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Determination of the Effective Mass of the Localized Electron in Dense Helium Gas from Space-Charge–Limited Currents*

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The effective mass of the localized electron in dense helium gas at 4.2°K has been determined from measurements of space-charge–limited currents. We derive a value of about 3 helium masses from our results.

From the early experiments of Levine and Sanders in 1962,[1] it was clear that the precipitous drop in electron mobility at 4.2°K with increasing density in helium was due to electron localization. One of the differences between dense helium and other disordered insulators is that the fluidity of the helium allows the localized states to be mobile. One can estimate the effective mass of the localized state from the low-mobility behavior provided one also knows the radius. However, there has not been any other experimental means of determining the effective mass in the gas. In this paper we show that the random velocity and therefore the effective mass can be obtained from a study of space-charge–limited currents. From the experimental results, we obtain a value of about 3 helium-atom masses at high gas density.

The basic idea of the experiment is very simple. If one has an electrode which can supply a finite current, then the reservoir of charge produced in an insulator will also be finite. The magnitude of the reservoir depends upon the injection current and the random velocity since the reservoir density is not altered at most voltages of experimental interest.[2] The effect of diffusion is to maintain this reservoir. At low applied voltages one obtains the square law, $j \propto V^2/L^3$, derived by Thomson and Thomson[3] but more commonly known as the Mott and Gurney[4] space-charge–limited–current law. At high voltages, while the density is not altered from its low-voltage value, the reservoir cannot supply sufficient charge during a transit time to modify the electric field significantly. In this case the current-voltage relationship is Ohmic, $j \propto V/L$. Adirovich[2] showed that the transition from the low-voltage, square-law regime to the high-voltage, Ohmic regime occurs when the voltage V is greater than a critical voltage V_c given by

$$V_c = 10(kT/e)(L/x_0)^2, \tag{1}$$

where $x_0 = (\epsilon kT/2\pi N_0 e^2)^{1/2}$ is the Debye length[5] with N_0 being the electron density in the reservoir. Equation (1) was also derived by Silver[6] from considerations of transient current. Equation (1) may be rewritten as

$$\tfrac{1}{5} C V_c = e N_0 L, \tag{2}$$

where C is the capacitance. Equation (2) directly relates the critical voltage V_c to N_0.

The electron density, N_0, is related to the net injected current j_0 by[7]

$$N_0 = \sqrt{6\pi}\, j_0/e\langle |v_0| \rangle, \tag{3}$$

where v_0 is the velocity of a species whose total energy is E and whose center-of-mass kinetic energy is $3kT/2$. The consequence of Eqs. (2) and (3) is that from a measurement of V_c and j_0 one can determine $\langle|v_0|\rangle$, the average of the magnitude of the velocity. Further since kT is known, then $\langle M^*\rangle$ defined dynamically by

$$\langle M^*\rangle = (3kT)/\langle|v_0|\rangle^2 \tag{4}$$

can also be determined.

The value of $\langle|v_0|\rangle$ one obtains may be a result of the random motion of both light and heavy localized electrons. Just as the mobility $\langle\mu\rangle$ can be written[8]

$$\begin{aligned}\langle\mu\rangle &= [\textstyle\int \mu(E)\eta(E)\,dE]/N_0\\ &= [\textstyle\int_f \mu_f(E)\eta_f(E)\,dE]/N_0\\ &\quad + [\textstyle\int_l \mu_l(E)\eta_l(E)\,dE]/N_0,\end{aligned} \tag{5}$$

where the subscripts f and l refer to free and localized electrons, respectively, so $\langle|v_0|\rangle$ can be written

$$\begin{aligned}\langle|v_0|\rangle &= [\textstyle\int |v_0(E)|\eta(E)\,dE]/N_0\\ &= [\textstyle\int_f |v_f(E)|\eta_f(E)\,dE]/N_0\\ &\quad + [\textstyle\int_l |v_l(E)|\eta_l(E)\,dE]/N_0.\end{aligned} \tag{6}$$

For simplicity Eq. (6) can be rewritten as

$$\langle|v_0|\rangle = (N_f/\langle M_f\rangle^{1/2} + N_l/\langle M_l\rangle^{1/2}) \times (3kT)^{1/2}/N_0, \tag{7}$$

where N_f and N_l are the concentrations of free and localized electrons, respectively, and $\langle M_f\rangle$ and $\langle M_l\rangle$ are their respective average masses. Similarly,

$$\langle\mu\rangle = (\langle\mu_f\rangle N_f + \langle\mu_l\rangle N_l)/N_0. \tag{8}$$

If the scattering cross section is not changing radically over the experimental conditions of temperature and density, then one would expect a precipitous drop in $\langle|v_0|\rangle$ to follow in a similar way the drop in mobility.

The current density j_0 is somewhat difficult to determine experimentally since it is the net rate of production of thermalized electrons in the insulator. One can measure j_0 at low gas densities ($\rho \approx 10^{20}$ cm^{-3}) and extrapolate to the densities of interest ($\rho \approx 10^{21}$ cm^{-3}) or calculate its value from the emission current into vacuum.[9,10] Both estimates give the same value for j_0 and these values are in agreement with injection experiments at 77°K by Morris.[9]

To obtain a finite reservoir of charge we have employed a "tunnel cathode" as an injecting electrode. The current injection characteristics into dense gases of these contacts have previously been reported.[10,11] To obtain space-charge limitation at reasonable voltages, we have achieved a stable injection current into vacuum at 4.2°K of about 5×10^{-7} A/cm^2. The experimental arrangement is similar to that previously described[11] and is not repeated here.

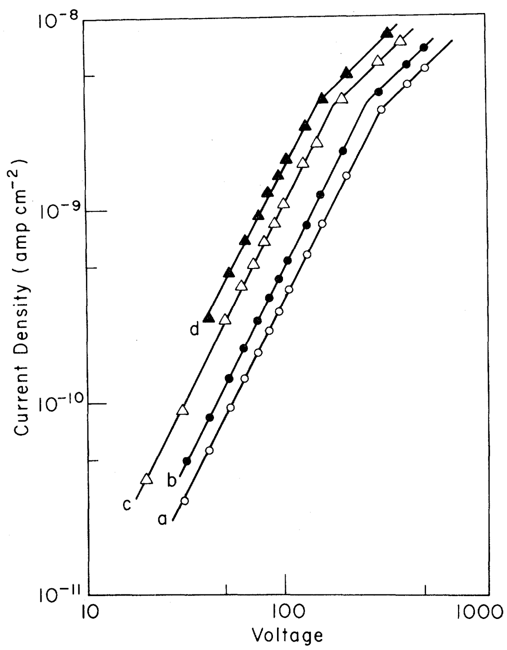

FIG. 1. Current density versus voltage for various helium gas densities. The gas densities are (curve a) 2.19×10^{21} cm^{-3}, (curve b) 1.86×10^{21} cm^{-3}, (curve c) 1.62×10^{21} cm^{-3}, and (curve d) 1.53×10^{21} cm^{-3}.

The current-voltage characteristics for several gas densities are shown in Fig. 1. The transition from V^2 to V dependence is quite sharp and the critical voltage V_c is observed to be a function of gas density. Since the average velocity is an inverse function of voltage, it is changing as well.

With this limited injection current, we could not observe space-charge-limited currents above $\langle\mu\rangle \simeq 1$. At higher $\langle\mu\rangle$ and lower density, the current is electrode limited. A more intense source or much larger electrode spacing would help.

Using the definition of $\langle M^*\rangle$ given by Eq. (4), we show in Fig. 2 the variation of $\langle M^*\rangle$ with density. At densities of 1.5×10^{21} cm^{-3} and above $\langle M^*\rangle$ tends to saturate to a value of about 3 helium-atom masses. Since at these higher densi-

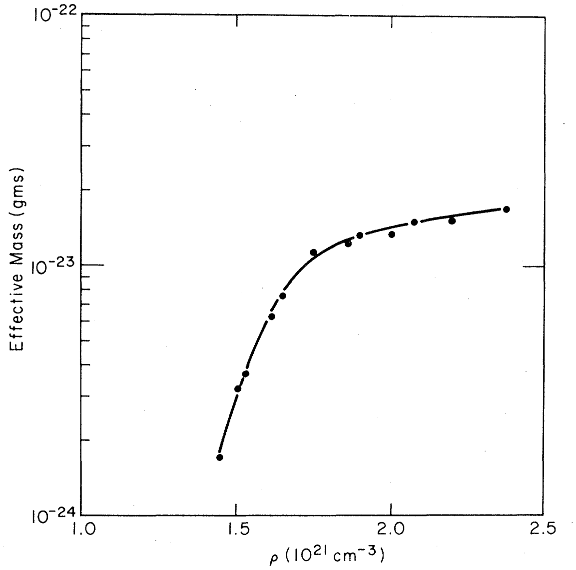

FIG. 2. Effective mass versus gas density. The values of $\langle M^* \rangle$ are obtained from $V_c^{\,2}$ as indicated in the text.

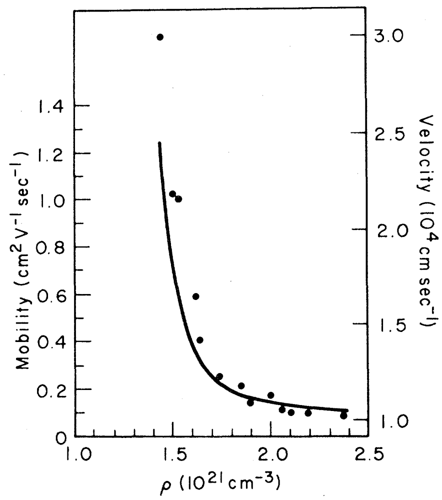

FIG. 3. Comparison of $\langle \mu \rangle$ versus ρ and $\langle |v_0| \rangle$ versus ρ. The solid curve is $\langle \mu \rangle$ from space-charge–limited–current curves. Solid circles are the measured values of $\langle |v_0| \rangle$.

ties, all of the average random velocity is due only to the localized electrons, the localized electrons have an average mass of $3M_{\mathrm{He}}$. Since this value does not change significantly with density above 1.5×10^{21} cm^{-3}, it is reasonable to assume that there is only one stable localized state.[12,13]

As indicated above, $\langle |v_0| \rangle$ and $\langle \mu \rangle$ are related to the relative populations of the extended and localized states. Consequently plots of $\langle \mu \rangle$ versus density and of $\langle |v_0| \rangle$ versus density will be similar providing that the scattering cross section is not a rapid function of density. This similarity is shown in Fig. 3. The curves for the mobility were obtained from the V^2/L^3 region of our space-charge–limited current and were in reasonable agreement with the results of Levine and Sanders.[14]

The small value of $3M_{\mathrm{He}}$ for the effective mass of the localized state is surprising. Calculations[3] suggested that about 130 helium atoms are displaced by the localized electron. In the hydrodynamic regime, one would then have expected a mass of about $65M_{\mathrm{He}}$. Our measured value of j_0, which is consistent with expectations from previous work,[9,10] would have to be smaller by a factor of 5 to bring the mass up to $65M_{\mathrm{He}}$. On the basis of the uncertainties in this experiment we can place limits on the value of the effective mass: $3M_{\mathrm{He}} \leq M^* < 5M_{\mathrm{He}}$. A more reasonable possibility is that under the conditions of our experiment the hydrodynamic regime is not reached. This suggests that the theoretical approach to the problem of electronic transport in dense helium fluid and other disordered insulators should carefully consider dynamic effects.

Since $\langle |v_0| \rangle$ is not a function of mean free path while $\langle \mu \rangle$ is, the space-charge–limited–current measurements are more sensitive to the relative population of the free and localized states than mobility measurements. We plan to use this technique to study localized states at higher temperatures as well as in other media.

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Positive-Ion Trapping on Vortex Lines in Rotating He II*

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The direct trapping of positive He ions on quantized vortex lines in rotating He II has been observed at temperatures below 0.7 K. Trapping lifetimes have been measured between 0.1 and 0.7 K, and agree with models of thermal escape and charge loss from vortex motion. An ion radius of 7.7±0.1 Å is found.

The trapping of ions on quantized vortices in superfluid helium has proven to be a very useful probe of the vortex properties and of the detailed structure of the ions.[1] The negative ion has been shown to be an electron which forms a bubble of radius[2] ~17 Å. The positive ion is thought to be a "snowball" structure in which electrostrictive pressures cause the He atoms to solidify around the positive charge to a radius[2] of 6–8 Å. The electron bubble is observed to become trapped on quantized vortex lines via a Bernoulli-pressure potential well.[3-5] This paper describes observations of *positive-ion* trapping on quantized vortex lines.[6,7]

The motivation for this work originates in the fact that ion trapping on vortex lines provides a very sensitive test of models describing both the size of the ion and the nature of the vortex core. It has been shown that trapped ions can leave the vortex via thermally activated processes.[3-5] This model predicts a trapping half-life $\tau_{1/2}$ which depends on temperature according to the relation

$$\tau_{1/2}=\tau_0 e^{U/kT}, \tag{1}$$

where k is Boltzmann's constant, T is the temperature, and τ_0 is a parameter which is determined mainly by the ion radius and mass. The depth U of the Bernoulli potential well depends on the ion radius. For negative ions, experiments[8] have shown that the temperature dependence of Eq. (1) is satisfied above 1.5 K with a binding energy U of 50 K. Below this temperature a lack of normal fluid damping allows the vortices to move more freely (presumably because of apparatus vibrations). The trapped charge is lost when a line encounters a boundary.[9,10]

Previous experiments[9,11] designed to observe positive-ion trapping on rectilinear vortices were unsuccessful. However, positive ions have been observed to be trapped on vortex rings and in strong electric fields the escape rate has been measured[12] and seems to agree with the thermal-activation model.[4] It was not expected that positive ions would be trapped at temperatures above 1 K because of the ion's small size.[3] Since the observed trapped lifetime is determined by the intrinsic ion-vortex interaction as well as line motion, it is desirable to observe both positive- and negative-ion trapping to help distinguish between the different charge loss processes.

Our measurements were made with a rotating dilution refrigerator.[13] The charge was trapped on vortex lines in a cylindrical region of rectangular cross section. The rotation axis runs through the cylinder axis. The sides of the container are split to allow the application of an electric field transverse to the vortex lines. No grids were placed within the cylinder. The walls were coated with resistance paint which, when voltage biased, allowed the ions to be moved along the lines towards a collector at the top of the cell.

The ions are produced near a 0.1-Ci tritium source located outside of the cell. Bias poten-