ed essentially by our present power supply capabilities.

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Model for Conversion of Intense-Electron-Beam Energy into Radially Converging Ion Fluxes*

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The pinch phenomenon in relativistic diodes is used with proper design of the anode surface to construct a theoretical scheme for ion acceleration. We analyze the early-time formation of the ion source, and give the approximate solutions for the steady state of ion and electron flow. Efficient conversion (> 50%) of electron current into ion current is predicted. The energetic ions are accelerated radially inwards in spherical geometry. Application to the generation of intense neutron pulses and pellet compression is discussed.

The concentration of relativistic electron currents by self-pinching in high-current diodes is a well-known phenomenon. Current densities upwards of 10^6 A/cm^2 have been reported.[1] The application of such current densities to pellet fusion is being actively studied.[2] However, the long electron range at the energies envisioned presents a drawback to deposition of energy and momentum in a target by the electrons directly. The higher energy densities available correlate with higher voltages, for which this drawback is more severe. For this reason we have considered the transformation of the electron energy into an imploding pulse of ions, for which the range and momentum transfer considerations are much more favorable.

Theoretical studies have been made for electron flow in relativistic diodes of arbitrary aspect ratio and indicate the relative importance of ion effects on the electron flow.[3] Here we alter the diode geometry to allow for efficient production of intense converging ion flow inside a spherical anode mesh. We show that efficient conversion of the incident electron energy into ion energy is made possible by electron space charge in the vacuum between the anode mesh and an absorber placed at the center of the sphere. In this geometry, fusion reactions between the accelerated ions (e.g., deuterons) and the target may be of significant interest.

One diode geometry which has azimuthal symmetry is that of an anode plane between two hollow cathodes. The spherical anode mesh replaces the central portion of the anode plane, as shown in Fig. 1. Regions I and II are idealized as initially vacuum regions. The target III is assumed floating. In the analysis that follows we treat the case of a flow of electrons radially inward with spherical symmetry. Recent experimental work[4] with a simple small spherical obstacle centered on the anode plane indicates quite uniform electron fluence over the surface of the sphere. We consider a spherical anode made of a mesh with a small fraction of the area covered by the wires

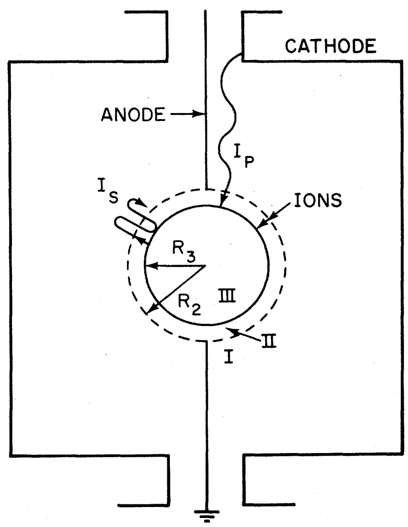

FIG. 1. Diode geometry consists of a spherical anode mesh (R_2) with the target (III) floating inside it. The two cylindrical hollow cathodes are the source of the primary electrons (I_P). Ions are emitted from the plasma mesh and accelerated inwards by the electron-rich cloud of the secondary electrons (I_S) which are reflected many times.

(fraction $P \ll 1$). In this case electrons from the cathode C pass through the spherical anode mesh, create a space charge in region II, and initially are reflected back into region I. Because of the electric and magnetic fields in region I, these electrons are reflected again into region II, and thus make several passes through the mesh. We neglect losses to the planar anode portion, on the basis of the strong magnetic confinement of the electron orbits. The anode mesh is rapidly ionized and is then free to emit ions into region II in a field-emission process which quickly becomes space-charge limited, i.e., the electric field at the surface vanishes: $E(R_2)=0$. The ions are accelerated inwards by the space-charge field of the electrons.

The initial time-dependent dynamics can be solved analytically but are of less interest here. The intermediate-time history (a few ion crossing times for region II) is a problem for computation. In what follows we treat the steady-state flow of ions and electrons self-consistently. In the steady state the primary electrons are only slowed down in region II and reach the abosrber III with a small fraction of their initial energy. The ions are accelerated by the electric field in region II that originates from the electron-rich cloud near the absorber. The net current into region III is zero because additional space-charge accumulation reduces the arrival of one or the other species; the charging process is thus self-adjusting.

On the surface of region III the electric field is zero because of plasma formation. This boundary emits electrons into region II. These "secondary" electrons cross the anode mesh with multiple reflections back from region I and simply add to the population of electrons in region II.[4] We solve Poisson's equation assuming that all charge flow is spherically symmetric and only electric fields exist in region II with the boundary conditions

$$E(R_2)=E(R_3)=0. \tag{1}$$

No magnetic fields are generated in region II because of spherical symmetry and zero total current. The total ion current as a fraction of the incident electron current, I_P, is computed.

The work reported here differs from current work by several other groups[5-7] in the following important respects: (1) Because no axial magnetic field is used in the electron-beam diode, focusing of the electrons is achieved, and hence current densities obtainable are orders of magnitude larger. In addition the geometry in this Letter is such that the ion flux is focused naturally and used immediately on the target. (2) In the usual "reflex triode," reflection of the primary electrons reduces the electron current significantly by space-charge effects; in the present work, the emission area is effectively isolated from the space-charge acceleration region of the ions. Primary electrons are not reflected, and thus the mathematical treatment is different.

The current circuit for the diode is closed by the planar portion of the anode, which takes electrons from the spherical mesh to compensate for its loss of ions. The treatment is relativistic for the electrons and we distinguish between two electron groups: (1) primary electrons and (2) secondary electrons followed for many reflections. In group (1) the electrons are slowed down in energy but very little in velocity and their radial velocity is nearly c—the velocity of light *in vacuo*. Their assumed current is denoted I_P. In group (2) the electrons leave the surface of region III with small thermal velocity, assumed zero. Their velocity as a function of radius is computed from energy conservation. After being reflected from region I, those electrons not absorbed by the mesh approach very near the surface of region III because they suffered only a small energy loss. The points of closest approach are idealized to be on the surface of region III.

The average number of times, N_2, these electrons appear in region II is computed by assuming a probability P of being absorbed in the wires of the anode mesh each time an electron crosses the anode mesh. Thus,

$$N_2 = 1 + 2\sum_{s=1}^{\infty}(1-P)^{2s} \simeq \frac{1}{P} - \frac{1}{2} (\text{for } P \ll 1). \quad (2)$$

The current of these electrons is denoted by I_S.

The velocity of the ions is computed from energy conservation assuming zero velocity at R_2. The ion current, I_3, is related to the electron currents by zero total current into region III:

$$|I_3| = |I_P| - |I_S|. \quad (3)$$

Poisson's equation is then

$$\frac{1}{R_2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = -4\pi\rho = -4\pi\sum_{k=1}^{3}\frac{J_k N_k}{|v_k|}, \quad (4)$$

where V denotes the local potential in region II and $V(R_2)=0$. The sum is over all charge species; J_k and v_k are the current densities and radial velocities; N_k is the number of times the kth species appears ($N_1 = 1$ for the primary electrons and $N_3 = 1$ for the ions). From spherical symmetry and steady-state conditions, one has

$$J_1 = -I_P/4\pi R^2, \quad J_2 = -I_S/4\pi R^2; \quad J_3 = J_2 - J_1,$$

$$v_1 = c, \quad v_2 = c(1 - 1/\gamma^2)^{1/2},$$

$$v_3 = c\left(\frac{2m_0 Z}{M}(\gamma_0 - \gamma)\right)^{1/2},$$

where

$$\gamma = 1 + |e(V+V_0)/m_0 c^2|$$

is the usual relativistic factor for the secondary electrons and $\gamma = \gamma_0$ at R_2. The mass of the accelerated ions is M and they are ionized Z times. The voltage at R_3 is just $-V_0$. V_0 is determined self-consistently and must be less than the diode voltage. (This places a lower bound on R_3 in order to avoid virtual cathodes in region II.) In the present analysis we consider the case

$$R_2 - R_3 \ll R_3. \quad (5)$$

We integrate Eq. (4) once exactly, using all the expressions of current densities and velocities given above and using (1). An expression is thus obtained for the ion current using (3):

$$\left|\frac{I_3}{I_P}\right| = \frac{N_2(\gamma_0+1)^{1/2} + (\gamma_0 - 1)^{1/2}}{N_2(\gamma_0+1)^{1/2} + (2M/m_0 Z)^{1/2}}, \quad (6)$$

giving 0.56 for protons accelerated to 2 MeV and $P = 0.03$ in Eq. (2). The second integration of Eq. (4) gives an implicit expression for the voltage V_0 through γ_0:

$$\frac{R_2 - R_3}{R_2}\left(\frac{2I_P e}{m_0 c^3}\right)^{1/2} = \int_1^{\gamma_0} d\gamma \left\{\gamma - 1 + N_2\left(1 - \frac{I_3}{I_P}\right)(\gamma^2 - 1)^{1/2} + \frac{I_3}{I_P}\left(\frac{2M}{m_0 Z}\right)^{1/2}[(\gamma_0 - \gamma)^{1/2} - (\gamma_0 - 1)^{1/2}]\right\}^{-1/2}. \quad (7)$$

It is easily seen that only when the primary current (I_P) is larger than $m_0 c^3/e$ (= 17 000 A) will the induced voltage be in the range of megavolts. Either the accelerating voltage (γ_0) or the left-hand side of Eq. (7) may be considered as the independent variable. In Table I we give the value of the left-hand side of Eq. (7) for several values of γ_0, using $N_2 = 10$ and $N_2 = 32.8$ ($P = 0.03$). The ions in this case were protons.

The above analysis may be extended to include scattering processes for the secondary electrons with similar results for Eqs. (6) and (7) as long as the voltage is a monotonic function of the radius. Time-dependent analysis has to be performed if the integrand of Eq. (7) is negative, as may occur if $R_2 - R_3 \sim R_3$.

The interaction of the absorber with the different forms of radiation incident upon it is clearly a highly complicated process. In the following we list the dominant phenomena and give order-of-magnitude estimates of some of their effects.

The flux of ions on the absorber may have possible applications. If, for example, the absorber

TABLE I. Values of the left-hand side of Eq. (7) versus γ_0, for two values of N_2.

γ_0 \ N_2	10	33
1.1	0.094	0.063
1.5	0.32	0.21
2.0	0.53	0.36
4.0	1.2	0.84
6.0	1.7	1.3
8.0	2.2	1.6
10.0	2.6	2.0
15.0	3.5	2.8
20.0	4.4	3.5

is a bare deuterium pellet and the accelerated ions are tritons they will undergo fusion reactions with a probability $\sim 10^{-3}$ per incident triton. Megavolt electron-beam current of order 1 MA for 30 nsec could then accelerate $\sim 10^{17}$ ions giving 10^{14} neutrons. In addition, heating of the absorber will increase the range of the tritons and enhance neutron production.

If the absorber is made of DT coated with a layer of high-Z material (e.g., gold) then one may follow the well-known schemes of pellet compression by laser beams or electron beams. Our scheme contains, however, certain important differences. The energy incident on the absorber is composed of (1) photons from the hot anode mesh, (2) energy delivered by the slowed primary electrons, and (3) energy in the ion flux.

(1) The anode mesh is heated by the scattering of the secondary electrons. Nearly half of the radiated energy from the anode mesh is incident on the absorber. At present, the spectrum of this radiation is unknown. The very thin anode mesh plasma may become also optically thin and radiate in the region of a few angstroms. This radiation would penetrate into the high-density region of the absorber plasma. The power would be at most of order $\frac{1}{2}I_S V_0$, which is of importance since it may ablate the outer portion of the high-Z layer.

(2) Because of the dispersion in direction of the primary electrons in the diode, they reach the anode shell with appreciable velocities perpendicular to the radial direction. The floating voltage at the absorber has to be made smaller than the diode voltage if most of the primary electrons are to be stopped in the absorber. For example, 1-MeV electrons having $0.3c$ perpendicular velocity will reach the absorber if $V_0 \leq 900$ kV. Their residual energy of 100 keV will be dissipated in the high-Z envelope of the pellet, defining its needed thickness (the ion range is only somewhat smaller). This thickness is 1.5 orders of magnitude smaller than needed for 1-MeV electrons hitting the absorber directly. The requirements on beam energy and power for pellet compression will thus be reduced accordingly. The power deposited by this flux of slowed-down primary electrons is about an order of magnitude less than that delivered by the ions.

(3) The ion energy loss in the high-Z material is mainly by ionization processes.[8] The local heating rate due to this process is concentrated about the Bragg peak near the end of the ion range. The material in the first portion of the range, although heated by ion energy deposition and thermal conduction, expands less rapidly than the Bragg-peak portion, and thus serves as a tamper to efficiently direct the momentum of the expanding "explosive" layer onto the imploding "pusher" layer of additional high-Z material.[9] Because the gradients of photon, electron, ion, and thermal pressure are determined by the ranges of the particles, absorption lengths, and radiative cooling rates, the dynamics of the ablation process are a problem for computer simulation and are not undertaken in this Letter.

We have shown here the possibility of generating and applying intense pulses of radially converging energetic ions, using known phenomena in electron-beam diodes, but modifying the central anode structure. Even with present-day electron beam sources (~1 MA, 1 MV, 50 nsec) the applications of this new ion pulse physics may prove quite far reaching.[10]

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