able because the charge carriers are bosons.

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[1]U. Bonse and M. Hart, Appl. Phys. Lett. 6, 155 (1965). That this design would work as a neutron interferometer was shown by H. Rauch, W. Treimer, and U. Bonse, Phys. Lett. 47A, 369 (1974).

[2]H. J. Bernstein, Phys. Rev. Lett. 18, 1102 (1967).

[3]Y. Aharonov and L. Susskind, Phys. Rev. 158, 1237 (1967).

[4]R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. 34, 1472 (1975). This experiment was proposed by A. W. Overhauser and R. Colella, Phys. Rev. Lett. 33, 1237 (1974).

[5]The importance of the residual phase δ was pointed out recently by M. A. Horne and A. Shimony, to be published.

Spectrum of Strange-Quark–Antiquark Bound States*

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A calculation is made of the energy levels of the bound states of a strange quark and its antiquark with an interaction which includes an attractive Coulomb potential term, a confining linear potential, and spin-orbit, quadratic-orbit, spin-spin, and quadratic-spin interactions. Good agreement is obtained with observed mesons, and the existence of other mesons is predicted.

Within the framework of the quark model, we regard the φ meson as the lowest $^3S_1$ bound state of a strange quark s and its antiquark $\bar{s}$, and the η' meson as the lowest $^1S_0$ bound state of this system. Other mesons are known which can be interpreted, as we shall see, as excited $s\bar{s}$ states. We shall consider the spectrum of $s\bar{s}$ bound states using a rough analogy with the states of positronium. Specifically, we assume that the s and $\bar{s}$ quarks are subject to an attractive Coulomb-like potential, a short-range interaction effective in S states only, a spin-orbit interaction, and an interaction which goes like the square of the orbital angular momentum L. We depart from the positronium analogy by omitting still other terms present in the electron-positron interaction (like the tensor force) and by including a linear potential which confines the quarks.

Linear and/or Coulomb-like potentials have been used previously by a number of authors[1-5] to describe bound states of a charmed quark and its antiquark. Gunion and Willey[6] have considered the spectrum of mesons made of $s\bar{s}$ quarks (and the spectrum of other hadrons) using a linear confining potential with spin-spin and spin-orbit interactions but without a Coulomb-like potential. De Rújula, Georgi, and Glashow[2] have considered the hadron spectrum in perturbation theory using Coulomb-like forces. The paper of De Rújula, Georgi, and Glashow contains a good discussion of the theoretical justification of Coulomb-like models.

Our treatment of the $s\bar{s}$ interaction differs from those given in previous works in two important ways. First, although we solve an ordinary Schrödinger equation, we partially include the effects of relativity by using relativistic kinematics. Second, we include terms present in the positronium interaction which have been omitted previously except as perturbations. It turns out that the $s\bar{s}$ coupling strength is sufficiently large that a perturbation treatment and the use of nonrelativistic kinematics are both inadequate approximations. In particular, although we obtain good agreement with experiment in our model, we cannot obtain this agreement if we evaluate the effect of the spin-orbit potential in perturbation theory.

The interaction H' responsible for the fine-structure splitting in positronium is given by[8]

(we set $\hbar = c = 1$)

$$H' = \frac{1}{2m^2 r}\frac{dV(r)}{dr}(3\vec{L}\cdot\vec{S} + L^2) + \frac{\pi\alpha}{m^2}\delta(\vec{r})(1 + S^2 + \tfrac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2) + h, \quad (1)$$

where α is the fine-structure constant, m is the electron mass, $V(r)$ is the Coulomb potential, $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are Pauli spin operators, $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$, and h contains terms which we shall neglect in adapting this interaction to the $s\bar{s}$ system.

The quantity h contains a tensor interaction plus terms involving derivatives of r. We omit the derivative terms because including them would cause the wave equation to have a form which is different from the Schrödinger equation. In particular, the equation would contain a fourth derivative. We omit the tensor interaction, first because including it mixes states of different L and thereby increases considerably the difficulty in solving the Schrödinger equation numerically, and second because at the present time, there are not enough data to enable us to determine the strength of this interaction. Even omitting h, we include in our calculation more terms than have previously been considered in nonperturbation treatments of the quark-antiquark system.

In the case of $s\bar{s}$, we modify $V(r)$ to contain a linear term as well as a Coulomb-like term. Specifically, we let

$$V(r) = -\alpha_s/r + \beta r, \quad (2)$$

where α_s and β are parameters. We also need to modify the spin-orbit interaction of Eq. (1) because this interaction contains the term $(1/r)dV/dr$ which, near the origin, goes like $1/r^3$. This causes no difficulty in the case of positronium, where H' is considered as a perturbation. However, in our case, where α_s is considerably larger than the fine-structure constant, we wish to solve the Schrödinger equation numerically including the $\vec{L}\cdot\vec{S}$ and L^2 terms. But it is well known that there are no bound-state solutions of the Schrödinger equation in an attractive $1/r^3$ potential. Therefore, we modify the $1/r^3$ potential at small distances by introducing a cutoff. Specifically, we make the replacement

$$\frac{1}{r}\frac{dV}{dr} \rightarrow \frac{\gamma}{r}\left(\frac{\alpha_s}{r^2 + a^2} + \beta\right), \quad (3)$$

where γ is a parameter giving the strength of the spin-orbit interaction and a is the cutoff parameter. If γ goes to 1 and a goes to 0, the term on the right-hand side becomes simply $(1/r)dV/dr$, where V is given by Eq. (2). Our justification for introducing the cutoff parameter is that the $1/r^3$ singularity is not present in the original Dirac equation but arises in its nonrelativistic reduction. In including the strength parameter, we are departing from the positronium analogy. It turns out that in our best fit to the data, γ is very nearly unity. For simplicity, we take the linear confining potential to be independent of spin. We find that in treating $s\bar{s}$ states, this simple assumption is adequate. Including a spin dependence in the linear term would necessitate our using an additional parameter which we could not determine from the data.

We make still another change in adapting Eq. (1) to the $s\bar{s}$ system. We treat the $\delta(\vec{r})$ term in the interaction phenomenologically by multiplying it by a parameter ϵ. It turns out that in order to get agreement with experiment, we need to take ϵ small, so that we can treat the $\delta(\vec{r})$ term as a perturbation.

Thus, our prescription to find the bound-state energy levels is as follows: We solve the Schrödinger equation

$$-\nabla^2\psi + mU\psi = k^2\psi$$

where k is the momentum and the phenomenological potential U is given by

$$U = -\frac{\alpha_s}{r} + \beta r + \frac{\gamma}{2m_s^2 r}\left(\frac{\alpha_s}{r^2 + a^2} + \beta\right)(3\vec{L}\cdot\vec{S} + L^2). \quad (4)$$

Here m_s is the mass of the strange quark. This procedure yields wave functions $\psi_{nlj}(r)$ and momentum eigenvalues k_{nlj}, where n is the radial quantum number, l is the orbital angular momentum, and j is the total angular momentum. In order to obtain the energy levels from the eigenvalues k_{nlj}, we use relativistic kinematics. Specifically, we let E_{nlj} be given by

$$E_{nlj}^2 = 4(m_s^2 + k_{nlj}^2). \quad (5)$$

Using the wave functions ψ_{nlj}, we evaluate the contribution to the energy from the interaction term

$$(\epsilon\pi\alpha_s/m_s^2)\delta(\vec{r})(1 + S^2 + \tfrac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

in perturbation theory. The contribution ΔE_{nlj} from this term to the energy is

$$\Delta E_{nlj} = \frac{\epsilon\pi\alpha_s}{m_s^2}(1 + S^2 + \tfrac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2)|\psi_{nlj}(0)|^2. \quad (6)$$

Of course $\psi_{nlj}(0)$ vanishes unless $l = 0$. The mass M_{nlj} of any meson which is a bound state of $s\bar{s}$ is then given by

$$M_{nlj} = E_{nlj} + \Delta E_{nlj}. \quad (7)$$

This expression for M_{nlj} contains six parameters, α_s, β, γ, a, ϵ, and m_s, which can be adjusted to fit the data.

In order to make contact with experiment, we need to identify the observed mesons which are likely candidates for excited states of the $s\bar{s}$ system. Such mesons must have isospin zero. Also, according to Zweig's rule,[9] they should have relatively narrow widths and should decay prominently into $K\bar{K}$ pairs. From the main meson table of Chaloupka *et al.*,[10] we see that likely candidates are the S^*, D, E, and f'. These mesons all have widths less than 60 MeV, and are considerably narrower than other mesons of comparable masses. In order to assign these mesons to the $s\bar{s}$ spectrum, we need to know their spins and parities. Unfortunately the spins and parities of the D and E mesons have not been definitely determined. However, we shall make tentative assignments as shown in Table I.

Altogether, we have identified six mesons as possible states of $s\bar{s}$: φ, η', S^*, D, E, and f'. Our expression for the meson masses contains six parameters. In varying these parameters to obtain a fit to the data, we have kept the mass of the s quark fixed at the value $m_s = 0.4$ GeV, the value estimated by Cheng and James.[11] The values of the other parameters turn out to be

$$\alpha_s = 1.375, \ \beta = 2.441 \text{ fm}^{-2}, \ \gamma = 0.8766,$$

$$a = 0.4374 \text{ fm}, \ \epsilon = 0.0291.$$

The calculated values of the masses are given in Table I. Thus, we have fit six levels by varying five parameters. However, even if we had needed to vary all six, we would still claim that "this is not an empty exercise,"[12] as it is not obvious *a priori* that our prescription is suitable to fit the energy levels. For example, if we had used perturbation theory to calculate the spin-orbit splitting, as has been done in all previous treatments where the spin-orbit interaction has been included, we would not have been able to get nearly as good agreement with the ^3P levels. This can be seen as follows: In perturbation theory, the splitting of the ^3P levels is proportional to $\vec{L}\cdot\vec{S}$. This gives a separation between $^3P_2$ and $^3P_1$ levels which is twice the separation between the $^3P_1$ and $^3P_0$ levels. In contrast, if the assignments we have made in Table I are correct, the experimental splitting of the $^3P_1$ and $^3P_0$ levels is larger than the $^3P_2$-$^3P_1$ splitting. It is worth while mentioning that we were unable to fit the ^3P levels with $\alpha_s = 0$, even when we solved the Schrödinger equation exactly. Thus, our model is the only one proposed so far, to our knowledge, which is able to obtain a fairly good fit to the ^3P levels.

As another example of the fact that the interaction we have chosen is useful to fit the energy levels, we mention the L^2 term. In our model, the magnitude of this term is fixed relative to the magnitude of the $\vec{L}\cdot\vec{S}$ term. Yet we find that there is just the right amount of L^2 interaction to make the ^3P levels come out right relative to the S levels. Without the L^2 term, the ^3P levels lie too

TABLE I. Experimentally observed mesons which we identify as states of $s\bar{s}$. Under the column labeled J^{PC} we give the preferred values of the spin, parity, and charge-conjugation parity of each meson, with values not excluded by experiment given in parentheses. Under the column labeled $^{2S+1}L_J$ we give the preferred values of the spin multiplicity, the orbital angular momentum, and total angular momentum of the $s\bar{s}$ state corresponding to the meson.

Meson	J^{PC}	$^{2S+1}L_J$	Mass (MeV)	Width (MeV)	Calculated mass (MeV)
η'	0^{+-}	$^1S_0$	957.6 ± 0.3	<1	957.7
S^*	0^{++}	$^3P_0$	993 ± 5	40 ± 8	993
φ	1^{--}	$^3S_1$	1019.7 ± 0.3	4.2 ± 0.3	1019.8
D	$1^{++}(0^-, 2^-)$	$^3P_1$	1286 ± 10	30 ± 20	1267
E	$0^{-+}(1^+, 2^+)$	$^1S_0$	1416 ± 10	60 ± 20	1414
f'	2^{++}	$^3P_2$	1516 ± 3	40 ± 10	1514

low relative to the S levels, and another adjustable parameter has to be included to take care of this.

In our treatment of the contact interaction, we found that it was necessary to introduce a small parameter $\epsilon = 0.0291$. This fact shows that the analogy between the $s\bar{s}$ bound states and positronium is a rough one. For this reason, we do not take too seriously our omission of a tensor interaction. While such an interaction with the same strength as α_s would substantially alter several energy levels, it is possible that a much weaker tensor interaction is appropriate. The strength of the tensor interaction is thus a parameter which we cannot determine with the present data. As we have seen, we have obtained a reasonably good fit with no tensor interaction at all.

We have treated the mesons φ, η', S^*, D, E, and f' as if they were pure $s\bar{s}$ states. According to other interpretations, these mesons may contain mixtures of nonstrange-quark–antiquark pairs. In particular, the η' is often regarded as primarily an SU(3) singlet, with only a small admixture of SU(3) octet.[12] If this is the case, then the η' contains a substantial admixture of nonstrange-quark–antiquark pairs. However, in our interpretation the mass of the s quark is substantially larger than that of the nonstrange quark and thereby breaks SU(3) symmetry. Therefore, even the η', which is probably the least "pure" of all the $s\bar{s}$ states, contains only a small amount of nonstrange quarks. In this picture the η is composed primarily of nonstrange quarks, and the fact that it is appreciably heavier than the pion must be attributed to isospin-dependent forces.

With the values of the parameters we have found, one can go on to predict the masses of other states of $s\bar{s}$. Some of these predicted levels are given in Table II. These mesons should all have isospin zero, have anomalously narrow widths, and have the property of decaying prominently into $\bar{K}K$ pairs, either with or without additional pions. In Table II we also give the expected prominent decay modes.

Of interest is our prediction of two mesons, one with $J^{PC} = 1^{+-}$ and the other with $J^{PC} = 1^{--}$, which are essentially degenerate with the E meson. Two other mesons, one with $J^{PC} = 0^{++}$ and one with $J^{PC} = 1^{--}$, are predicted to have only slightly higher masses. The energy between 1400 and 1470 MeV should be a very rich one to explore experimentally, if our model is a good one. As the meson masses increase, the production cross sections will decrease, so that the mesons of higher mass will be increasingly more difficult to find. We recommend a careful search for mesons of masses between 1400 and 1800 MeV, as the real test of our model will be whether or not mesons with the predicted quantum numbers are discovered with masses near the predicted values.

TABLE II. Some mesons predicted by the $s\bar{s}$ model. We include states with principal quantum number ≤ 3. All these states have isospin zero and G parity equal to C.

J^{PC}	$^{2S+1}L_J$	Mass (MeV)	Prominent decay modes[a]
1^{+-}	$^1P_1$	1414	$K\bar{K}\pi$
1^{--}	$^3D_1$	1416	$K^+K^-, K_SK_L, K\bar{K}\pi$
0^{++}	$^3P_0$	1427	$K^+K^-, K_SK_S, K_LK_L, K\bar{K}\pi\pi$
1^{--}	$^3S_1$	1450	$K^+K^-, K_SK_L, K\bar{K}\pi$
1^{++}	$^3P_1$	1557	$K\bar{K}\pi$
2^{--}	$^3D_2$	1577	$K\bar{K}\pi$
2^{-+}	$^1D_2$	1638	$K\bar{K}\pi$
1^{+-}	$^1P_1$	1650	$K\bar{K}\pi$
0^{-+}	$^1S_0$	1658	$K\bar{K}\pi$
1^{--}	$^3S_1$	1688	$K^+K^-, K_SK_L, K\bar{K}\pi$
2^{++}	$^3P_2$	1722	$K^+K^-, K_SK_S, K_LK_L, K\bar{K}\pi$
3^{--}	$^3D_3$	1741	$K^+K^-, K_SK_L, K\bar{K}\pi$

[a]Decays with additional pions are also expected.

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[1]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[2]A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975).

[3]T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, Phys. Rev. Lett. 34, 365 (1975).

[4]E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lande, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975).

[5]B. J. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 168, 706 (1975).

[6]J. F. Gunion and R. S. Willey, to be published.

[7]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[8]A. J. Akhiezer and V. B. Berestetsky, *Quantum Electrodynamics* (Interscience, New York, 1965).

[9]G. Zweig, unpublished.

[10]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).

[11]T. P. Cheng and P. B. James, Phys. Rev. Lett. 34, 917 (1975).

[12]The quote is from Ref. 4, where three parameters were used to fit two pieces of data.

[13]See, e.g., J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

Two-Particle Correlations in Inclusive and Semi-inclusive $\pi^- p$ Reactions at 200 GeV/c*

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Significant two-particle correlations of dynamical origin are observed in 200 GeV/c $\pi^- p$ inclusive interactions. This is demonstrated by comparison with kinematic correlations calculated from an independent-particle-emission model. Two distinct correlation types are observed: (a) unlike-particle correlations with correlation length ~ 1.3 rapidity units independent of azimuthal separation, and (b) like-particle correlations with correlation length ~ 0.4 rapidity units which are observed only for small azimuthal separations.

Previous studies of two-particle rapidity correlations in pp [1] and $\pi^\pm p$ [2] interactions at Fermilab and the CERN Intersecting Storage Ring have established the existence of significant correlations between particles at small rapidity separations. Conclusions differ, however, as to whether the correlations are the same for like and unlike particles, as to their semi-inclusive and azimuthal dependence, and indeed as to whether the correlations are of dynamical origin or are an artifact of either energy-momentum conservation or the convolution of uncorrelated spectra for different topologies. We present new data, with 2–5 times the previous statistics, which demonstrate that correlations for identical and nonidentical particles exist as distinct entities, for both inclusive and semi-inclusive reactions, with different azimuthal behavior. Comparison with an independent-particle-emission model shows that these correlations have dynamical significance, and are not merely a consequence of energy-momentum conservation.

We have taken some 120 000 sets of bubble- and spark-chamber photographs of 200-GeV/c $\pi^- p$ interactions, using the Experiment 2B hybrid spectrometer arrangement at Fermilab. Four dual wide-gap optical spark chambers are set up downstream of the 30-in. bubble chamber to permit the measurement of fast, forward secondaries with high precision. An additional 40 000 photographs were obtained by operating the wide-gap spark-chamber system behind the Fermilab–University of California at Berkeley "bare chamber" experiment 137. The data reported here are from these runs and consist of ~ 10 000 inelastic events of all topologies.

We define the normalized two-particle and single-particle densities ρ_2 and ρ_1 as

$$\rho_2(y_1,y_2)=\frac{1}{\sigma_{\text{inel}}}\frac{d^2\sigma}{dy_1\,dy_2},\quad \rho_1(y)=\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma}{dy},$$

where y_1 and y_2 are the center-of-mass rapidities of particles 1 and 2. We then use the correlation