metals, MacKenzie, Jackman, and Thrane[1] reported mean lifetmes that range from 106 psec in Fe to 415 psec in Cs. The data reproduced in the upper graph of Fig. 1 exhibit a systematic dependence on the atomic number of the metal. The trends could be indicative of the overlap of the positron wave function with the core electrons although, as the authors[1] point out, the constant lifetime in the rare earths suggests that the positron is, in fact, insensitive to the filling of the inner shell.

The theory of positron annihilation in metals asserts that the lifetime is determined predominantly by the density the electron gas acquires at the site of the positron between the ion cores in screening the positron charge, and only to a minor extent by the overlap with the ion cores. We wish to demonstrate that the data reported by MacKenzie, Jackman, and Thrane agree with this interpretation. Lifetimes can become as low as 90 psec in some fully annealed metals, because the electron density available for annihilation at the site of the positron through positron-electron correlation is dictated by the effective number of electrons per atom that participate in the dominant plasmon mode through the dynamic interaction of the valence electron gas with the ion cores. The parabolic part of the angular correlation between the two annihilation-gamma quanta, of course, is linked to the valency of the metal.

The collective effects on positron lifetimes can be large. For example, in the noble metals some eight electrons per atom participate to give a plasmon resonance energy $\hbar\omega_p = 25$ eV from which one predicts a positron lifetime of 90 psec,

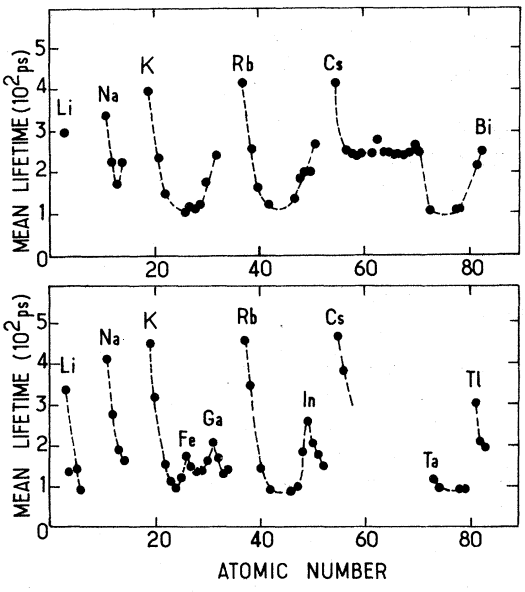

FIG. 1. Mean positron lifetimes in metals as a function of their atomic number. Upper graph: experimental values given by MacKenzie, Jackman, and Thrane (Ref. 1). Lower graph: values calculated according to Eqs. (1) and (2) from metal-plasmon excitation frequencies. The dashed curves are drawn to guide the eye.

whereas for a valency 1 the lifetime should be 300 psec. The practical import for the selection of envelope materials in the preparation of positron sources for precision lifetime measurements is evident.

Mean lifetimes, τ, were calculated by approximating the effect of positron-electron correlation with the density enhancement factor[2] $h(r_s) = 1 + \frac{1}{6}(r_s^3 + 10)$, which represents very closely the most recent many-body calculations[3,4] in the r_s range of interest. The symbol r_s denotes the Wigner-Seitz radius per electron, in units of $a_0 = \hbar^2/me^2$, in the responding electron gas of density ρ such that $\frac{4}{3}\pi r_s^3 a_0^3 \rho = 1$. Then

$$\tau = \tfrac{1}{12} r_s^3 [1 + \tfrac{1}{6}(r_s^3 + 10)]^{-1} \simeq [2 + 134\rho a_0^3]^{-1} \text{ nsec}, \quad (1)$$

where $\frac{1}{12} r_s^3$ nsec is the Sommerfeld approximation. We insert the effective values of ρ, or r_s, determined from experimental plasma frequencies,[5-7] $\omega_p = (4\pi e^2 \rho/m)^{1/2}$, by the relations

$$\hbar\omega_p = (96.4 \text{ eV})(\rho a_0^3)^{1/2} = (47.1 \text{ eV}) r_s^{-3/2}, \quad (2)$$

as listed in a compilation of available data.[8]

The results given in the lower portion of Fig. 1 clearly duplicate the striking features of the experimental data. Additional details not displayed by the positron lifetimes, particularly in transition metals, may fall within the uncertainities of the proper choice of ω_p which enters Eq. (1) as ω_p^2. Such materials usually have no unique resonance frequency of magnitude close to that expected for the valence-electron-gas plasma. Rather, the valence-electron plasma interacts strongly with single-particle core transitions, resulting in a band of collective frequencies higher than the plasma frequency of the valence-electron gas. Small differences remain also for metals with weak coupling between the conduction electrons and the ion cores, such as the alkali metals. Inasmuch as the positron wave function overlaps with the ion cores, Eq. (1) gives an upper bound for the lifetime.

In summary, the dependence of positron lifetimes in metals on atomic number is linked *prima facie* to the virtual excitation of coupled valence-electron-plasma and single-particle modes in the collective response of the metal electrons to screen the positron charge.

*Work supported by the National Science Foundation and the French Atomic Energy Commission.

[1]I. K. MacKenzie, T. E. Jackman, and N. Thrane, Phys. Rev. Lett. 34, 512 (1975).

[2]W. Brandt and J. Reinheimer, Phys. Lett. 35A, 109 (1971).

[3]P. Bhattacharyya and K. S. Singwi, Phys. Rev. Lett. 29, 22 (1972).

[4]D. N. Lowy and A. D. Jackson, Phys. Rev. B 12, 1689 (1975).

[5]J. Daniels, C. V. Festenberg, H. Raether, and K. Zeppenfeld, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1970), Vol. 54, p. 77.

[6]M. Glicksman, Solid State Phys. 26, 275 (1971).

[7]F. C. Brown, Solid State Phys. 29, 1 (1974).

[8]D. Isaacson, *Compilation of r_s Values*, National Auxiliary Publication Service Document No. 02698, to be ordered from ASIS NAPS, c/o Microfiche Publications, 305 E. 46th St., New York, N. Y. 10017, remitting $1.50 for microfiche or $5.00 for photocopy up to 30 pages and $0.15 for each additional page over 30. Estimate 10 pages.