the net frequency shift observed in the laboratory is a combination of this red shift plus a blue shift due to the overall expansion of the upstream plasma. The increased red shift may be sufficient to yield a net red shift in the frequency of the reflected light.

For simplicity we have discussed backscatter, but similar results should also obtain for sidescatter (the momentum transfer is less by a factor of $\sqrt{2}$). It should be noted that 2D effects could further reduce the time over which a large reflectivity can occur. For example, the incident light may form narrow filaments, expelling the plasma laterally.[15]

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Ion-Induced Pinch and the Enhancement of Ion Current by Pinched Electron Flow in Relativistic Diodes*

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A new model for time-dependent and steady-state ion and electron flow in large-aspect-ratio diodes is constructed. The electron trajectories are computed with use of the self-consistent fields calculated during the initial ion motion. The dynamic formation of a tightly pinched electron flow is qualitatively explained. Very large ion currents, nearly equal to the electron current, are predicted for flat solid cathodes, when steady-state flow is achieved.

Dynamic formation of pinched electron flow has been observed in large-aspect-ratio diodes with hollow[1] and solid cathodes.[2-4] In the hollow-cathode geometry a ring of electrons hitting the anode plane is observed to collapse radially, with typical velocities of ~0.3 cm/nsec, forming a pinch of 1–3 mm diameter. Very similar phenomena are also observed when solid cathodes are used.[3] In early time, before the anode plasma has been formed, only electron flow will exist and its properties are well described by the laminar flow model of Goldstein, Davidson, Siambis, and Lee[5] (GDSL). This model predicts only weak pinching of the electron flow, for solid cathodes, and is in agreement with experimental observations[3] of the electron flow before the pinch collapses. At later time, the electrons striking the anode plane generate a plasma by a complicated process which is not yet fully understood but which is experimentally observed.[6] Earlier theories[4,5] have relied on the direct action of this anode plasma to focus the electron flow to a tight pinch. Since the anode plasma expands with a typical velocity of only a few centimeters per microsecond, the above-mentioned models are unable to explain the experimentally observed fast velocities of the collapsing pinch. The anode plasma, however, acts as a source of ions which are quickly accelerated into the diode region by the large electric field. We incorporate the dynamics of these ions into a new model which describes the initial time-dependent emission of ions from the anode plasma and their effect on the electric field near the anode. The modification of the electric field results in a radial drift of the electron orbits. This radial drift, combined with the gradual generation of plasma on the anode, explains the transition from laminar to pinched electron flow. Finally, an estimate of the ion current density for steady-state flow predicts a geometric enhancement of the total ion current above the Langmuir results for bipolar flow.

Consider the electron and ion flow in a flat-faced, axisymmetric diode (see Fig. 1) with large aspect ratio, $R/D \gg 1$, where R is the cathode radius and D is the distance between the cathode and anode plasmas. The GDSL model predicts the current density distribution of the electron emission from the cathode plasma and the current density of the relativistic electrons hitting the anode plane. At large radii, the electrons graze the anode plane with $P_z/P_r \sim O(D/R)$ where $\vec{P}$ is the electron mechanical momentum and the net force on an electron near the anode plane is $\vec{E}+(\vec{v}\times\vec{B})/c \simeq O(D/R)\vec{E}$. The fraction of the total current emitted from beyond the radius R_1 is $(\gamma_0-1)/\gamma_0$, where R_1 is the radius at which the outermost electron hits the anode plane and $\gamma_0 \equiv 1-eV_0/m_0c^2$, with V_0 the diode voltage. Thus, for $V_0 = 2$ MV, 80% of the current is emitted from large radii. The electrons striking the anode plane generate a plasma which acts as a source of ions. The heating of the anode material is

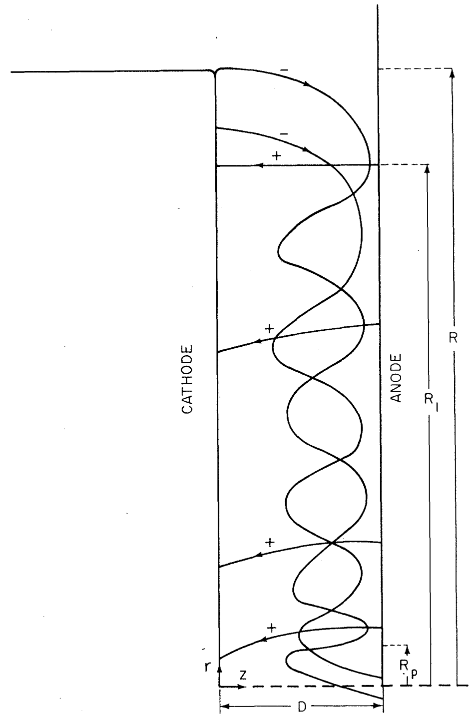

FIG. 1. Diode geometry with schematic electron (−) and ion (+) trajectories.

more efficient at large radii because of the grazing incidence of electrons on the anode plane. A dense anode plasma is thus expected to form there first. The initial motion of the ions, accelerated from the anode plasma, forms an ion sheath which modifies the electric field. We will follow the dynamics of the ion sheath as a function of time and show that the electric field drops (linearly in the sheath) from its value given outside the sheath by GDSL to zero on the anode plasma. This linear drop of the electric field across the ion sheath is in sharp contrast to the steady-state Child situation where the electric field goes to zero as $x^{1/3}$, where x is the distance from the anode plasma. This is not surprising since the initial ion motion is far from steady state.

While the anode plasma contains a diamagnetic electron flow which excludes the magnetic field, no such effect exists in the ion sheath, allowing for the full penetration of the magnetic field. When an electron enters the ion sheath with grazing incidence (i.e., $P_z/P_r \ll 1$) it will be reflected back into the diode by the unaffected magnetic field but reduced electric field. These electrons then move radially inwards until they reach a radius where no dense anode plasma has yet been produced. There, the enhanced electron flow hits the anode plane until enough ions are formed to allow further radial drift of the electron flow from large radii. The necessary ion-sheath thickness to provide such electron reflection is reached in times which are short enough (< 1 nsec) to explain the fast collapsing pinch. Also, a qualitative explanation for the collapsing hollow ring, as observed experimentally[1] for hollow cathodes, is provided by the above picture. The following quantitative analysis applies only for solid cathodes at large radii.

The dynamics of the ion sheath and electron orbits is now computed under the following assumptions. Using the electrostatic approximation to solve for the time-dependent electric field as a function of the distance x from the anode plasma with $E(x=0)=0$ we find, for $t>0$, $E(x,t)=\int_0^x 4\pi\rho(x',t)dx'$, where $t=0$ is the time of anode-plasma generation. This plasma is assumed to be dense enough to provide a sufficient reservoir of ions so as to reduce the diode electric field to zero in the anode plasma. The electric field in the diode is of order ~1 MV/cm and thus prevents anode-plasma electrons from following the ions. We follow the ion motion only for short times so that the ion front $x_F(t) \ll D$. Since the electric field is zero on both the cathode and anode plasmas, the total charge of the ions is nearly equal to the charge of the electrons present in the diode gap. The fraction of the electron charge in the very thin ion sheath is thus neglible compared to the ion charge. Outside the ion sheath, the electron charge dominates and results in an electric field $E(x_F)$ which is nearly constant in time. The total number of ions in the sheath during its early expansion is thus fixed. Solving the dynamics of N test ions, with equal intervals of space charge between them, one finds for the ith ion that $x_i(t)=x_i^0+v_i^0 t + \frac{1}{2}a_i t^2 \simeq \frac{1}{2}a_i t^2$, where $a_i=(i/N)ZeE(x_F)/M$, and the ions are ionized Z times and have mass M. At a given time, therefore, the ion distances are spaced in proportion to i which means a uniform ion density, n, for $0 \leq x \leq x_F(t)$. This density decreases as a function of time, $Zen = E(x_F)/4\pi x_F$, where $x_F(t)=x_N(t)$. An estimate for $E(x_F)$ is provided by GDSL and is

$$E(x_F)=\frac{V_0}{D}\left(\frac{\gamma_0+1}{\gamma_0-1}\right)^{1/2}\ln[\gamma_0+(\gamma_0^2-1)^{1/2}]. \quad (1)$$

It is easy to see that inside the constant-density ion sheath the electric field drops linearly to zero from its value $E(x_F)$.

We follow the orbit of an electron that enters at x_F with velocity $v_z/c \ll 1$ and large radial momentum. For a thin ion sheath we may neglect relative changes in the radial momentum and the magnetic field along the electron trajectory. The magnetic force is thus $-eE(x_F)$. The electron also undergoes little change of potential energy; hence, $\gamma \simeq \gamma_0$. The axial force equation for an electron is thus

$$\gamma_0 m_0\, dv_z/dt = eE(x_F)(z+x_F-D)/x_F, \quad (2)$$

which can be readily integrated. Electrons entering the ion sheath with an axial velocity v_0 will be reflected if their turning point lies inside the ion sheath. The turning-point distance is easily computed from Eqs. (1) and (2) and is less than x_F when

$$x_F > D\left(\frac{v_0}{c}\right)^2 \frac{\gamma_0}{(\gamma_0^2-1)^{1/2}\ln[\gamma_0+(\gamma_0^2-1)^{1/2}]}. \quad (3)$$

This ion-sheath thickness is achieved for typical diodes during ~1 nsec for $v_0/c < \frac{1}{6}$ and $D \sim 0.5$ cm. The electrons then follow complicated orbits inside the diode, drifting radially inwards with multiple reflections until the diode axis is reached and a tight pinch results. Thus, at small radii, the electron flow pattern includes accumulation of charge and the GDSL model does not apply any more. The electron and ion motion, before steady state is achieved, is presently under investigation using a time-dependent computer code.[7]

As yet no analytic theory exists describing the evolution of the electron and ion flow towards steady state. We can, however, draw a few conclusions about the flow properties in steady state. When calculating the ratio of ion current to electron current in the steady state the following considerations apply. The current for each species is equal to the amount of its charge inside the diode divided by its average crossing time. For space-charge-limited flow the total charge inside the diode is nearly zero. The current ratio is, therefore, inversely proportional to the crossing times. For strict one-dimensional motion of electrons from cathode and ions from anode, the crossing length is equal to D, the anode-cathode gap, and the ratio of crossing times depends only on the species velocities, giving $T_e/T_i = \langle D/v_e\rangle/\langle D/v_i\rangle = (m_e/M)^{1/2}$ which is the well-known Langmuir result for nonrelativistic electrons. Now, for strongly pinched electron flow, the ion crossing length remains equal to D, while the electrons move radially inwards and their crossing-length scale is of the order of the diode radius (R). The crossing time for the electrons is thus longer by a factor R/D relative to the one-dimensional flow. The ratio of the ion current is thus enhanced by the factor R/D, giving

$$\frac{I_i}{I_e} \simeq \frac{R}{D} \frac{\langle 1/v_e \rangle}{\langle 1/v_i \rangle} \tag{4}$$

which may clearly be above unity. If a very hollow cathode is used, D is then the gap distance in the hole, and the ion current is reduced. A more rigorous computation of the electron-ion current ratio is given by finding a lower bound for the ion charge density as a function of radius. For space-charge-limited flow from the cathode and the anode the electric field is zero at both surfaces and hence $\int_0^D \rho_i \, dz \approx \int_0^D |\rho_e| \, dz$.

We have shown that the electrons emitted at large radius reach the anode plane at small radius within the pinch. These electrons constitute the major contribution to the electron current. For radii $r > R_p$, where R_p is the radius of the pinch on the anode, the current flux through a cylinder of radius r and length D is

$$I_e = \int_0^D 2\pi r \rho_e v_{er} \, dz = 2\pi r c \int_0^D (\rho_e v_{er}/c) \, dz. \tag{5}$$

Since $|v_{er}/c| < 1$ (in reality this fluid velocity may be *much* smaller than c) we have $I_e < 2\pi r c \int_0^D |\rho_e| \times dz$. For steady-state ion flow, the axial ion current density J_i is only a function of radius. This gives the ion charge density, assuming no trapped ions (no virtual anodes): $\int_0^D \rho_i dz = \int_0^D (J_i/v_i) dz = J_i \langle D/v_i \rangle$. From the above one finds

$$J_i > \frac{I_e}{2\pi r} \frac{\langle (v_i/c)^{-1} \rangle^{-1}}{D}. \tag{6}$$

This $1/r$ dependence (excluding $r < R_p$, of course) gives the ratio of the ion current emitted from a disk of radius r to the total electron current:

$$I_i(r)/I_e > (r/D) \langle (v_i/c)^{-1} \rangle^{-1} \tag{7}$$

For a nearly constant electric field that drops to zero near the cathode and anode on a distance scale much smaller than D, the ion velocity scales as $v_i \sim (D-z)^{1/2}$ and this gives $I_i/I_e > \frac{1}{2}(R/D) \times (2eV_0/Mc^2)^{1/2}$. If the accelerated ions are protons and $V_0 \simeq 2$ MV then a diode with aspect ratio $R/D \simeq 20$ will generate an ion-to-electron current ratio of the order $I_i/I_e > 0.65$. The total diode current consists of the sum of the electron and ion currents and is self-limited by the total magnetic field. For the above numerical example the total current is above 1 MA giving about 0.5 MA of ion current.

The above estimates clearly indicate the possibility of using high-power electron-beam accelerators to generate ion beams efficiently. A plasma jet will, of course, be formed if these MeV ions are extracted through a meshed cathode and may be focused geometrically by use of a hemispherical diode. Two such diodes could provide a source of megampere ion currents, e.g., 5 MV-8 MA, which could be used to irradiate high-Z pusher-tamper spheres containing deuterium and tritium. Ion beams deposit their energy efficiently and the above power level is near breakeven.[8] Subsequent to the completion of this work we learned of the unpublished work of J. W. Poukey who found similar ion currents in computer simulations of diodes.

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