very sensitive to the value of the temperature coefficient of E_{g2}. The value of the temperature coefficient of $E_{g2} = 0.17 - 4\times 10^{-4} T$ eV is from Andreev[13] and Andreev and Radionov.[14] The multiplicity of VB(Σ) was assumed to be 12.

In summary we have shown that the secondary valence-band maxima in PbTe have ⟨100⟩ orientation although this is not the symmetry direction on which they are believed to be located. This fact is in agreement with a model[9] of the Fermi surface of SnTe which has Σ maxima similar to PbTe. It must be pointed out, however, that by investigating β one is only able to determine the direction of the axis of rotation of the ellipsoids and not the location in k space. The somewhat ambiguous situation about the location of the secondary valence-band maxima (generally believed to be Σ) therefore remains.

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COMMENTS

How to Test a Colored-Quark Model for ψ Particles*

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We present crucial tests of our assignment for the ψ particles in a colored-quark model. They include observations of (1) the predominant direct decays, $\psi(3.1)$ and $\psi(3.7) \to \omega + X_{\text{vac}}$; (2) the chain decays, $\psi(4.1) \to \psi(3.1) + X \to \mu^+ + \mu^- + X$ (X is an arbitrary hadronic state and X_{vac} is that with the vacuum quantum numbers); (3) the monochromatic meson emissions, $\psi(4.1) \to \pi + (\rho, \rho^0)$, $K + (K^*, \rho^0)$; and (4) the radiative decays, $\psi \to \gamma + 2n\pi$, $\gamma + \eta$.

In our previous paper (I),[1] we have assigned[2] the recently discovered vector-meson resonances,[3] $\psi(3.1)$, $\psi(3.7)$, and $\psi(4.1)$, and a predicted one, $\psi(4.9)$, to certain states in an irreducible representation of $SU(3) \otimes SU(3)_c$ symmetry, based on a generalized three-triplet quark model. [$SU(3)_c$ stands for the symmetry of an additional internal degree of freedom for hadrons, the color.[4,5]] In this paper, we shall propose a number of crucial tests of our assignments which can be

TABLE I. Hierarchy of interactions of new particles. The word "hadrons" in Example denotes a color-singlet hadronic state.

SU(3) SPACE

Interaction	Strength	Transformation Property	Selection Rules	Symmetry
Strong	0(10)	(1,1)	$\Delta\vec{I}=\Delta Y=0$	SU(3)
(a) Semi-Strong	0(1)	F_8, D_8	$\Delta\vec{I}=\Delta Y=0$	SU(2)xU(1)
(b) Singlet-Octet Mixing	0(1)	$(\omega_8, 1)$	$\Delta\vec{I}=\Delta Y=0$	SU(2)xU(1)
(c) Electro-Magnetic	0(α)	$(\rho^0+\frac{1}{\sqrt{3}}\omega_8, 1)$	$\Delta I_3=\Delta Y=0$ $\Delta\vec{I}\neq 0$	

SU(3)$_c$ SPACE

Interaction	Strength	Transformation Property	Selection Rules	Symmetry	Example
Strong	0(10)	(1,1)	$\Delta\vec{I}_c=\Delta Y=0$	SU(3)$_c$	$\psi(4.1)\rightarrow\psi(3.1)$ +hadrons
(a′) Semi-Strong	0(1)	F_{c_8}, D_{c_8} ?	$\Delta\vec{I}_c=\Delta Y_c=0$	SU$_c$(2)x U$_c$(1)	Non-Degeneracy between $\psi(3.1)$ and $\psi(3.7)$
(c_1') Color-Singlet-Octet Mixing	0(α)?	$(1, \omega_8)$	$\Delta\vec{I}_c=\Delta Y=0$		$\psi(3.7)\rightarrow$hadrons
(c_2') Color-Isospin Breaking	0(α)?	$(1, \omega_8)$x $(1, \rho^0)$	$\Delta I_{c_3}=\Delta Y_c=0$ $\Delta\vec{I}_c\neq 0$		$\psi(3.7)\rightarrow\psi(3.1)$ +hadrons
$(c_3')=(c_1')+(c_2')$	0(α)?	$(1, \rho^0)$	$\Delta I_{c_3}=\Delta Y_c=0$ $\Delta\vec{I}_c\neq 0$		$\psi(3.1)\rightarrow$hadrons
(c_4') Electro-Magnetic	0(α)	$(1, \rho^0+\frac{1}{\sqrt{3}}\omega_8)$	$\Delta I_{c_3}=\Delta Y_c=0$ $\Delta\vec{I}_c\neq 0$		$\psi(3.1)\rightarrow\gamma$+hadrons

performed immediately. Although our discussions are valid for an arbitrary three-triplet quark model with SU(3)⊗ SU(3)$_c$ symmetry, they may also be restricted to the Han-Nambu quark model.[4]

In a SU(3)⊗ SU(3)$_c$ colored-quark model, the color-nonsinglet part of the electromagnetic current transforms as $J_c^{\text{em}}\sim(1,\rho^0)+(1/\sqrt{3})(1,\omega_8)$, where a and b in (a,b) denote some members of irreducible representations of SU(3) and SU(3)$_c$, respectively. As is shown in I, the color-nonsinglet part can accommodate four ψ particles. (ψ particles are defined as color-nonsinglet vector mesons with the photon quantum numbers.) For the purpose of this paper, we summarize in Table I the hierarchy of interactions of the ψ particles which we have already discussed in I.

We have assumed that the breaking of SU(3)$_c$ symmetry is similar to that of SU(3): (a) The mass degeneracy among members of a SU(3) multiplet is lifted by a medium-strong interaction, conserving the isospin and hypercharge, whose Hamiltonian transforms as a linear combination of F_8 and D_8. (a′) Similarly, the mass degeneracy among members of a SU(3)$_c$ multiplet is lifted by a new medium-strong interaction, conserving the color isospin and hypercharge, whose Hamiltonian transforms, for example, as a linear combination of $F_{c,8}$ and $D_{c,8}$. [$F_{c,i}$ and $D_{c,i}$, where $i=1,2,3,\ldots,8$, are SU(3)$_c$ operators corresponding to the SU(3) operators F_i and D_i, respectively.] This new medium-strong interaction splits a single ψ state into two. (b) Some states in a singlet and octet representation of SU(3) can be mixed by another medium-strong interaction conserving the isospin and hypercharge (ω-φ mixing). This doubles the number of ψ particles and makes the four ψ's: $\psi(3.1)\equiv(\omega,\rho^0)$, $\psi(3.7)\equiv(\omega,\omega_8)$, $\psi(4.1)\equiv(\varphi,\rho^0)$, and $\psi(4.9)\equiv(\varphi,\omega_8)$. (b′) It is a basic assumption in our picture that the color singlet-octet mixing, conserving the color isospin and hypercharge, is extremely small compared to the usual ω-φ mixing. (c) The color-singlet electromagnetic interaction violates isospin conservation. (c′) Similarly, there are various possible color-isospin-breaking interactions. It is tempting to conjecture that they are all caused by the color-octet electromagnetic interaction. We can not, however, rule out the existence of completely new interactions which also violate color-isospin conservation.

The breakings of the mass degeneracies described above, together with ω-φ mixing, make $9\times 8=72$ color-octet vector mesons. Some of these new vector mesons already discussed in I are illustrated in Fig. 1 with their possible decay modes. All of the tests which we shall propose in the following are related to these decay

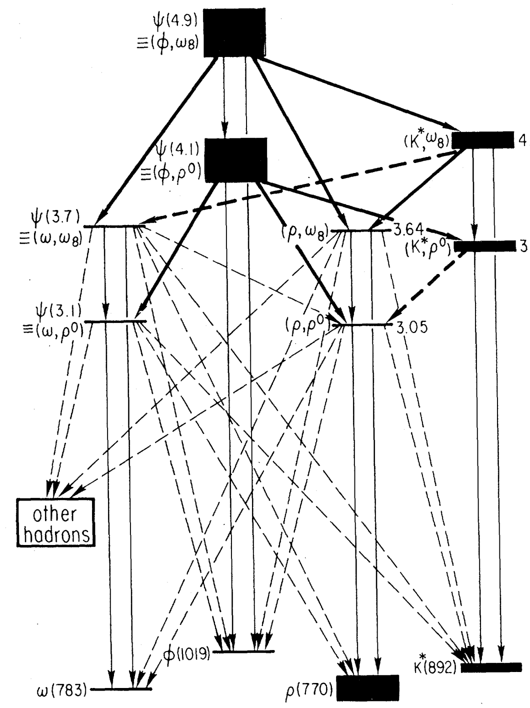

FIG. 1. Two nonets of color-octet vector mesons and their decay schemes. The vertical scale corresponds to the mass and width (for broad resonances only) of each resonance. The heavy lines correspond to decays allowed by strong interactions, the heavy broken lines to strong decays which may be suppressed or even forbidden by phase space, the regular lines to decays which are suppressed because of the violation of color-quantum-number conservation but enhanced by the vacuum-dominance hypothesis, and the regular broken lines to decays which are suppressed because of the violation of color-quantum-number conservation and not enhanced by the hypothesis.

modes.

(1) *Predominant decays of* $\psi(3.1)$ *and* $\psi(3.7)$.—We introduce a supplementary hypothesis on the decays of color-nonsinglet vector mesons, the vacuum-dominance hypothesis: Among suppressed decays of color-nonsinglet vector mesons, violating color-quantum-number conservation, decays of the type $(a,b)\to(a,b')+X_{\mathrm{vac}}$ are enhanced. [X_{vac} is an arbitrary hadronic state $\sim(1,1)$ with the vacuum quantum numbers.] This hypothesis is consistent with the experimental observation[6] that $\psi(3.7)\to\psi(3.1)+2\pi$ with the two-pion system in an S-wave $I=0$ state is a dominant decay mode of $\psi(3.7)$. According to this hypothesis, we predict that the following decay modes have large branching ratios compared to other modes which do not satisfy the hypothesis: (a) $\psi(3.1)\to\omega+X_{\mathrm{vac}}$, (b) $\psi(3.7)\to\psi(3.1)+X_{\mathrm{vac}}$, and (c) $\psi(3.7)\to\omega+X_{\mathrm{vac}}$. The decay mode $\psi(3.1)\to\omega+\pi^++\pi^-$ has also been observed[7] and, in fact, its branching ratio is larger than those of any other decay modes of $\psi(3.1)$ so far observed. We expect further that considerable fractions of the vacuum-dominated decays have resonating X_{vac}—for example, $\psi(3.1)\to\omega+\epsilon(700)\to5\pi$ and $\psi(3.1)\to\omega+S^*(993)\to3\pi+K+\bar{K}$ or 5π. In the latter mode, the invariant-mass spectrum of $K\bar{K}$ should have a peak at 993 MeV. If such a peak is observed, it is further desirable to measure the angular momentum of the $K\bar{K}$ system to discriminate S^* against φ.

Notice that the predominance of decay modes (a) and (c) is in sharp contrast with predictions in other models. For example, if $\psi(3.1)$ transforms as $c\bar{c}$, where c is the fourth quark associated with the SU(4) symmetry, we naively expect that the rate for $\psi(3.1)\to\varphi+X$ (where X is an arbitrary hadronic state) is comparable to that for $\psi(3.1)\to\omega+X$.[8]

The vacuum-dominance hypothesis also predicts many other decays of color-octet vector mesons shown in Fig. 1. For example, $(\rho,\rho^0)\to\rho+X_{\mathrm{vac}}$ is expected to be a most enhanced decay mode of the (ρ,ρ^0) meson.

(2) *Chain decays of* $\psi(4.1)$ *and* $\psi(4.9)$.—In our assignment, $\psi(4.1)$ decays strongly into eight channels: $\psi(4.1)\to\psi(3.1)+X$, $(K^*,\rho^0)+X$, $(\rho,\rho^0)+X$. Since $\Gamma(\psi(4.1)\to\mu^++\mu^-+X)/\Gamma(\psi(4.1)\to\mathrm{all})=[\Gamma(\psi(4.1)\to\psi(3.1)+X)/\Gamma(\psi(4.1)\to\mathrm{all})]\times[\Gamma(\psi(3.1)\to\mu^++\mu^-)/\Gamma(\psi(3.1)\to\mathrm{all})]$ is estimated to be roughly $\frac{1}{100}$, it is possible to detect the chain decay $\psi(4.1)\to\psi(3.1)+X\to\mu^++\mu^-+X$ by searching for a $\psi(3.1)$ peak in the invariant-mass spectrum of noncollinear $\mu^+\mu^-$ pairs. Similarly, the branching ratio for the chain decay $\psi(4.9)\to\psi(3.7)+X\to\psi(3.1)+X\to\mu^++\mu^-+X$ is estimated to be roughly $\frac{1}{200}$. Observation of this mode provides an unambiguous test for the existence of $\psi(4.9)$.

(3) *Monochromatic pion and kaon emissions.*—Detection of monochromatic pions and kaons in the decay products of $\psi(3.7)$, $\psi(4.1)$, and $\psi(4.9)$ signals production of colored vector mesons which do not couple directly to a photon. The relevant decays are (a) $\psi(3.7)\to\pi^{\pm,0}+(\rho^{\mp,0},\rho^0)$; (b) $\psi(4.1)\to\pi^{\pm,0}+(\rho^{\mp,0},\rho^0)$; (c) $\psi(4.1)\to K^\pm+(K^{*\mp},\rho^0)$, $K^0+(\bar{K}^{*0},\rho^0)$, or $\bar{K}^0+(K^{*0},\rho^0)$; (d) $\psi(4.9)\to\pi^{\pm,0}+(\rho^{\mp,0},\omega_8)$; and (e) $\psi(4.9)\to K^\pm+(K^{*\mp},\omega_8)$, $K^0+(\bar{K}^{*0},\omega_8)$, or $\bar{K}^0+(K^{*0},\omega_8)$. Since the decay $\psi(3.7)\to\pi+(\rho,\rho^0)$ is not enhanced by the vacuum-dominance hypothesis, we expect the branching ratio $\Gamma(\psi(3.7)\to\pi+(\rho,\rho^0))/\Gamma(\psi(3.7)\to\mathrm{all})$ to be

rather small ($\ll 20\%$). Even a very rough estimate of the single-pion energy spectrum at the position of the $\psi(3.7)$ resonance is very useful since it will set an upper bound on the branching ratio.

(4) *Monochromatic photon emissions.*—As is stressed in I, the absence of radiative decays of $\psi(3.1)$ or $\psi(3.7)$ would become a very serious problem for our picture. Especially, the radiative decays $\psi(3.1)$ or $\psi(3.7)\to\gamma+2n\pi$ ($n=1,2,3,\ldots$), $\gamma+\eta$, or $\gamma+\eta'$ are all allowed but the decays $\psi(3.1)$ or $\psi(3.7)\to\gamma+(2n-1)\pi$ are forbidden. For the two-body decays into $\gamma+\eta$ and $\gamma+\eta'$, monochromatic photons will give a clear signal.

(5) *Doubly charged mesons.*—If we restrict ourselves to the Han-Nambu quark model, the electromagnetic charge of a state is given by $Q=I_3+\frac{1}{2}Y+I_{c,3}+\frac{1}{2}Y_c$. Thus some colored vector mesons such as $(K^{*\pm},K^{*\pm})$ and $(\rho^{\pm},\rho^{\pm})$ are doubly charged. This point may be useful in searches for such new mesons in hadron-hadron collisions.[9]

(6) *Color-nonsinglet pseudoscalar mesons and baryons.*—Since dynamics of colored states is considered to be quite different from that of color-singlet states in our picture, we can not reliably guess the masses of $9\times 8=72$ possible color-octet pseudoscalar mesons. However, if one such meson is found, our hypothesis introduced in I will predict the masses of all other pseudoscalar mesons with the same color quantum numbers. A similar consideration is applicable to possible colored baryons.

In conclusion, we stress that measurements of the branching ratios for the decays $\psi(3.1)\to\omega+X$, $\psi(3.7)\to\varphi+X$, $\psi(3.7)\to\omega+X$, $\psi(3.7)\to\varphi+X$, and $\psi(3.1)$ or $\psi(3.7)\to\gamma+2n\pi$, $\gamma+(2n-1)\pi$, $\gamma+\eta$, or $\gamma+\eta'$ are very important. We emphasize a sharp contrast in predictions:

$$\frac{\Gamma(\psi(3.1)\to\omega+X)}{\Gamma(\psi(3.1)\to\varphi+X)}\begin{cases}\gg 1, & \mathrm{SU}(3)\otimes\mathrm{SU}(3)_c \text{ symmetry;}\\ \cong O(1), & \mathrm{SU}(4) \text{ symmetry.}\end{cases}$$

A similar prediction holds for the decays $\psi(3.7)\to\omega+X$ and $\psi(3.7)\to\varphi+X$. Finally, detection of the chain decays $\psi(4.1)\to\psi(3.1)+X\to\mu^++\mu^-+X$ and $\psi(4.9)\to\psi(3.7)+X\to\psi(3.1)+X\to\mu^++\mu^-+X$ are extremely interesting since the former proves that $\psi(4.1)$ is indeed a resonance while the latter proves the existence of $\psi(4.9)$.

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