The following experiment tends to support this conclusion.

Figure 3 shows the change in the photoconductivity spectra with increasing impurity concentration. The position of the peak is seen to shift to higher energies.

Such a shift has been predicted by Nishimura[6] for the D^- state. He showed, by using the tight-binding approximation, that the interaction between the D^- states leads to the formation of the D^- band, and the energy separation between the conduction band and the D^- band spreads with increasing density of neutral donors.

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Combined Resonances in Hot-Electron Magnetophonon Oscillations in InSb

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A full series of combined cyclotron- and spin-resonance transitions is observed in magnetophonon oscillations in n-InSb under hot-electron conditons. These originate from the emission of LO phonons. Probabilities of combined resonances induced by polar and nonpolar coupling with optical phonons are calculated for the spin-orbit interaction mechanism.

The magnetophonon effect, also known as Gurevich-Firsov oscillations,[1] serves as a powerful tool in the investigations of band structure and its behavior in a magnetic field, as well as of the nature of electron-phonon interaction in semiconductors. The magnetoresistance oscillations, due to inelastic transitions induced by optical phonons between Landau sub-bands with the same spin orientation, occur at magnetic fields corresponding to the condition $N\omega_c = \omega_L$, $N = 1, 2, \ldots$, with $\omega_c = eH/m^*c$ and ω_L denoting cyclotron frequency and phonon frequency, respectively. The results obtained by some authors on InSb and InAs,[2] and previously attributed to transitions involving a change in spin orientation, can be shown to occur at the wrong fields for spin-flip transitions if the band nonparabolicity is taken into account, but at the correct field for the first extremum in a series $N\omega_c = 2\omega_L$.[3,4] The latter condition has been interpreted by Stradling and Wood[4] as due to two-phonon scattering processes. It seems, however, that the additional extrema can be reinterpreted in terms of one-phonon pseudoresonances proposed by Peterson,[5] which occur at $(2N+1)\omega_c = 2\omega_L$. Recently Morita, Takano, and Kawamura[6] reported combined-resonance transitions in the transverse photoconductivity of InSb, not attempting, however, to propose the physical mechanism responsible for the spin-flip.

Here we report direct evidence of phonon-induced electron transitions between Landau subbands with different spin orientations, observed in the magnetophonon effect under hot-electron conditions. The experiments were performed in the longitudinal configuration with n-InSb having a carrier concentration of $n = 2\times10^{14}$ cm^{-3} and a mobility of 4.72×10^5 cm^2/V sec (at 77 K) in the temperature range 10–20 K. The dc pulses of 1 μsec duration and a repetition rate of 50 Hz were applied to the samples, thus avoiding lattice heating at high electric fields. A sampling technique, making it possible to choose the exact time t of measurement after the rise of the voltage pulse,[7] in connection with an improved tuned amplification technique was employed in order to extract the oscillations from the monotonic nonoscillatory magnetoresistance, avoiding complications due to higher harmonics. The oscillatory part was recorded with the reversed sign.

The results for $T = 20$ K are shown in Fig. 1, the arrows indicating calculated resonant magnetic fields. The numbers 2, 3, ... denote spin-conserving transitions, $2^+ \to 0^+$, $3^+ \to 0^+$, etc., while $2'$, $3'$, ... represent combined processes involving spin-flip, $2^- \to 0^+$, $3^- \to 0^+$, etc., and $3''$, $4''$, ... correspond to combined resonances, $3^+ \to 0^-$, $4^+ \to 0^-$, etc. It can be seen that, in addition to the usual spin-conserving transitions, full series of combined resonances occur. The resonant magnetic fields obey the relation $\lambda_n^{s} - \lambda_0^{s'} = \hbar\omega_L = 23.9$ meV, corresponding to emission of single optical phonons. We have also observed

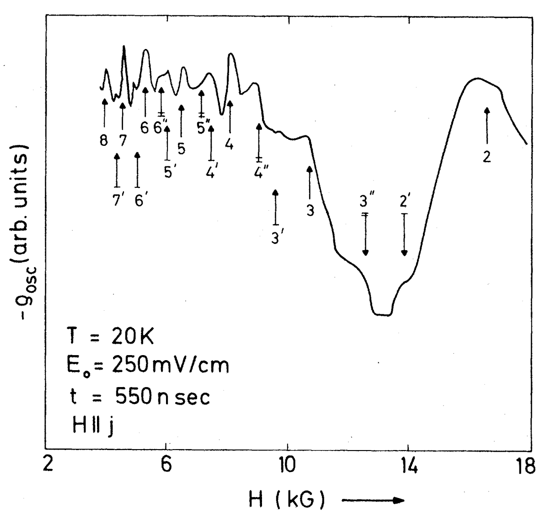

FIG. 1. Oscillatory longitudinal magnetoresistance of InSb in the hot-electron regime versus magnetic field. Arrows indicate spin-conserving (2, 3, 4, ...) and two series of spin-flip ($2'$, $3'$, $4'$, ... and $3''$, $4''$, $5''$...) transitions due to LO-phonon emission.

magnetophonon oscillations at $T = 11$ K, which can be attributed to simultaneous emission of two transverse phonons from the zone boundary near the X point, the quality of the data not allowing us, however, to ascribe unambiguously the transitions involved.

The electron energies and wave functions are calculated in the three-level model (the Γ_6 level separated by the energy gap $\epsilon_0 = 0.236$ eV from the degenerate Γ_8 level, which in turn is split off by the spin-orbit interaction $\Delta = 0.9$ eV from the Γ_7 level), solved for the presence of an external magnetic field by Yafet.[8] For InSb one can use an approximation $\Delta \gg \epsilon_0$, and the energies are[9]

$$\lambda_{nk_z}^{\pm} = -\tfrac{1}{2}\epsilon_0 + [(\tfrac{1}{2}\epsilon_0)^2 + \epsilon_0 D_{nk_z}^{\pm}]^{1/2}, \tag{1}$$

with

$$D_{nk_z}^{\pm} = \hbar\omega_c(n+\tfrac{1}{2}) + \hbar^2 k_z^2/2m^* \pm \tfrac{1}{2}g^*\mu_B H, \tag{2}$$

where $m^* = 0.0139m_0$ and $g^* = -52.0$ are the effective mass and the Landé factor at the band edge. Plus and minus superscripts correspond to the two projections of the total angular momentum j (which hitherto and hereafter we call for brevity "spin") on the magnetic field direction. The wave functions for electron energies away from the band edge are mixtures of spin-up and spin-down components, as well as s-like and p-like periodic parts.[8,9] We use the wave functions given in Ref. 9 to calculate probabilities of combined resonances induced by emission of LO phonons.[10] For polar electron-phonon coupling, described by the Fröhlich Hamiltonian, the square of the matrix element for the transition between states $|n, k_x, k_z, -\rangle$ and $|0, k_x', k_z', +\rangle$ is

$$\tfrac{9}{4}A(b_0^+b_n^-)^2\frac{1}{q^2}\frac{(\hbar\omega_c)\hbar^2/2m^*}{D_0^+D_n^-}\frac{1}{n!} \times \exp(-\xi)\xi^{n-1}(k_z\xi - q_z n)^2, \tag{3}$$

while for combined transitions between states $|n, k_x, k_z, +\rangle$ and $|0, k_x', k_z', -\rangle$, one obtains

$$\tfrac{9}{4}A(b_0^-b_n^+)^2\frac{1}{q^2}\frac{(\hbar\omega_c)\hbar^2/2m^*}{D_0^-D_n^+}\frac{1}{n!} \times \exp(-\xi)\xi^{n+1}(k_z - q_z)^2, \tag{4}$$

where $A = (4\pi\alpha/V)(\hbar\omega_L)^2(\hbar/2m\omega_L)^{1/2}$, with α denoting the polar coupling constant, V the crystal volume, and $\vec{q}$ the phonon wave vector. The coefficients b depend on energy according to $3(b_n^{\pm})^2 = \lambda_n^{\pm}/(\epsilon_0 + 2\lambda_n^{\pm})$. The momentum conservation $k_x' = k_x - q_x$ and $k_z' = k_z - q_z$ is satisfied. The definition $2\xi = (q_x^2 + q_y^2)L^2$ is used, with $L = (\hbar c/$

$eH)^{1/2}$. As follows from Eqs. (3) and (4), the transitions are forbidden for $k_z = k_z' = 0$.

A nonpolar interaction of optical phonons with electrons in the conduction band of InSb is also possible, as a result of mixing of p-like periodic parts into the electron wave functions. The initial Hamiltonian of the interaction is taken from the formulation of Bir and Pikus.[11] For nonpolar coupling the combined transitions are not forbidden for $k_z = k_z' = 0$ and we consider namely this case. The square of the matrix element for the transition between states $|n, k_x, 0, +\rangle$ and $|0, k_x', 0, -\rangle$, due to longitudinal phonon emission, is

$$\frac{9}{16} B \frac{d_0^2}{a_0^2} (b_n^- b_0^+) \frac{(\hbar\omega_c)^2}{D_0^+ D_n^-} \frac{1}{n!} \times \exp(-\zeta)\zeta^n(-\zeta + n + 1)^2, \quad (5)$$

where $B = \hbar/2VMN\omega_L$, $d_0 \simeq 35$ eV is the optical deformation potential,[12] a_0 is the lattice constant, M is the reduced mass of the ions, and N is the number of unit cells in a unit volume. For transitions between states $|n, k_x, 0, -\rangle$ and $|0, k_x', 0, +\rangle$ one obtains an expression almost identical with Eq. (5).

A full theoretical description of the hot-electron magnetophonon effect is very involved and even the problem of maxima versus minima behavior is not quite clear.[5,13] All existing theoretical models indicate that collision broadening of Landau levels, the electric field strength, and the relative contribution of elastic scattering processes to the total scattering strongly affect the resonant behavior. The combined resonances in our data, although weaker than the spin-conserving ones, are surprisingly strong. Detailed theoretical comparison of amplitudes for spin-conserving and combined resonances is hard to carry out because of the fact that the matrix elements for the latter have a strong k_z dependence, cf. Eqs. (3) and (4), whereas in spin-conserving processes the k_z dependence is weak. Still, because of the appearance of the coefficients b in the spin-flip case (they do not appear in spin-conserving transitions, since in this case the spin mixing in the wave functions is not necessary) one expects the spin-flip resonances to be much weaker. The contribution of elastic scattering, due to ionized impurities and acoustic phonons, to the total scattering is different for spin-conserving and spin-flip processes,[14] which probably produces a relative enhancement of the combined-resonance amplitudes. The two series of combined resonances are nearly comparable in strength, which is theoretically reflected by the similarity of expressions for both polar and nonpolar spin-flip transitions.

It should be mentioned that Stradling and Wood[4,15] did not observe spin-flip magnetophonon resonances in a search in both Ohmic and hot-electron regimes. It seems that the short-time sampling technique applied in our experiments, which allowed us to avoid lattice heating entirely, is crucial for the reported observation. Also the tuned amplification technique[16] has been decisive for the quality of our data.

It is worth noting that both polar and nonpolar mechanisms considered above could also give rise to combined resonances of the longitudinal magnetoresistance in the Ohmic region, in which k_z relaxation is required.

While the revised version of this work was being prepared for publication, a recent paper of Amirkhanov, Bashirov, and Elizarov[17] came to our attention, in which the authors report an observation of three combined resonances in transverse and longitudinal magnetophonon oscillations in InSb under Ohmic conditions.

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Temperature-Dependent Susceptibility of the Symmetric Anderson Model: Connection to the Kondo Model

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The temperature-dependent impurity susceptibility for the symmetric Anderson Hamiltonian is discussed for all physically relevant values of its parameters U (the Coulomb correlation energy) and Γ (the d-level width). For $U > \pi\Gamma$, the results, for temperatures less than $U/10k_B$, map onto those of the spin-$\frac{1}{2}$ Kondo Hamiltonian with an effective exchange constant given by $\rho J_{\text{eff}} = -8\Gamma/\pi U$. At higher temperatures the results show the formation of the local moment from the free orbital.

We report here the results of the first calculation of the temperature-dependent impurity susceptibility for a magnetic impurity in a nonmagnetic metal describable by the symmetric Anderson model[1] over the full, physically relevant range of the parameters of that model. The calculation is based on the numerical renormalization-group techniques developed by Wilson for the spin-$\frac{1}{2}$ Kondo Hamiltonian.[2,3] The results are expected to be accurate to a few percent.

Among the noteworthy features of the results are the following: (1) Within certain ranges of the parameters there exists a temperature region where the model has a local moment, as indicated by a Curie-Weiss susceptibility. (2) At low enough temperatures this moment is always compensated by the conduction electrons and the impurity susceptibility approaches a constant at zero temperature. (3) The low-temperature susceptibility maps precisely onto that of the spin-$\frac{1}{2}$ Kondo Hamiltonian.

The Hamiltonian for the symmetric Anderson model is[1]

$$H_A = \sum_{ks} \epsilon_k n_{ks} - \tfrac{1}{2} U \sum_s n_{ds} + U n_{d\uparrow} n_{d\downarrow} + V \sum_{ks} (c_{ds}^\dagger c_{ks} + c_{ks}^\dagger c_{ds}). \qquad (1)$$

We assume that the conduction electrons (whose energies are measured relative to the Fermi level) are in an isotropic band of constant density of states per spin ρ within a bandwidth $2D$. The *symmetry*[4] of the Hamiltonian is that while the first electron in the localized d orbital has energy $-U/2$, because of the Coulomb repulsion energy U, a second electron in the orbital has energy $+U/2$. The coupling V between the localized d orbital and the conduction band leads to a d-level width[1]

$$\Gamma = \pi \rho V^2. \qquad (2)$$

When $\Gamma \ll U$ one expects[1,5] the model to display a local moment and to be describable by a spin-$\frac{1}{2}$ Kondo Hamiltonian

$$H_K = \sum_{ks} \epsilon_k n_{ks} - J \vec{s}(0) \cdot \vec{S}_d, \qquad (3)$$