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Collective Nuclear States as Representations of a SU(6) Group*

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We propose a description of collective quadrupole states in even-even nuclei in terms of representations of a boson SU(6) group. We show that within this model both the vibrational and the rotational limit can be recovered.

The purpose of this note is to point out that the group SU(6) of six-dimensional special unitary transformations might provide the appropriate framework for a *unified* description of collective nuclear states. Restricting ourselves to even-even nuclei we observe that the main features of the collective nuclear motion are (i) the quadrupole ($L=2$) character of the excitations, and (ii) the near equality of the vibrational and rotational frequencies which does not allow a clear-cut distinction between the two different types of motion. An additional and important difference from other many-body systems is the limited number of particles available in each sef-consistent shell which introduces a cutoff in the vibrational and rotational bands.

To illustrate the usefulness of the SU(6) group in classifying the variety of observed spectra we construct a specific model which (i) has the properties mentioned above, (ii) can be shown to have analytic solutions corresponding to the vibrational and rotational limits, and (iii) reproduces, after slight rearrangement, the Hamiltonian derived by Janssen, Jolos, and Dönau,[1] using the Lie algebra of pair operators. The algebraic structure of this model was first discussed by Arima[2] who pointed out the existence of the two limits, to be described below, and by Taruishi[3] who numerically investigated the vibrational-rotational transition. In addition to suggesting a wider range of applicability of the model, we stress here (i) the presence of approximately unbroken symmetries for which there seems to be compounding experimental evidence, and (ii) the large set of analytic relations which can be derived in these limiting situations using standard group-theoretical methods[4-6] and which are the new and most useful aspect of the SU(6) boson group approach.

To begin with, we claim that a number of positive-parity states can be generated in even-even nuclei as states of a system of N bosons having no intrinsic spin but able to occupy two levels, a ground-state level with angular momentum $L=0$, and an excited state with angular momentum $L=2$. In the case in which the two levels are degenerate and there is no interaction between bosons, the five components of the excited $L=2$ state, called d for convenience, and the single component of the ground $L=0$ state, called s, span a six-dimensional vector space which provides the basis for the representations of the unitary group U(6). Disregarding phase transformations we can reduce U(6) to the unitary unimodular group SU(6). The representations of SU(6) are characterized by the symmetry properties of the wave function. For bosons the only allowed representations are the totally symmetric ones, belonging to the partition $[N]$ of SU(6). In the absence of interaction and for zero splitting between s and d levels, all states belonging to $[N]$ are degenerate. The residual interaction between bosons and the energy difference $\epsilon=\epsilon_d-\epsilon_s$ split the degeneracy and give rise to a definite spectrum. The spectrum is defined by ϵ, by the seven two-body matrix elements $\langle d^2L|V|d^2L\rangle$ $(L=0, 2, 4)$, $\langle d^20|V|s^20\rangle$, $\langle ds2|V|ds2\rangle$, $\langle ds2|V|d^22\rangle$, $\langle s^20|V \times|s^20\rangle$, and by the partition $[N]$ of SU(6) to which it belongs, nine parameters in all. The energy levels can be found by diagonalizing the model Hamiltonian

$$H=\epsilon\sum_m d_m{}^\dagger d_m+\sum_{L=0,2,4}\langle d^2L|V|d^2L\rangle[(d^\dagger d^\dagger)^{(L)}(dd)^{(L)}]^{(0)}+\langle ds2|V|d^22\rangle[(d^\dagger d^\dagger)^{(2)}(ds)^{(2)}+(d^\dagger s^\dagger)^{(2)}(dd)^{(2)}]^{(0)}$$
$$+\langle d^20|V|s^20\rangle[(d^\dagger d^\dagger)^{(0)}(ss)^{(0)}+(s^\dagger s^\dagger)^{(0)}(dd)^{(0)}]^{(0)}$$
$$+\langle ds2|V|ds2\rangle[(d^\dagger s^\dagger)^{(2)}(ds)^{(2)}]^{(0)}+\langle s^20|V|s^20|V|s^20\rangle[(s^\dagger s^\dagger)^{(0)}(ss)^{(0)}]^{(0)}. \tag{1}$$

Here $d^\dagger$ (d) and $s^\dagger$ (s) are the creation (annihilation) operators for bosons in the $L=2$ and $L=0$ state, the zero of the energy has been chosen in such a way that $\epsilon_s=0$, and the parentheses denote angular momentum couplings. To the extent that Eq. (1) describes collective states there are associated transition operators. The quadrupole operator is defined in terms of the two reduced matrix elements $(d\|\vec{\mathrm{Q}}\|d)$ and $(d\|\vec{\mathrm{Q}}\|s)$ and given by

$$\vec{\mathrm{T}}_m{}^{(2)}=(d\|\vec{\mathrm{Q}}\|s)[(d^\dagger s)_m{}^{(2)}+(s^\dagger d)_m{}^{(2)}]+(d\|\vec{\mathrm{Q}}\|d)[(d^\dagger d)_m{}^{(2)}]. \tag{2}$$

It is worthwhile mentioning at this stage that the Hamiltonian of Eq. (1) is equivalent to that derived by Janssen, Jolos, and Dönau.[1] In fact within the basis states $|s^{N-n_d}d^{n_d}[N]\chi LM\rangle$, where χ is whatever quantum number is needed to specify uniquely the states, the $s^\dagger$ and s operators can be replaced by c-number functions of n_d:

$$H=\epsilon n_d+\textstyle\sum_L\langle d^2L|V|d^2L\rangle[(d^\dagger d^\dagger)^{(L)}(dd)^{(L)}]^{(0)}$$
$$+\langle ds2|V|d^22\rangle\{[(d^\dagger d^\dagger)^{(2)}d]^{(0)}(N-n_d)^{1/2}+(N-n_d+1)^{1/2}[d^\dagger(dd)^{(2)}]^{(0)}\}$$
$$+\langle d^20|V|s^20\rangle\{[d^\dagger d^\dagger]^{(0)}\tfrac{1}{2}[(N-n_d)(N-n_d-1)]^{1/2}+\tfrac{1}{2}[(N-n_d+1)(N-n_d+2)]^{1/2}[dd]^{(0)}\}$$
$$+\langle ds2|V|ds2\rangle(N-n_d)n_d/\sqrt{5}+\langle s^20|V|s^20\rangle\tfrac{1}{2}(N-n_d)(N-n_d-1), \tag{3}$$

yielding, after a slight redefinition of the parameters, the Hamiltonian of Ref. 1.

We now show that for different choices of the parameters $\epsilon, \ldots,$ the Hamiltonian and transition operator of Eqs. (1) and (2) produce both vibrational- and rotational-like spectra. As these parameters change, the SU(6) model spans the entire variety of observed spectra. We begin by considering the case in which the energy ϵ is much larger than all interaction terms in Eq. (1). In that case the Hamiltonian is invariant under separate transformations among the five components of the $L=2$ state. Thus the states are characterized by the number of bosons occupying the $L=2$ level, n_d, and an (approximately) unbroken SU(5) symmetry emerges from the decomposition $\mathrm{SU}(6)\supset\mathrm{SU}(5)\otimes\mathrm{U}(1)$. The quantum number $n_s=N$

$-n_d$ plays no role in this case. The representations of SU(5) contained in $[N]$ are all the symmetric representations $[n_d=0]$, $[n_d=1]$, $[n_d=2]$, up to $[n_d=N]$. We have discussed previously[6] the quantum numbers which are needed to specify uniquely these states through the decomposition $\mathrm{SU}(5)\supset \mathrm{O}^+(5)\supset \mathrm{O}^+(3)$. In this limit the wave functions are labeled by $|[N][n_d]vn_\Delta LM\rangle$ and the energy spectrum is given by

$$E([N][n_d]vn_\Delta LM)=\epsilon n_d. \tag{4}$$

This is a vibrational spectrum cut off at $n_d=N$. In Ref. 6 we have shown that the $\mathrm{SU}(5)\supset \mathrm{O}^+(5)$ symmetry is preserved even if we introduce the two-body interaction terms $\langle d^2L|V|d^2L\rangle (L=0, 2, 4)$, in the sense that the different representations of SU(5) are split but not admixed by these terms.

The other limiting situation occurs when the energy ϵ is small and of the same order of magnitude of the two-body matrix elements. In particular if both the splitting ϵ and the two-body matrix elements correspond to those of a quadrupole-quadrupole interaction in a major oscillator shell $V=-\kappa\sum_{i,j}\vec{Q}_i\cdot\vec{Q}_j$, where κ is the strength of the interaction and Q_i the quadrupole operator of the ith boson, another approximate symmetry occurs.[4] The related wave functions serve now as a representation space for the groups $\mathrm{SU}(6)\supset\mathrm{SU}(3)\supset \mathrm{O}^+(3)$, and they are characterized by the quantum numbers $|[N](\lambda,\mu)KLM\rangle$, where (λ, μ) label the representations of SU(3) belonging to the partition $[N]$ of SU(6). Since $[N]$ is totally symmetric the decomposition $\mathrm{SU}(6)\supset\mathrm{SU}(3)\supset \mathrm{O}^+(3)$ is easy to carry out.[4] In this limit the energy levels are given by

$$E([N](\lambda, \mu)KLM)=9\kappa\left[\frac{L(L+1)}{12}-\frac{C(\lambda, \mu)}{9}\right] \tag{5}$$

where C is the Casimir operator of SU(3), $C(\lambda, \mu) =\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)$. For the symmetric representations, $C(\lambda, \mu)$ is only a function of N. Thus the entire spectrum is given in terms of the single parameter κ and of the partition $[N]$ of SU(6). The spectrum of Eq. (5) is shown in Fig. 1 for $N=8$. It is a rotational spectrum cut off at $L=2N$. The β, γ, and higher bands appear here in a natural way as representations of the boson SU(3) group. A survey of the available data appears to indicate that there is a large number of rotational-like nuclei whose spectrum is approximated by the boson SU(3) spectrum of Eq. (5). An example is shown in Fig. 2. Others can be found in the neighboring nuclei as well as in deformed nuclei in the Dy and W regions.

Quadrupole transitions can be calculated by taking matrix elements of the operator of Eq. (2) between eigenstates of the Hamiltonian Eq. (1). In

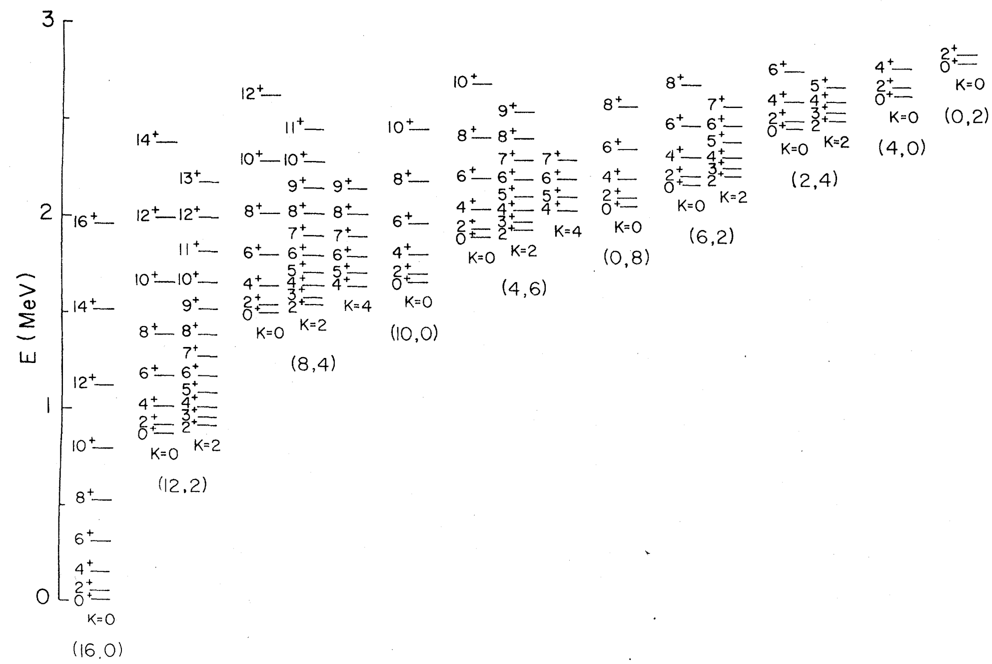

FIG. 1. Decomposition of the representation [8] of SU(6) in representations (λ,μ) of SU(3). The orthogonal basis of Vergados, Ref. 5, is used in the decomposition of $\mathrm{SU}(3)\supset \mathrm{O}^+(3)$.

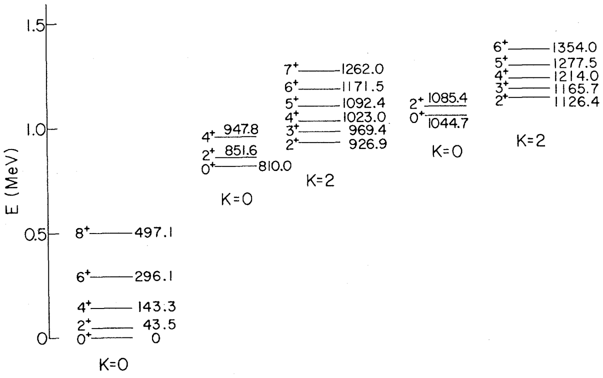

FIG. 2. Low-lying positive-parity bands in ^{234}U. The experimental energies are from the Nuclear Data Sheets.

the two limiting situations these matrix elements are easily constructed. Since we have already discussed the vibrational case[6] we consider here only the rotational limit. Again if the two reduced matrix elements in Eq. (2) correspond to those of the quadrupole operator in a major harmonic oscillator shell $\vec{Q}=\alpha\sum_i\vec{Q}_i$ the resulting $T_m^{(2)}$ is an operator of SU(3). Thus matrix elements between states of different representations (λ,μ) of SU(3) vanish and those between states of a given representation depend on the strength α and on the quantum numbers which label the representation. Formulas for these matrix elements can be derived using the methods described by Elliott.[4]

It is interesting to note that the quadrupole transitions between γ and β bands although retarded are not completely forbidden in the SU(3) limit. This is because both β and γ bands belong to the same representation $(2N-4, 2)$ of SU(3). As a consequence the ratio $B(E2\, 2_\gamma \to 0_\beta)/B(E2\, 2_\gamma \to 0_g)$ is expected to be large, since the transition $2_\gamma \to 0_g$ is strictly forbidden in the SU(3) limit. There is some evidence that this might be the case.[7]

In any event we believe that a description of collective states in terms of a SU(6) model might be appropriate, especially in the two limiting situations in which the approximate symmetries $O^+(5)$ and SU(3) occur. For nuclei whose spectrum is not too far from these exact symmetries it might be useful to use the respective unperturbed wave functions and energies, Eqs. (4) and (5), as a basis for a perturbative treatment. For example, a small breaking of the SU(3) symmetry yields for the ground-state band

$$\langle [N](2N,0)LM|H|[N](2N,0)LM\rangle = A + BL(L+1) + CL^2(L+1)^2. \qquad (6)$$

We have obtained analytic expressions for the coefficients A, B, and C in terms of ϵ and of the two-body matrix elements $V_L = \langle [2](4,0)L|V|[2](4,0)L\rangle$. We have also derived similar analytic relations for other bands, as well as for transition rates, in either case of an approximate $O^+(5)$ or SU(3) symmetry, and we will present them in a forthcoming longer paper. For the other, transitional, nuclei a diagonalization of Eq. (1) may be needed, although it is not excluded that other subgroups of SU(6) can be found which correspond to some other typical situation.

Finally we point out that a Hamiltonian similar to Eq. (1) has been derived by Kerman and Koonin[8] using the time-dependent Hartree-Fock approach. The transition from vibrational to rotational nuclei has also been studied by Moszkowski[9] in a two-dimensional version of Eq. (1).

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