# Proton-Nucleus Interactions at High Energies and Scaling*

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In proton-nucleus interactions at 200 GeV in nuclear emulsion we have found that in the projectile region the multiplicity is independent of the target nucleus, while in the target region it is approximately proportional to the nuclear thickness. After carefully excluding the elastic and the coherent events, we find that the secondary particles produced in $p$-nucleus collisions obey the same Koba-Nielsen-Olesen scaling as found previously in $p$-$p$ collisions.

Recently there have been intense experimental and theoretical efforts to explore the mechanisms of hadron production in strong interactions. Experiments using nucleons as targets give data only on the asymptotic states produced, while experiments with nuclei as targets offer the unique opportunity of studying the space-time development of the final state of a nucleon-nucleon interaction.[1]

Recent experimental results[2-5] at 200–300 GeV in nuclear emulsion and in other different targets[6] have favored Gottfried's model.[1] We have found that the average number $\langle n_s\rangle$ of relativistic charged shower particles ($\beta > 0.7$) grows slowly with the atomic weight, $A$, of the target.[3,4] The rise is primarily in the target-fragmentation region, while in the projectile region, the multiplicity is the same for heavy-nuclei targets as for hydrogen.[4] For nucleon-nucleus interactions, if we represent the multiplicity ratio by $R_A = \langle n_s\rangle_{(pA)}/\langle n_s\rangle_{(pp)} = A^\alpha$, where $\langle n_s\rangle_{(pp)}$ and $\langle n_s\rangle_{(pA)}$ are average shower-particle multiplicities in $p$-$p$ and $p$-$A$ collisions and $\alpha$ is a constant, we have found that $R_{\rm CNO} = 1.41$, $R_{\rm em} = 1.74$, and $R_{\rm AgBr} = 1.8$ with $\alpha \approx 0.13$, a very weak dependence on mass number $A$. These relations are independent of energy at $E \geqslant 100$ GeV, and in nuclear emulsion we have found a linear relation of the form $R_{\rm em} = a + bN_h$, where $a = 1$, $b = 0.06$, and $N_h$ is the number of heavily ionizing evaporation prongs ($\beta < 0.7$), a simple measure of the size of the target nucleus or its degree of excitation. This scaling relation is true for $E \geqslant 100$ GeV and we tested[4] it at 200, 300, and 1000 GeV proton energies. This scaling law is very useful to predict the multiplicities from nuclear targets, once the multiplicity from $p$-$p$ collisions at the same energy is known. We have also found that the ratio $R_E = \langle n_s\rangle_{E_1}/\langle n_s\rangle_{E_2}$ of the mean multiplicities of fast charged secondaries produced on a nuclear target at energies $E_1$ and $E_2$ (when $E_1, E_2 \geqslant$ GeV) is constant in nuclear emulsion as a function of $N_h$ and this value is the same as for $p$-$p$ interactions (e.g., if $E_1 = 300$ GeV and $E_2 = 200$ GeV then for overall emulsion $R_E = \langle 16.0\rangle_{300}/\langle 13.6\rangle_{200} \approx 1.17$ and for $p$-$p$ interactions $R_E = \langle 8.86\rangle_{300}/\langle 7.65\rangle_{200} \approx 1.15$). In the analysis of charged multiplicity distributions in $p$-$p$ interactions, it has been shown[7] that the dispersion $D = [\langle n_s^2\rangle - \langle n_s\rangle^2]^{1/2}$ is a linear function of the average charged multiplicity $\langle n_s\rangle$, i.e., $D = a\langle n_s\rangle - b$, where $a = b = 0.58$. Wroblewski[7] has predicted that $\langle n_s\rangle/D$ will tend towards an asymptotic value of 1.7, and experimentally it is found[8] to be ~2. For 200- and 300-GeV $p$-nucleus interactions, by neglecting the elastic and coherent events, we find $D = 8.0 \pm 0.5$ and $8.5 \pm 0.5$ with $\langle n_s\rangle = 13.6 \pm 0.5$ and $16.0 \pm 1.5$, respectively, and in each case the value of $\langle n_s\rangle/D$ is ~1.8, just as found in $p$-$p$ interactions at these energies.[9] In order to see the development of the shower particles with mass $A$ and energy $E$, we divide the shower particles into inner ($\tan\theta < 1/\gamma_c$, where $\gamma_c$ is the c.m. Lorentz factor) and outer ($\tan\theta \geqslant 1/\gamma_c$) cones in the laboratory system. We show in Figs. 1(a) and 1(b) the distributions of shower particles produced in light-element (CNO) and heavy-element (AgBr) groups, respectively, for 200-GeV protons, and in Figs. 1(c) and 1(d) for 300-GeV protons, for different inner and outer cones. In both beams we find that the particles produced in the inner cones remain practically constant in each group for different $N_h$ values, such that $(R_A)_{\rm inner} = \langle n_s\rangle_{(pA)}/\langle n_s\rangle_{(pp)} \sim 1$, while for outer cones, in both cases, $\langle n_s\rangle$ increases slowly with $N_h$, i.e., $(R_A) = C_A'$, where $C_A'$ is constant as a function of energy and is $>1$. Thus we see that the whole increase of $\langle n_s\rangle$ with $N_h$ within the same group, CNO or AgBr, comes from the slow particles produced only in the outer cone and this increases with the increase in energy.

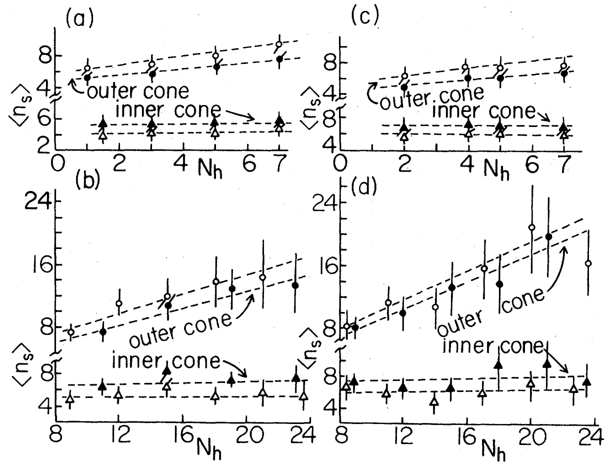

FIG. 1. $\langle n_s \rangle$ versus $N_h$ for inner cone (triangles) and outer cone (circles) (a) for CNO at 200 GeV, (b) for AgBr at 200 GeV, (c) for CNO at 300 GeV, and (d) for AgBr at 300 GeV. In (a) and (b), the inner cone is defined by $\tan\theta \le 0.07$ (open points) or 0.1 (solid points) and the outer cone by $\tan\theta > 0.07$ (solid) or 0.1 (open); in (c) and (d), the respective values are 0.06 and 0.08. Dotted lines are drawn by free hand.

Recently the scaling laws proposed by Feynman and Yang have been extended theoretically to "semi-inclusive reactions" by Koba, Nielsen, and Olesen[10] (known as KNO scaling) and this has been observed experimentally by Slattery[11] and others.[12] The KNO scaling law can be stated in either of two ways:

$$\sigma_n/\sigma_{\rm inel} = \langle n \rangle^{-1}\psi(n/\langle n \rangle) \text{ or } \langle n^q \rangle = C_q \langle n \rangle^q, \tag{1}$$

where

$$\langle n^q \rangle = \sum_{n=1}^{\infty} n^q \sigma_n/\sigma_{\rm inel}$$

is the $q$th moment of the multiplicity distribution for $q = 2, 3, 4, \ldots$, $\sigma_n$ is the partial cross section for producing $n$ charged particles,[13] $\sigma_{\rm inel}$ is the total inelastic cross section (excluding coherent events), $\langle n \rangle$ is the average charged-particle multiplicity, and $\psi$ and $C_q$ are energy-independent functions. We have already proved for different parameters the similarity that exists between $p$-$p$ and $p$-nucleus interactions at 200–300 GeV and now we shall extend this discussion to the KNO scaling law.

First, we look at the white-star events,[3] i.e., events with $N_h = 0$ (no black prong) or $N_h = 1$ (one black prong in the forward hemisphere). In Fig. 2(a) we plot $\langle n \rangle(\sigma_n/\sigma_{\rm inel})$ versus $n/\langle n \rangle$ for white stars with even multiplicities (i.e., for quasi $p$-$p$ interactions) after neglecting all elastic events.

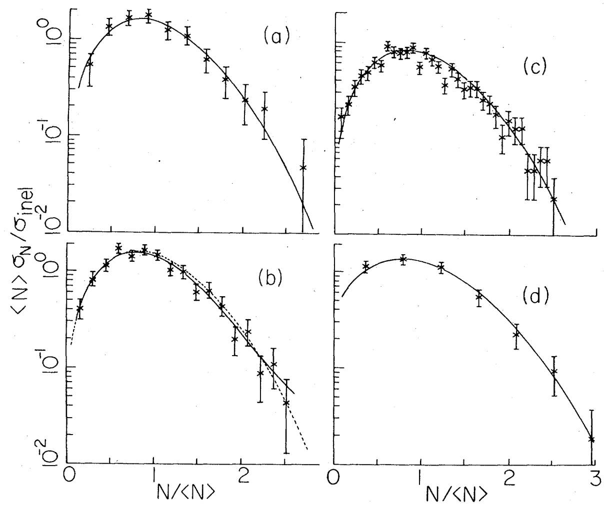

FIG. 2. (a) Plot of $\langle n \rangle(\sigma_n/\sigma_{\rm inel})$ versus $n/\langle n \rangle$ for $p$-nucleus even charged multiplicities for white stars ($N_h = 0, 1$) at 200 GeV. The functional form of the curve is presented in the text [Eq. (2)]. (b) KNO scaling for all events with even multiplicity for $N_h \ge 0$. The solid and dotted curves are empirical fits to the data, given by Eqs. (3) and (2), respectively. (c) KNO scaling for all events with all multiplicities for $N_h \ge 0$. The curve is given by Eq. (2) normalized to our data. (d) KNO scaling for all events with even multiplicities for $N_h \ge 0$ for $\pi^-$-nucleus interaction at 16.0 GeV. The curve is a fit to Eq. (8), Ref. 12, for $\alpha = 0.65$.

Our experimental points lie on a universal curve given by Slattery[11] which he fitted with the empirical function

$$\psi(z = n/\langle n \rangle) = (3.97z + 33.7z^3 - 6.64z^5 + 0.332z^7) \times \exp(-3.04z). \tag{2}$$

The overall $\chi^2$ is 6.99 for eleven data points. In Fig. 2(b) are shown the experimental data points for all events with an even number of produced charged particles with $N_h \ge 0$; these are fitted with a function given by

$$\psi(z) = e^{-\beta z} \sum_{m=0}^{M} \alpha_m z^m,$$

and the curve is shown in Fig. 2(b) where $\beta$ and $\alpha_m$, $m = 0, 1, \ldots, M$, are chosen so as to minimize the overall $\chi^2$. Following Slattery[11] we have chosen only odd values of $m \le 7$. The solid curve shown in Fig. 2(b) is described by the formula

$$\psi(z) = (3.80z + 34.97z^3 - 8.86z^5 + 0.745z^7) \times \exp(-3.055z), \tag{3}$$

with the two constraints having the same values

TABLE I. Experimental values for the ratios $\langle n^q \rangle / \langle n \rangle^q$ for $p$-$p$ and $p$-nucleus interactions at 200 GeV.

| $q$ | HBC[a] | White stars[b] | Even events[c] | All events[d] |
|---|---|---|---|---|
| 2 | 1.258 ± 0.019 | 1.232 | 1.229 | 1.259 |
| 3 | 1.856 ± 0.065 | 1.778 | 1.871 | 1.946 |
| 4 | 3.08 ± 0.18 | 2.896 | 3.298 | 3.458 |
| 5 | 5.60 ± 0.46 | 5.18 | 6.52 | 6.84 |

[a]Slattery's data from hydrogen bubble chamber.
[b]$p$-$p$ events from white stars in emulsion.
[c]$p$-nucleus even events from emulsion.
[d]All events from emulsions.

(2) as given by Slattery.[11] In the same figure we show by dotted lines the theoretical curve given by Slattery from Eq. (2). The values of $\chi^2$ in fitting the seventeen data points with Eqs. (2) and (3) are 10.62 and 9.37, respectively, which are very close to one another. In Fig. 2(c) is shown the KNO scaling for proton-nucleus events for all multiplicities with $N_h \geq 0$, excluding elastic and coherent events.[2] The overall $\chi^2$ between the theoretical curve given by Eq. (2), normalized to our data, and the experimental points is 35.9 for 34 data points. Thus we see that a universal behavior of the multiplicity distribution in $p$-nucleus interactions is just of the same form as in nucleon-nucleon interactions. Furthermore, we show in Table I the experimental values of the ratio $\langle n^q \rangle / \langle n \rangle^q = C_q$ (energy-independent parameters) for $q = 2$–$5$: (i) for events with even multiplicity with $N_h = 0, 1$ (white stars), (ii) for all events ($N_h \geq 0$) with even multiplicity, (iii) for all events with $N_h > 0$. These values are compared with the bubble-chamber data from the 205-GeV proton beam. We find that the values of the first few moments observed in our data are very close to those observed in $p$-$p$ interactions at this energy. Recently it has been found that the KNO scaling law given by Eq. (1) is not so well satisfied at low energies and it is modified to a new scaling law given by Eq. (8) of Ref. 12. In order to check this new scaling law in pion-nucleus interactions, we used our data from 16.0-GeV $\pi^-$ interactions in nuclear emulsion for even multiplicities with $N_h \geq 0$ and the results are shown in Fig. 2(d). We find that for $\alpha = 0.65$, the fit of the experimental data with KNO scaling given by Eq. (8) of Ref. 12 is very good, giving $\chi^2 \approx 1.17$ for seven data points.

In conclusion, we can say that for $p$-nucleus interactions, the increase in multiplicity comes only in the fragmentation region of the target, and at energies of 200–300 GeV the nuclear multiplicity follows a scaling law. Proton-nucleus interactions follow the same KNO scaling law as $p$-$p$ interactions. For low energies the modified form of KNO scaling works very well.[14]

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[13]Produced charge particles $n$ include the light and grey tracks only. For white stars only, $n$ includes all tracks, as in $p$-$p$ interactions.

[14]After the completion of this work on KNO scaling, we received a preprint from Professor W. D. Walker where he has used the KNO scaling in $\pi^-$-Ne collisions at 10.5 and 200 GeV. We are thankful to Professor Walker for this information before its publication.