

tivation cross section is lowered by $\approx 30\%$, as indicated by the Los Alamos $\pi^+ -^{12}\text{C}$ remeasurement.¹⁰

The cross sections for excitation of the 6.131-MeV, 3^- level in ^{16}O are nearly equal, as expected for inelastic scattering from a $T=0$ target.

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¹See, e.g., the work reviewed by D. S. Koltun, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3, Chap. 2.

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($^6\text{Li}, d$) Reaction on fp -Shell Nuclei and Alpha-Transfer Distorted-Wave Born-Approximation Analysis

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Strongly structured ($^6\text{Li}, d$) angular distributions exhibiting systematic variations with A for $A=40$ to $A=90$ target nuclei are well reproduced by α -stripping calculations using a single set of potential parameters. Zero-range and finite-range distorted-wave Born-approximation results are found very similar both in the shapes of the angular distributions and in the relative magnitudes. The question of nuclear-structure dependence in the angular distributions is discussed.

The ($^6\text{Li}, d$) reaction has been studied on a series of fp -shell nuclei at the University of Rochester.^{1,2} In this note we want to demonstrate the capability of the α -transfer distorted-wave Born

approximation (DWBA), in a very simple parameterization, to reproduce the highly structured angular distributions for the full range of nuclei investigated. Exact recoil finite-range calcula-

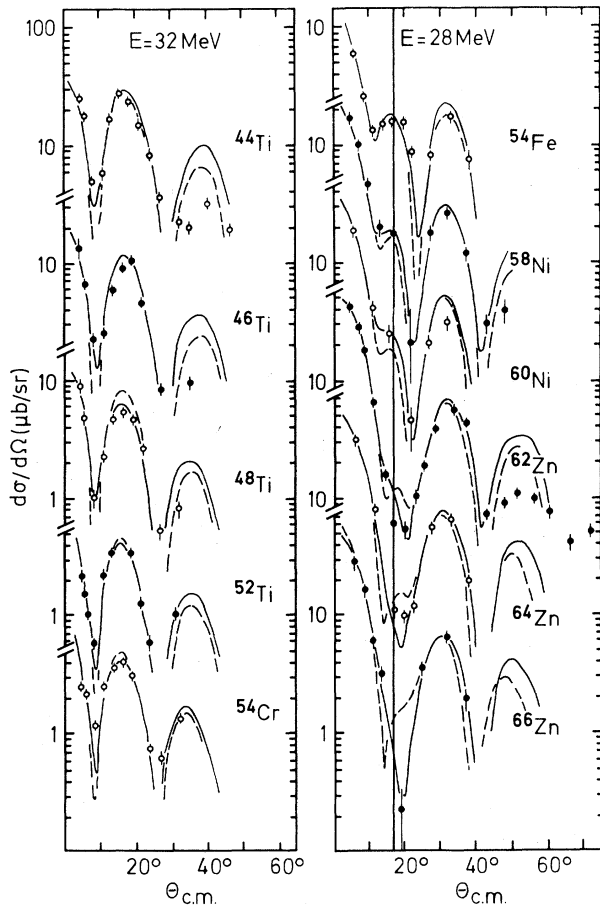


FIG. 1. Angular distributions for $L=0$ transitions to the ground states of the final nuclei indicated. The curves represent α -stripping calculations in zero-range (dashed curves) and finite-range DWBA (solid curves).

tions are compared with zero-range calculations. Further the question of possible nuclear-structure dependence of the angular distributions, recently raised,³ is discussed.

In Fig. 1 we present angular distributions for 0^+ final ground states. They have in common the forward maximum typical of $L=0$ transfer. Note however the systematic changes—particularly near the second maximum—with increasing A and Z . These details are well reproduced by the calculations described below.

The DWBA matrix element of the α -transfer reaction $A(^6\text{Li}, d)B$ is given by

$$T_{\text{DWBA}} = \langle \chi_{Bd} \varphi_{\alpha A}^B | V^D | \varphi_{\alpha d}^A \chi_{A\alpha} \rangle,$$

where φ refers to the bound-state wave functions and χ to the distorted waves in the channels indi-

TABLE I. Potential parameters.^a

| Channel | V | r_r | a_r | W_D | r_i | a_i | r_c |
|------------------|-----|----------------------|-------|-------|-------|-------|---------|
| $^6\text{Li}, A$ | 250 | b | 0.65 | 30 | b | 0.65 | 2 |
| d, B | 95 | 1.14 | 0.8 | 10 | 1.4 | 0.8 | 2 |
| | | 1.10 ^c | | | | | |
| α, A | | 1.33 | 0.65 | | | | 2 |
| α, d | | $R=4.4$ | 0.65 | | | | $R_c=R$ |
| V^D | | $R=3.0$ ^d | 0.65 | | | | |

^aForm factors: Woods-Saxon for real potentials; Woods-Saxon derivative for imaginary potentials. Well depths in MeV; lengths in fm.

^b $R_r = R_i = 0.9(A^{1/3} + 1.9)$.

^c1.10 for Ca isotopes.

^dSee Ref. 8.

cated. The exit-channel distorted wave χ_{Bd} was generated with the average deuteron optical potential from the elastic-scattering analysis of Newman *et al.*⁴ For the entrance channel a strongly absorbing potential with a “deep” real well $V(^6\text{Li}) \approx V(d) + V(\alpha)$ was selected from a variety of discrete potential families.⁵ The usual $r_0 A^{1/3}$ dependence of radii was adopted, except for the ^6Li channel where the relation $R_{A6} = r_0(A^{1/3} + c)$ was used which takes account of the finite size of the ^6Li projectile ($c \approx 6^{1/3}$).

The bound-state wave functions φ were simple wave functions of relative motion of the cluster pairs d, α and A, α in a Woods-Saxon well with the depth adjusted to reproduce the corresponding α -separation energies. The radial quantum numbers N and L were determined as usual by the relation of oscillator-energy conservation in a Talmi-Moshinsky transformation from single-nucleon to cluster coordinates. With the assumption that the clusters be in their intrinsic ground states one obtains $2(N-1) + L = 12$ for $(1f2p)^4$ final states. The orbital momentum L is equal to the final spin in the present cases of 0^+ target nuclei.

For $^6\text{Li}_{g.s.}(1^+)$ the above prescription allows $NL = 2S$ and $NL = 1D$. The latter component is however believed to be very weak,⁶ and a $2S$ -type wave function—as also predicted by the antisymmetrized-cluster model of Kudeyarov *et al.*⁷—was used in the present calculations.

The interaction potential V^D is in post representation the difference between the exit-channel total interaction and the potential generating the exit-channel distorted wave. In analogy to the approximation $V^D \approx V(r_{cn})$ usually made for a sin-

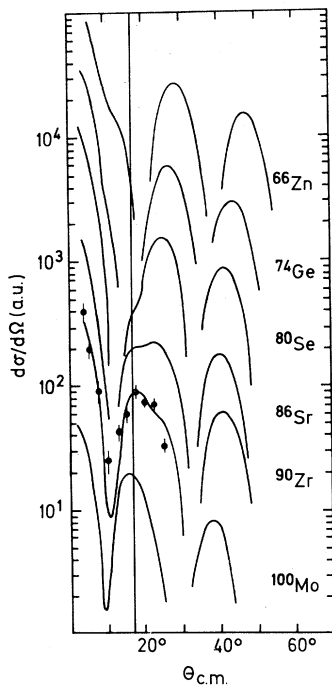


FIG. 2. Angular distributions for $L=0$ ground-state transitions. The target nuclei are indicated. The energy is 32 MeV. The curves represent finite-range DWBA calculations.

gle-nucleon-transfer reaction $A([c+n], c)$, we assume⁸ here $V^D \approx V(r_{\alpha\alpha})$.

With the code LOLA,⁹ exact finite-range DWBA calculations have been performed using the potential set given in Table I. This potential set turns out to suffice for the description of the angular distributions of the full range of nuclei studied. Note particularly the systematic trend observed experimentally at around 18° (Fig. 1); it is almost perfectly reproduced by the calculations.

To investigate the trends further we have extended the calculations to heavier nuclei up to $A=100$. Some examples are displayed in Fig. 2. The predictions are nicely confirmed by the experimental observations for ^{90}Zr .

The successful fitting of distributions for different orbital momenta up to $L=6$ is illustrated in Fig. 3 for ^{44}Ti states (for example). Similar success is observed for other nuclei.² The assignment $J^\pi = 6^+$ for the state at 4.0 MeV was uncertain; the present result now supports this assignment.

We emphasize again that all results are obtained with one parameter set. In particular there was no need to readjust the bound-state ra-

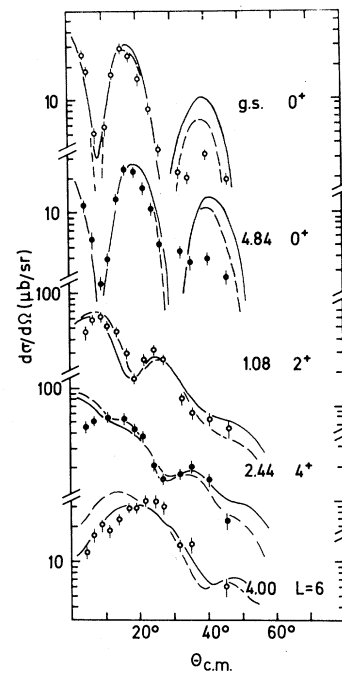


FIG. 3. Angular distributions for the reaction $^{40}\text{Ca}(^6\text{Li}, \alpha)^{44}\text{Ti}$ at 32 MeV. The curves represent α -stripping calculations in zero-range (dashed curves) and finite-range DWBA (solid curves).

dus from nucleus to nucleus. This is contrary to a recent report³ where—with a different set of optical parameters—such adjustments were necessary. It was suggested there that variations of subshell occupancy, not taken account of in the cluster wave function (same N and L), were responsible. The present results, however, indicate that such microscopic structure differences are of minor importance as far as the shapes of the angular distributions are concerned. This is supported by the fact that angular distributions for the *same* nucleus (i.e., for the same optical potential) do not differ significantly, in spite of the necessarily different subshell occupancies involved. Several such examples exist. Figure 3 displays the angular distributions for the $L=0$ transitions to the ground state and to the state at 4.84 MeV of ^{44}Ti . The former state is primarily of $(1f_{7/2})^4$ configuration whereas the latter contains considerable $(2p_{3/2})^4$ strength.¹ Nevertheless no differences of the kind observed for different target nuclei are seen in these angular distributions.

The S state of relative cluster motion in $^6\text{Li}_{g.s.}$ with its maximum amplitude at zero distance

TABLE II. Relative spectroscopic factors derived via zero-range (ZR) and exact finite-range (EFR) DWBA α -stripping calculations.

| Final state | ⁴⁴ Ti | ⁴⁶ Ti | ⁴⁸ Ti | ⁵² Ti | ⁵⁴ Fe | ⁶⁶ Zn | ⁴⁴ Ti | | |
|-------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------------|----------------------|----------------------|
| | Ground states | | | | | | 1.08, 2 ⁺ | 2.44, 4 ⁺ | 4.00, 6 ⁺ |
| S(ZR) | 1 | 0.42 | 0.30 | 0.15 | 0.10 | 0.30 | 0.33 | 0.19 | 0.14 |
| S(EFR) | 1 | 0.33 | 0.20 | 0.15 | 0.11 | 0.31 | 0.26 | 0.15 | 0.10 |

usually is assumed to justify the zero-range approximation: $V_{\alpha d}^D \varphi_{\alpha d} = D_0 \delta(r_{\alpha d})$. Of considerable practical interest then is the question how the results of computer-time-saving zero-range (ZR) DWBA calculations compare with the results of exact finite-range (EFR) DWBA. ZR calculations have been performed with the code DWUCK¹⁰ using the relevant parameters of Table I. As seen in the figures, ZR and EFR angular distributions turn out to be quite similar. Moreover the relative magnitudes are practically the same, as shown in Table II for some of the cases studied. The reason for the similarity of ZR and EFR results is not evident. To what extent it is related to the specific choice of the optical-potential parameters remains to be studied in detail. Here we stress this result because of its economic practical use.

A very striking feature of the results presented in Table II, we finally want to point at, is the drastic decrease of the spectroscopic factors with increasing neutron number in the Ti isotopes. A similar decrease is observed for low-lying 2⁺ states as will be discussed in more detail in a forthcoming paper. The exploration of these effects and their attribution to nuclear structure first of all demanded an adequate description of kinematic effects. This is felt to be achieved by the present calculations which, to an unexpected degree, predict the characteristic angular distributions for a wide range of masses, Q values, and L transfers without parameter adjustments to the individual cases.

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