PHYSICAL REVIEW LETTERS

VOLUME 35 27 OCTOBER 1975 NUMBER 17

Improved Upper Limit on the Photon Rest Mass

Aaron Barnes and Jeffrey D. Scargle
Theoretical Studies Branch, Ames Research Center, National Aeronautics and Space Administration, Moffett Field, California 94035
(Received 14 August 1975)

By standard arguments, observed hydromagnetic waves in the Crab Nebula imply an upper limit to the photon rest mass of $\mu \lesssim 10^{-16} - 10^{-15}\ \mathrm{cm}^{-1}$, $m_{\mathrm{ph}} \lesssim 3\times 10^{-54} - 3\times 10^{-53}$ g (uncertainty < factor 100), smaller than the previous best limits by a factor 10^4–10^5. However, the standard wave arguments used here and elsewhere are reexamined and found not to be rigorous, although the limits given are probably valid in order of magnitude.

It has been recently suggested that observations of hydromagnetic waves in interplanetary space[1] and in the earth's magnetosphere[2] imply an improved upper limit on the photon rest mass. Motions of the features in the Crab Nebula called "wisps" have been identified as the propagation of magnetoacoustic waves.[3] In this note, it is shown that application of standard theoretical arguments[1,2,4-6] to the wisps in the Crab dramatically improves the upper bound on the photon rest mass. However, it is also pointed out that there are inconsistencies in the standard arguments, so that, although the photon-mass limits given in Refs. 1 and 2 and in the present note are probably valid in order of magnitude, some doubt remains.

First, we present the standard argument and its application to the Crab. It is assumed that massive photons are governed by the Proca field equations [Ref. 6, Eqs. (2)–(19)] containing the parameter $\mu^2 = (2\pi/\lambda_C)^2$, where $\lambda_C = h/cm_{\mathrm{ph}}$ is the Compton wavelength of the photon; $\mu^2 = 0$ gives the usual Maxwell equations. The Lorentz force law and Faraday's law are unchanged by finite μ.

The usual way of calculating small-amplitude plasma waves begins with the assumption of small-amplitude fluctuating electromagnetic fields, related by Faraday's law, superposed on an *infinite, uniform* background plasma, and continues with the solution of the equations of motion for plasma particles. The Fourier-analyzed current density $\delta\vec{J}$ is related linearly to the Fourier-analyzed electric field $\delta\vec{E}(\vec{k},\omega)$ as

$$\delta\vec{J}(\vec{k},\omega) = (\omega/4\pi i)[\overleftrightarrow{K}(\vec{k},\omega) - \overleftrightarrow{I}]\cdot\delta\vec{E}(\vec{k},\omega). \qquad (1)$$

The tensor $\overleftrightarrow{K}(\vec{k},\omega)$ is usually called the dielectric tensor.[7] So far, the calculation is independent of μ. The final step is to require that Eq. (1) be consistent with the electrodynamic equations, which do involve μ. The consistency condition is the dispersion relation

$$\det\left[\vec{N}\vec{N}\left(1+\frac{\mu^2}{k^2}\frac{N^2}{N^2-1}\right) - N^2\left(1+\frac{\mu^2}{k^2}\right)\overleftrightarrow{I} + \overleftrightarrow{K}\right] = 0, \qquad (2)$$

where $\vec{N} = \vec{k}c/\omega$. Solution of Eq. (2) for various plasma conditions typically shows that, below a cutoff frequency (which is proportional to μ), waves of a given mode are evanescent[1,2,4-6]; thus the observation of propagating waves of known frequency gives an upper limit on μ. We now apply this idea to the wisps of the Crab Nebula.

Observations of the central region of the Crab Nebula (see Ref. 3 and references therein) indicate that the plasma consists of an ultrarelativistic electron component and a tenuous lower-energy background component, embedded in a magnetic field that is relatively uniform on the scale $\sim 10^{18}$ cm. Quasiperiodic disturbances ($\omega \sim 10^{-6}$ sec^{-1}) generated in the vicinity of the pulsar propagate across the magnetic field out into the nebula. This series of "wisps" has been identified as a sequence of magnetoacoustic waves in which the wave compressions produce local enhancements of the synchrotron radiation.[3] The kinetic theory of small-amplitude hydromagnetic waves in relativistic plasma has been worked out in detail,[8] so that the appropriate dispersion relation (2) is easily written.

The dispersion relation for magnetoacoustic waves propagating transverse to the background magnetic field is

$$\left(\frac{\omega}{kc}\right)^2\left[1+\frac{4\pi}{B^2}(\mathcal{E}+P_\perp)\right] = 1+\frac{2\pi P_\perp}{B^2}(4-\zeta)+\frac{\mu^2}{k^2}, \tag{3}$$

where B is the background magnetic field strength, $P_\perp$ is the total plasma pressure transverse to $\vec{B}$, $\mathcal{E}$ is the total energy density (including rest mass) of matter in the plasma, and ζ is a numerical factor between 0 and 1. Equation (3) implies wave evanescence ($k^2 < 0$) unless

$$\omega > \omega_c = \mu c[1+(4\pi/B^2)(\mathcal{E}+P_\perp)]^{-1/2}. \tag{4}$$

However, the ratio $4\pi\mathcal{E}/B^2$ is not known directly from observation. It is better to derive analogous inequalities involving the directly observed group velocity $V_g = d\omega/dk$. Straightforward manipulation of Eq. (3) permits us to derive three different inequalities, none of which depends on $4\pi\mathcal{E}/B^2$ explicitly:

$$\mu < [k\omega(1+\beta)/V_g]^{1/2}, \tag{5}$$

$$\mu < (1+\beta)^{1/2}\omega/V_g, \tag{6}$$

$$\mu < (1+\beta)^{1/2}kc/V_g. \tag{7}$$

Here $\beta = 8\pi P_\perp/B^2$.

Estimates of the quantities ω, k, and V_g can be obtained from measurements of photographs of the Crab Nebula.[3] Circular frequencies in the range 10^{-7} sec$^{-1} \lesssim \omega \lesssim 4\times 10^{-6}$ sec^{-1} are associated with the motions of the "driving piston" (wisp 1). Measurements of the position of the wave crests on numerous plates, taken over nearly 20 yr, give $k \approx (1.3\times 10^{-16}$ cm$^{-1})(1.5/D)$ and $V_g/c \approx 0.04(D/1.5)$ for the inner wisps, near the pulsar, and $k \approx (3.7\times 10^{-17}$ cm$^{-1})(1.5/D)$ and $V_g/c \approx 0.016(D/1.5)$ for the outer wisps near the edge of the nebula. Here D is the distance to the Crab in kiloparsecs, and is almost certainly between 1.0 and 2.5.) The upper bounds on μ, using these estimates and inequalities (5)–(7), are given in Table I.

The uncertainties in ω, k, and V_g arise mostly from ambiguities in interpretation, rather than from measurement errors. For practical reasons the plates have not been taken often enough to allow unambiguous determination of the time histories of the rapidly moving features. This makes assessment of the uncertainties difficult, but it is unlikely that the values given here are incorrect by more than a factor of about 3 to 5. The major uncertainty in the limit on μ is in the unobservable quantity β. The usual assumption is that $\beta \approx 1$, based on rough dynamical arguments. Fortunately, the results are not sensitive to β: If $\beta \ll 1$, there is virtually no change in the limit, and, even in the very unlikely event that β is as large as 100, the limit is only a factor of 10 worse (larger) than for $\beta \approx 1$. Thus, taken at face value, Table I indicates that $\mu \lesssim 10^{-16}$–10^{-15} cm^{-1} ($m_{\rm ph} \lesssim 3\times 10^{-54}$–$3\times 10^{-53}$ g). These numbers represent an improvement of 10^4 to 10^5 on the upper limits given by Hollweg[1] and Lanzerotti,[2] based

TABLE I. Upper bounds on μ from Crab Nebula observations.

Inequality	Inner wisps	Outer wisps
(5)	$[(1-7)\times 10^{-16}\ \text{cm}^{-1}](1+\beta)^{1/2}(1.5/D)$	$[(0.9-6)\times 10^{-16}\ \text{cm}^{-1}](1+\beta)^{1/2}(1.5/D)$
(6)	$[(0.8-30)\times 10^{-16}\ \text{cm}^{-1}](1+\beta)^{1/2}(1.5/D)$	$[(2-80)\times 10^{-16}\ \text{cm}^{-1}](1+\beta)^{1/2}(1.5/D)$
(7)	$(3.3\times 10^{-15}\ \text{cm}^{-1})(1+\beta)^{1/2}(1.5/D)^2$	$(2.3\times 10^{-15}\ \text{cm}^{-1})(1+\beta)^{1/2}(1.5/D)^2$

on observed interplanetary and magnetospheric waves. Even the most pessimistic assumptions about β, ω, k, and V_g would raise the new limit by less than a factor of 100. Thus, this limit is a dramatic improvement over previous ones.[9]

It remains to discuss the appropriateness of the standard argument made above (and in Refs. 1 and 2). The spacecraft observations correlate directly measured plasma and magnetic fluctuations, so that there is little doubt that the fluctuations discussed in Refs. 1 and 2 are hydromagnetic waves. The identification of the wisps in the Crab as hydromagnetic waves is supported by the possible identification of the mechanism by which the pulsar excites the waves,[10] and by the confirmation[11] of the prediction[8] that the high-energy x rays would be concentrated in the regions of the wisps, based on a model for the heating of the plasma by collisionless damping of hydromagnetic waves.

Alternatively, it is conceivable that the wisps are wave packets consisting of a carrier wave of relatively high frequency modulated by a low-frequency envelope. According to this picture, the carrier wave is not resolved in the plates and only the envelope of the wave packet is seen.[12,13] Although this possibility cannot be excluded, it has difficulties. If dispersion is not to destroy the wave packets, the carrier frequency would have to lie below the gyrofrequencies of most of the relativistic particles ($\lesssim 1$ Hz), well below the pulsar rotation frequency (30 Hz). There is no known motion in the system that would drive a large-amplitude wave at frequencies below 1 Hz, except for the piston itself. Furthermore, the carrier wave would be of large amplitude and probably form shocks after a few wave periods. Such a packet of shocks might lose its integrity by dissipation and/or dispersion.

We must now consider whether conditions appropriate for Eq. (1) are satisfied. The background plasma seems to be fairly uniform in the solar wind, and probably also in the Crab. However, the waves are not of small amplitude in either system. Nevertheless, it is unlikely that nonlinear effects invalidate the qualitative conclusions, since (i) in $\mu = 0$ plasma physics, the propagation properties of large- and small-amplitude waves are similar in most respects,[14] and (ii) finite μ enters the problem through the field equations, which are rigorously linear.

There remains a theoretical difficulty that seems not to have been noticed before. An essential assumption of the standard argument is that the background plasma is infinite and uniform. However, one simple consequence of the Proca equations is that, in general, either a static magnetic field varies appreciably over distances of order μ^{-1}, or there is a large background current $\vec{J}_0 \simeq (c/4\pi)\mu^2\vec{A}$ [$J_0 \gg (c/4\pi)|\mathrm{curl}\vec{B}|$]. In the former case, the dispersion relation (2) can be valid only if $|\vec{k}| \gg \mu$; in the latter case, currents modify the dielectric tensor[15] so that $\overleftrightarrow{K}(\vec{k},\omega)$ is no longer independent of μ, and a dispersion relation based on $\overleftrightarrow{K}(\mu=0)$, such as in Refs. 1, 2, 4–6, and Eq. (3) of the present paper, will not be correct. In either case, the dispersion relation breaks down for $|\vec{k}| \lesssim \mu$. In this sense, the upper limits of μ given in this paper and in Refs. 1 and 2 are open to doubt.

On the other hand, one may argue plausibly that these limits are valid at least in order of magnitude. According to the Proca equations, it is difficult to set up electromagnetic fields, presumably including oscillatory ones, that do not decay in a distance of order μ^{-1} or less. This suggests that "oscillations" of frequency ω whose $\mu = 0$ dispersion relation predicts $k(\omega) \lesssim \mu$ are likely to decay rather than to propagate. This impression has been reinforced by formal calculation for the special case of a cold plasma embedded in a magnetic field generated by a current sheet.[16] It turns out that there are critical frequencies $\omega_c(\mu)$ such that oscillatory solutions are forbidden if $\omega < \omega_c$, where ω_c is essentially identical with that obtained from the standard argument. The problem is much more complicated for more general plasmas, and has not been solved. Nevertheless, the cold-plasma results give confidence that the cutoff frequencies found by the "standard argument" are correct in order of magnitude. Preliminary calculations of the other extreme case, in which the plasma is fairly uniform because of a large background Proca current, indicate that such a plasma is unstable with respect to some hydromagnetic wave modes.[16]

Altogether, we find that the standard arguments for inferring upper limits to the photon rest mass from cosmic hydromagnetic waves give plausible, but not rigorous, limits. If the plausibility of such limits is accepted, existence of magnetoacoustic waves in the Crab Nebula apparently implies the lowest limit now available. If hydromagnetic-wave arguments are excluded, the best limit probably is found from studies of the stationary geomagnetic field, at least 10^5 times higher.[17]

[1]J. V. Hollweg, Phys. Rev. Lett. 32, 961 (1974).
[2]L. J. Lanzerotti, Geophys. Res. Lett. 1, 229 (1974).
[3]J. D. Scargle, Astrophys. J. 156, 401 (1969).
[4]M. A. Gintsburg, Astron. Zh. 40, 703 (1963) [Sov. Astron. AJ 7, 536 (1964)].
[5]V. L. Patel, Phys. Lett. 14, 105 (1965).
[6]A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. 43, 277 (1971).
[7]E.g., see T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
[8]A. Barnes and J. D. Scargle, Astrophys. J. 184, 251 (1973).
[9]A lower limit has been suggested by E. Williams and D. Park, Phys. Rev. Lett. 26, 1651 (1971), from consideration of the galactic magnetic field. However, there are a number of difficulties in their argument, and their limit cannot be considered well established (see Ref. 6).
[10]J. D. Scargle and E. A. Harlan, Astrophys. J. Lett. 159, L143 (1970); J. D. Scargle and F. Pacini, Nature (London), Phys. Sci 232, 144 (1971).
[11]G. R. Ricker, A. Scheepmaker, S. G. Ryckman, J. E. Ballantine, J. P. Doty, P. N. Downey, and W. H. G. Lewin, Astrophys. J. Lett. 197, L83 (1975).
[12]This possibility was mentioned to us by R. H. Miller (private communication).
[13]Such aliasing would not be a problem for the case of spacecraft observations, which usually are made by magnetometers designed to filter out frequencies that they cannot resolve (B. F. Smith, private communication).
[14]E.g., see A. Barnes and J. V. Hollweg, J. Geophys. Res. 79, 2302 (1974).
[15]See, e.g., N. A. Krall, in *Advances in Plasma Physics, 1*, edited by A. Simon and W. B. Thompson (Interscience, New York, 1968), pp. 153–199.
[16]A. Barnes, unpublished.
[17]After submission of this Letter we received a preprint of the paper "Limit on the Photon Mass Deduced from Pioneer 10 Observations of Jupiter's Magnetic Field" by L. Davis, Jr., A. S. Goldhaber, and M. M. Nieto. This paper contains what appears to be the best limit obtained from *in situ* observations, $\mu \lesssim 2 \times 10^{-11}$ cm^{-1}. Also, L. J. Lanzerotti recently brought some other references to our attention. One of these [J. C. Byrne and R. R. Burman, J. Phys. A: Gen. Phys. 6, L12 (1973)] gives limits comparable to the present paper. Their argument is based on estimates of the maximum current density in the interstellar medium and is very qualitative; we find this argument to be plausible but perhaps not totally convincing.

New Narrow Resonances and Separate Localization of Ordinary and Color SU(3)*

O. W. Greenberg
Department of Physics and Astronomy, Center for Theoretical Physics, University of Maryland, College Park, Maryland 20742
(Received 28 July 1975)

I interpret $\psi(3095)$ and $\psi'(3684)$ as space and color excitations in a model in which a quark is a composite of a Fermi, spin-$\frac{1}{2}$, ordinary SU(3) object and a Bose, spin-0, color SU(3) object, and a meson is a four-body object. Color conservation and the absence of dipole transitions from the space mode make these two ψ's narrow for nonradiative and for radiative decays, respectively, to the usual hadrons.

Since the discovery of narrow photon- and hadron-excited resonances[1] above 3 GeV in electron-positron and hadron-hadron collisions, many attempts have been made to interpret these narrow resonances in terms of new hadronic degrees of freedom, such as charm, color, heaviness, etc.[2] These interpretations lead to the prediction of other resonances at comparable mass which have internal quantum numbers which cannot be realized in the Gell-Mann–Zweig quark model, as well as to the prediction of radiative decays to mesons. The failure, up to now, to detect such resonances, and the small observed widths for radiative decays have cast doubt on both the charm and color interpretations of the narrow resonances.

The main idea of the present article is to supplement the new internal degree of freedom, in our case color,[3] by new space or mechanical degrees of freedom in order to build a model of these resonances. I suggest that the usual SU(3) charges and the color SU(3) charges are localized in separate regions of space (for short, at different "points") so that a colored quark nonet is a two-body system[4] ($Q\overline{C}$), Q being a spin-$\frac{1}{2}$ SU(3)-triplet Fermi object carrying the usual SU(3) quantum numbers and C being a spin-0[5] SU(3)-triplet Bose object carrying the color